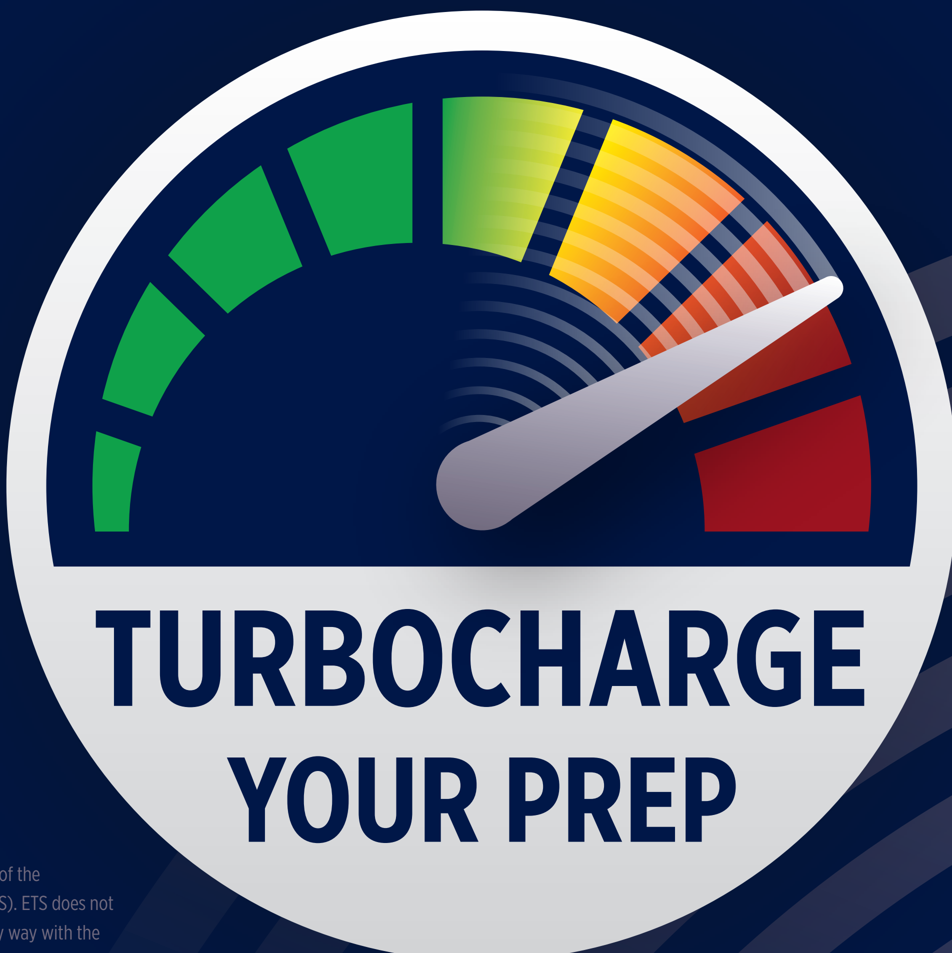
Manhattan Review®

3rd Edition

GRE® Quantitative Question Bank

Joern Meissner



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The Turbocharge Your GRE Series consists of 13 guides that cover everything you need to know for a great score on the GRE. Widely respected among GRE educators worldwide, Manhattan Review's GRE prep books offer the most professional GRE instruction available anywhere. Now in its updated 3rd edition, the full series is carefully designed to provide GRE test-takers with exhaustive GRE preparation for optimal test scores. Manhattan Review's GRE prep books teach you how to prepare for each of the different GRE testing areas with a thorough instructional methodology that is rigorous yet accessible and enjoyable. You'll learn everything necessary about each test section in order to receive your best possible GRE scores. The full series covers GRE verbal, quantitative, and writing concepts from the most basic through the most advanced levels, and is therefore a great study resource for all stages of GRE preparation. Students who work through all books in the series significantly improve their knowledge of GRE subject matter and learn the most strategic approaches to taking and vanquishing the GRE.

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About the Company

Manhattan Review's origin can be traced directly back to an Ivy League MBA classroom in 1999. While teaching advanced quantitative subjects to MBAs at Columbia Business School in New York City, Professor Dr. Joern Meissner developed a reputation for explaining complicated concepts in an understandable way. Prof. Meissner's students challenged him to assist their friends, who were frustrated with conventional test preparation options. In response, Prof. Meissner created original lectures that focused on presenting standardized test content in a simplified and intelligible manner, a method vastly different from the voluminous memorization and so-called tricks commonly offered by others. The new methodology immediately proved highly popular with students, inspiring the birth of Manhattan Review.

Since its founding, Manhattan Review has grown into a multi-national educational services firm, focusing on preparation for the major undergraduate and graduate admissions tests, college admissions consulting, and application advisory services, with thousands of highly satisfied students all over the world. Our GRE material is continuously expanded and updated by the Manhattan Review team, an enthusiastic group of master GRE professionals and senior academics. Our team ensures that Manhattan Review offers the most time-efficient and cost-effective preparation available for the GRE. Please visit www.ManhattanReview.com for further details.

About the Founder

Professor Dr. Joern Meissner has more than 25 years of teaching experience at the graduate and undergraduate levels. He is the founder of Manhattan Review, a worldwide leader in test prep services, and he created the original lectures for its first test preparation classes. Prof. Meissner is a graduate of Columbia Business School in New York City, where he received a PhD in Management Science. He has since served on the faculties of prestigious business schools in the United Kingdom and Germany. He is a recognized authority in the areas of supply chain management, logistics, and pricing strategy. Prof. Meissner thoroughly enjoys his research, but he believes that grasping an idea is only half of the fun. Conveying knowledge to others is even more fulfilling. This philosophy was crucial to the establishment of Manhattan Review, and remains its most cherished principle.

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Chapter 1

Welcome

Dear Students,

Here at Manhattan Review, we constantly strive to provide you the best educational content for standardized test preparation, as we make a tremendous effort to keep making things better and better. This is especially important with respect to an examination such as the GRE. A typical GRE aspirant is often confused with so many test-prep options available. Your challenge is to choose a book or a tutor that prepares you for attaining your goal. We cannot say that we are one of the best; it is you who has to be the judge.

There are umpteen books on Quantitative Reasoning for GRE preparation. What is so different about this book? The answer lies in its approach to deal with the questions. Solution of each question is dealt with in detail. There are over hundred questions that have been solved through Alternate Approaches. You will also find a couple of questions that have been solved through as many as four approaches. The objective is to understand questions from multiple aspects. Few seemingly scary questions have been solved through Logical Deduction or through Intuitive approach.

The book boasts of 500 questions on every topic that has been tested on the GRE. It has 125 questions on each type: Multiple Choice, Select One or Many, Numeric entry, and Quantitative Comparison.

Apart from books on 'Word Problem', 'Algebra', 'Arithmetic', 'Geometry', 'Permutation and Combination', and 'Sets and Statistics' which are solely dedicated on GRE-Quantitative Reasoning question types, the book on 'Fundamentals of GRE math' is solely dedicated to develop your math fundamentals.

The Manhattan Review's 'Quantitative Reasoning Question Bank' book is holistic and comprehensive in all respects. Should you have any queries, please feel free to write to me at info@manhattanreview.com.

Happy Learning!

Professor Dr. Joern Meissner
& The Manhattan Review Team

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Chapter 2

Question Bank

2.1 Multiple Choice Questions

1. $(99,998)^2 - (2)^2 =$

(A) $10^{10} - 4$

(B) $(10^5 - 4)^2$

(C) $10^4(10^5 - 4)$

(D) $10^5(10^4 - 4)$

(E) $10^5(10^5 - 4)$

2. $\sqrt{3\sqrt{80} + \dfrac{3}{9 + 4\sqrt{5}}} =$

(A) 3

(B) $3\sqrt{2}$

(C) $2\sqrt{3}$

(D) $3\sqrt{3}$

(E) $4\sqrt{3}$

3. $\left(\dfrac{1}{4 - \sqrt{15}}\right)^2 =$

(A) $16 + 16\sqrt{15}$

(B) $31 - 8\sqrt{15}$

(C) $31 + 8\sqrt{15}$

(D) $32 - 4\sqrt{15}$

(E) $32 + 4\sqrt{15}$

4. $5^3 + 5^3 + 5^3 + 5^3 + 5^3 =$

(A) 5^4

(B) 5^6

(C) 6^5

(D) 5^8

(E) 5^9

5. $\dfrac{\left(\frac{1}{3}\right)^{-2}}{3^{-2}} =$

(A) 9

(B) 81

(C) $3\sqrt{3}$

(D) $\sqrt{3}$

(E) 3

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6. A sports apparel company makes basketball shirts in 15 sizes. For each of the 14 sizes, the ratio of the length of a shirt to that of its next larger size shirt is a fixed constant. The sizes are ordered by increasing the length of the shirts. If the length of the smallest size shirt is 20 inches, and the length of the largest size shirt is 40 inches, then what is the length of the 8th size shirt?

(A) $20\sqrt{2}$
(B) $20\sqrt[14]{2^5}$
(C) $20\sqrt[4]{2^7}$
(D) $20\sqrt{2^8}$
(E) $20\sqrt{2^{14}}$

7. If $\sqrt{2k+3} = k+2$, which of the following is/are true?

I. $k^k = k$
II. $|k| = -k$
III. $k^0 = -k$

(A) Only I
(B) Only I and II
(C) Only I and III
(D) Only II and III
(E) I, II and III

8. If k is a two-digit positive integer with tens digit x and units digit y, then $k^2 - (x+y)^2$ must be divisible by which of the following?

(A) 2
(B) $6y$
(C) $9x$
(D) $9xy$
(E) 27

9. If k and $\left(0.0025 \times 0.025 \times 0.00025 \times 2^k \times 5^l\right)$ are integers, what is the least possible value of $(k+l)$?

(A) 6
(B) 12
(C) 16
(D) 18
(E) 20

10. If $k < 1$, which of the following must be negative?

(A) $k^2 - 6k + 8$
(B) $k^2 - 2k + 1$
(C) $|k| - k^2$

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(D) $2^k - 3$

(E) k^4

11. A machine can be repaired for \$1,200 and will last for one year, while the new machine would cost for \$2,800 and will last for two years. The average cost per year of the new machine is what percent greater than the cost of repairing the current machine ?

(A) 7%

(B) 10%

(C) 16.67%

(D) 18.83%

(E) 20%

12. An item is levied a sales tax of 10 percent on the part of the price that is greater than \$200. If a customer paid a sales tax of \$10 on the item, what was the price of the item?

(A) \$200

(B) \$250

(C) \$300

(D) \$360

(E) \$400

13. A particular sales tax rate is \$0.82 per \$50. What is the rate, as a percent, which is thrice as much as the rate mentioned?

(A) 492%

(B) 49.2%

(C) 4.92%

(D) 1.23%

(E) 0.055%

14. Cyclist P increases his speed from 10 miles per hour to 25 miles per hour in the last lap, while another Cyclist Q increases his speed from 8 miles per hour to 24 miles per hour in the last lap. By what percent is the percent increase in speed of Cyclist Q more than the percent increase in speed of Cyclist P?

(A) 33.33%

(B) 50%

(C) 66.67%

(D) 75%

(E) 100%

15. The population of Country X is 120,108,000 and its land area is 2,998,000 square kilometers. The population of Country Y is 200,323,000 and its land area is 7,899,000 square kilometers. The population density is defined as the population per square kilometer of land area. The population density of Country X is approximately what percent greater or lesser than that of Country Y?

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(A) 60%
(B) 50%
(C) 45%
(D) 37%
(E) 15%

16. A shopkeeper could sell only $\left(\frac{4}{5}\right)^{\text{th}}$ of the stock at the rate of \$3 per item. If 100 itms were unsold, what was the total amount he received from the sale?

(A) \$240
(B) \$1,200
(C) \$1,250
(D) \$1,300
(E) \$1,500

17. A shopkeeper procured 1,600 boxes of a chocolate brand at a cost of \$10 per box. If he sold $\left(\frac{3}{4}\right)^{\text{th}}$ of the boxes for one and half times their procurement cost and sold the remaining boxes at a loss of 25 percent of their procurement cost, what was the shopkeeper's gross profit on the total sale?

(A) \$5,000
(B) \$5,500
(C) \$6,000
(D) \$6,500
(E) \$7,500

18. A merchant sells only two brands of cakes, brand X and brand Y. The selling price of a brand X cake is \$20, which is 40 percent of the selling price of a brand Y cake. If the merchant sells 900 pieces of cakes, and $\left(\frac{2}{3}\right)^{\text{rd}}$ of which are brand Y, what is merchant's total revenue from the sale of cakes?

(A) \$14,000
(B) \$18,000
(C) \$24,000
(D) \$36,000
(E) \$40,000

19. A trader bought a consignment at a purchase price of \$800 and sold it for 20% less than the marked price. If the trader made a profit equivalent to 30% of the purchase price, what was the marked price of the consignment?

(A) \$1,000
(B) \$1,200
(C) \$1,300
(D) \$1,350
(E) \$1,500

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20. A small textile company buys few machines to stitch garments, costing a total of $10,000. The per unit cost of each garment is $2.50 and is sold for $4.50. How many units of the garments must be sold to achieve break-even (A phenomenon when all the investment and production costs are recovered by the sales revenue)?

(A) 2,000

(B) 3,500

(C) 4,500

(D) 5,000

(E) 6,000

21. A shopkeeper sells a commodity with a profit of 30 percent on the cost of the commodity. If the selling price were increased by $100, it would yield a profit of 40 percent of the commodity's cost. What was the initial selling price of the commodity?

(A) $1,000

(B) $1,200

(C) $1,300

(D) $1,400

(E) $1,800

22. A pen manufacturer produces pens at a cost of $6.00 each for the first 200 pens and $4.50 for each additional pen. If 1,000 pens were produced by the manufacturer and sold for $9.00 each, what was the manufacturer's gross profit?

(A) $2,500

(B) $3,500

(C) $4,200

(D) $5,550

(E) $6,600

23. A manufacturer's gross profit on an item was 20 percent of the cost of the item. If the manufacturer increased the selling price of the item from $60 to $65, while kept the cost of the item same, then the manufacturer's profit on the item after the price increase was what percent of the cost of the item?

(A) 12%

(B) 15%

(C) 20%

(D) 24%

(E) 30%

24. A school has a student-to-teacher ratio of 25 to 2. The average (arithmetic mean) annual salary for teachers is $42,000. If the school pays a total of $3,780,000 in annual salaries to its teachers, how many students does the school have?

(A) 900

(B) 1,000

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(C) 1,125
(D) 1,230
(E) 1,500

25. The average (arithmetic mean) score of a class was 70. If the boys' average was 65 and that of girls' was 80, what could be the number of boys and girls, respectively, in the class?

(A) 8; 9
(B) 18; 27
(C) 9; 18
(D) 18; 9
(E) 5; 2

26. A cookery course is divided into two groups. In group X, the average score in the test was 76. In group Y, the average score in the test was 70. If the average score of all 30 trainees in the course was 74, how many trainees are in group X?

(A) 10
(B) 17
(C) 20
(D) 25
(E) 28

27. A store has 500 kgs of tea in stock, 30 percent of which is dust. If the store adds another 200 kgs of tea of which 40 percent is dust, approximately what percent, by weight, of the store's tea contains dust?

(A) 31%
(B) 33%
(C) 34%
(D) 35%
(E) 47%

28. A class has 4 sections P, Q, R and S, with their average weights of the students in them are 45lb, 50lb, 55lb and 65lb, respectively. What is the maximum possible number of students in section R if there are 40 students in all sections combined and the average weight of the all students across all the sections is 55lb? It is known that each section has at least one student.

(A) 18
(B) 20
(C) 25
(D) 35
(E) 37

29. The average of seven numbers is 20. The average of the first four numbers is 19 and that of the last four is 24. What is the value of the fourth number?

(A) 23

(B) 25

(C) 32

(D) 43

(E) 63

30. The total expenses of organizing a party has a fixed expense of $250 as the rent of the place where the party is to be organized and a variable expense depending on the number of guests attending the party. For 10 guests the total expense was estimated to be $650. What is the estimated total expense for 20 guests?

(A) $800

(B) $900

(C) $1,050

(D) $1,250

(E) $1,300

31. A beaker was filled with a mixture of 40 liters of water and a liquid chemical in the ratio of 3 : 5, respectively. If 2 percent of the initial quantity of water and 5 percent of the initial quantity of liquid chemical evaporated each day during a 10-day period, what percent of the original amount of mixture evaporated during this period?

(A) 22.22%

(B) 33.33%

(C) 38.75%

(D) 44.44%

(E) 58.33%

32. In Ghazal's doll collection, $\left(\frac{3}{5}\right)^{\text{th}}$ of the dolls are Barbie dolls, and $\left(\frac{4}{7}\right)^{\text{th}}$ of the Barbies were purchased before the age of 10. If 90 dolls in Ghazal's collection are Barbies that were purchased at the age of 10 or later, how many dolls in her collection are non-Barbie dolls?

(A) 70

(B) 90

(C) 140

(D) 154

(E) 192

33. The ratio of the ages of A and B is 7 : 11. Which of the following can be the ratio of their ages after 5 years?

(A) 1 : 3

(B) 9 : 20

(C) 4 : 15

(D) 3 : 5

(E) 2 : 3

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34. A company assembles two kinds of phones: feature and smartphone. Of the phones produced by the company last year, $\left(\frac{2}{5}\right)^{th}$ were feature phones and the rest were smartphones. If it takes $\left(\frac{8}{5}\right)^{th}$ as many hours to produce a smartphone as it does to produce a feature phone, then the number of hours it took to produce the smartphones last year was what fraction of the total number of hours it took to produce all the phones?

(A) $\frac{8}{31}$

(B) $\frac{11}{31}$

(C) $\frac{12}{17}$

(D) $\frac{13}{34}$

(E) $\frac{15}{34}$

35. At a garment shop, the ratio of the number of shirts to the number of trousers is 4 to 5, and the ratio of the number of jackets to the number of shirts is 3 to 8. If the ratio of the number of sweaters to the number of trousers is 6 to 5, what is the ratio of the number of jackets to the number of sweaters?

(A) 9 to 25

(B) 1 to 3

(C) 1 to 4

(D) 3 to 5

(E) 6 to 5

36. At a church prayer, $\left(\frac{3}{5}\right)^{th}$ of the members were males and $\left(\frac{3}{5}\right)^{th}$ of the male members attended the prayer. If $\left(\frac{7}{10}\right)^{th}$ of the female members attended the prayer, what fraction of the members who did not attend the prayer are males?

(A) $\frac{1}{4}$

(B) $\frac{3}{7}$

(C) $\frac{2}{3}$

(D) $\frac{9}{10}$

(E) $\frac{6}{19}$

37. Amy, Betty, and Chris paid a total of \$135 for a common party. If Amy paid $\left(\frac{3}{5}\right)^{th}$ of what Chris paid, Betty paid \$51 and Chris paid the rest, what fraction of the total amount did Chris pay?

(A) $\frac{3}{7}$

(B) $\frac{1}{5}$

(C) $\frac{2}{15}$

(D) $\frac{7}{18}$

(E) $\frac{11}{18}$

38. A trip of 900 miles would have taken 1 hour less if the average speed for the trip had been greater by 10 miles per hour. What was the average speed for the trip?

(A) 40 miles per hour

(B) 45 miles per hour

(C) 60 miles per hour

(D) 75 miles per hour

(E) 90 miles per hour

39. A truck traveled 336 miles per full tank of diesel on the national highway and 224 miles per full tank of diesel on the state highway. If the truck traveled 4 fewer miles per gallon on the state highway than on the national highway, how many miles per gallon did the truck travel on the state highway?

(A) 6

(B) 8

(C) 10

(D) 12

(E) 15

40. A bike traveling at a constant speed takes 5 minutes longer to travel 10 miles than it would take to travel 10 miles at 60 miles per hour. At what speed, in miles per hour, is the bike traveling?

(A) 36

(B) 40

(C) 42

(D) 48

(E) 50

41. A biker increased his average speed by 10 miles per hour in each successive 10-minute interval after the first interval. If in the first 10-minute interval, his average speed was 30 miles per hour, how many miles did he travel in the fourth 10-minute interval?

(A) 4

(B) 5

(C) 8

(D) 10

(E) 15

42. An aircraft flew 600 miles to a town at an average speed of 400 miles per hour and made the return trip following the same route at an average speed of 500 miles per hour. Which of the following is aircraft's approximate average speed, in miles per hour, for the trip?

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(A) 420
(B) 444
(C) 450
(D) 467
(E) 483

43. An assembly machine produces 2,000 units of product per hour. Working 15 hours each day, another machine, thrice as efficient, will produce how many units of the same product in 10 days?

(A) 90,000
(B) 100,000
(C) 200,000
(D) 600,000
(E) 900,000

44. A water pump began filling an empty swimming pool with water and ran at a constant rate till the swimming pool was full. At sometime, the pool was $\frac{1}{2}$ full, and $2\frac{1}{3}$ hours later, it was $\frac{5}{6}$ full. How many hours would it take the pump to fill the empty pool completely?

(A) 4
(B) $5\frac{1}{3}$
(C) 7
(D) $7\frac{1}{5}$
(E) $8\frac{1}{3}$

45. Two taps can fill a cistern in 20 minutes and 30 minutes, respectively. Initially, only the first tap was opened; after x minutes, the second tap was also opened. If it took a total of 15 minutes for the cistern to be filled, what is the value of x?

(A) 5.0
(B) 7.5
(C) 9.0
(D) 10.0
(E) 12.5

46. An empty swimming pool with a capacity of 5,760 gallons is filled by a pipe at the rate of 12 gallons per minute. There is an emptying pipe which, in 9 hours, can empty the pool, which is $\frac{3}{4}$ full. How many hours does it take to fill the pool, which is already half filled, if both the pipes are kept open?

(A) 6
(B) 12
(C) 24
(D) 36
(E) 72

47. A solution leaks out of a jar at the rate of p liters for every q hours. If the solution costs 10 dollars per liter, what is the cost, in dollars, of the amount of the solution that will leak out in r hours?

(A) $\frac{pr}{10q}$

(B) $\frac{10q}{pr}$

(C) $\frac{10p}{qr}$

(D) $\frac{10pr}{q}$

(E) $\frac{10qr}{p}$

48. There are two different scales, the X-scale and the Y-scale; they are related linearly. Measurements on the X-scale of 10 and 30 correspond to measurements on the Y-scale of 30 and 60, respectively. What measurements on the X-scale corresponds to a measurements of 90 on the Y-scale?

(A) 20

(B) 40

(C) 50

(D) 60

(E) 70

49. If Suzy had thrice the amount of money that she has, she would have exactly the money needed to purchase four pencils, each costing $1.35 per piece and two erasers, each costing $0.30 per piece. How much money does Suzy have?

(A) $1.50

(B) $2.00

(C) $2.25

(D) $2.50

(E) $2.75

50. The population of a country increases at the rate of 30,000 people every month. The population of the country in 2012 was 360 million. In which year would the population of the country be 378 million?

(A) 2060

(B) 2061

(C) 2062

(D) 2063

(E) 2064

51. An Ice-cream parlor buys milk-cream cartons, each containing $2\frac{1}{2}$ cups of milk-cream. If the restaurant uses $\frac{1}{2}$ cup of the milk-cream in each serving of its ice-cream, what is the least number of cartons needed to prepare 98 servings of the ice-cream?

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(A) 9
(B) 19
(C) 20
(D) 21
(E) 24

52. Few marbles are put into 8 pouches such that each pouch contains at least one marble. At the most 4 pouches can contain the same number of marbles, and no two of the remaining pouches can contain an equal number of marbles. What is the least possible number of marbles in the 8 pouches?

(A) 8
(B) 17
(C) 18
(D) 24
(E) 30

53. A sum of money invested under simple interest, amounts to $1,200 in three years and $1,500 in five years. What is the rate at which the sum of money was invested?

(A) 10%
(B) 15%
(C) 20%
(D) 25%
(E) 45%

54. The difference, after two years, between compound interest and simple interest on a sum of money invested at the same rate of interest, is $18. If the simple interest accumulated on the sum after two years is $180, what is the rate of interest at which the sum of money was invested?

(A) 36%
(B) 30%
(C) 25%
(D) 20%
(E) 10%

55. Andrew borrows two equal sums of money under simple interest at 5% and 4% rate of interest. He finds that if he repays the former sum on a date six months before the latter, he will have to pay the same amount of $1,100 in each case. What is the total sum that he had borrowed?

(A) $750
(B) $1,000
(C) $1,500
(D) $2,000
(E) $4,000

56. A sum of $100,000 was invested in two deposits at simple interest rates of 3 percent and 4 percent, respectively. If the total interest on the two deposits was $3,600 at the end of one year, what fractional part of the 100,000 was invested at 4 percent?

(A) $\frac{5}{8}$

(B) $\frac{1}{5}$

(C) $\frac{2}{3}$

(D) $\frac{3}{5}$

(E) $\frac{3}{7}$

57. In the beginning of the year, 35 percent of company X's 120 customers were retailers, and after the 24-month period, 25 percent of its 240 customers were retailers. What was the simple annual percent growth rate in the number of retailers?

(A) 14.28%

(B) 21.43%

(C) 24.0%

(D) 30.0%

(E) 37.25%

58. A basket contains five apples, of which one is spoiled and the rest are good. If Henry is to select two apples from the basket simultaneously and at random, what is the probability that the two apples selected will include the spoiled apple?

(A) $\frac{1}{20}$

(B) $\frac{1}{10}$

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

(E) $\frac{3}{5}$

59. At the start of an experiment, a population consisted of x organisms. At the end of each month, after the start of the experiment, the population size increased by twice of its size at the beginning of that month. If the total population at the end of five months is greater than 1,000, what is the minimum possible value of x?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

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60. If the function f is defined by $f(p) = p^2 + \dfrac{1}{p^2}$ for all non-zero numbers p, then $\left(f\left(-\frac{1}{\sqrt{p}}\right)\right)^2 =$

(A) $f(p) + 2$

(B) $\dfrac{2}{f(p^2)}$

(C) $\left(\frac{1}{f(\sqrt{p})}\right)^2$

(D) $1 - (f(\sqrt{p}))^2$

(E) $f(p) - 2$

61. If $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x}{x^2+1}$, for all $x > 0$, what is the minimum value of $f(g(x))$?

(A) 0

(B) $\dfrac{1}{2}$

(C) 1

(D) $\dfrac{3}{2}$

(E) 2

62. $C_n^m = \dfrac{m!}{(m-n)!.n!}$ for non-negative integers m and n, with $m \geq n$. If $C_3^5 = C_x^5$ and $x \neq 3$, what is the value of x?

(A) 0

(B) 1

(C) 2

(D) 4

(E) 5

63. A color "code" is defined as a sequence of three dots arranged in a row. Each dot is colored either "red" or "black." How many distinct codes can be formed?

(A) 4

(B) 5

(C) 6

(D) 8

(E) 10

64. A daily store stocks two sizes of mugs, each in four colors: black, green, yellow and red. The store packs the mugs in packages that contain either three mugs of the same size and the same color or three mugs of the same size and of three different colors. If the order in which the colors are arranged is not considered, how many different packings of the types described above are possible?

(A) 4

(B) 10

(C) 16

(D) 20

(E) 30

65. A pizza-seller offers six kinds of toppings and two kinds of breads for its pizzas. If each pizza contains an equal number of kinds of toppings and an equal number of kinds of breads, how many different pizzas could the pizza-seller offer?

(A) 9

(B) 10

(C) 16

(D) 18

(E) 27

66. A botanist designates each plant with a one-, two- or three-letter codes, where each letter is one among the 26 letters of the alphabets. If the letters may be repeated and if the same letters used in a different order convey a different code, how many different plants can the botanist uniquely designate with these codes?

(A) 2,951

(B) 9,125

(C) 16,600

(D) 17,576

(E) 18,278

67. A box contains exactly 22 balls, of which 11 are white and 11 are black. If two balls are to be picked from this box at random and without replacement, what is the probability that both balls will be black?

(A) $\frac{1}{6}$

(B) $\frac{5}{21}$

(C) $\frac{5}{11}$

(D) $\frac{11}{40}$

(E) $\frac{17}{40}$

68. A box contains 12 balls, 7 of them are red and 5 of them are green. If 3 balls are to be selected at random from the box, what is the probability that 2 balls will be red and 1 ball will be green?

(A) $\frac{7}{44}$

(B) $\frac{7}{22}$

(C) $\frac{51}{100}$

(D) $\frac{21}{44}$

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(E) $\frac{7}{9}$

69. A particular characteristic or a feature in a large population has a distribution that is symmetric about the mean m. If 68 percent of the distribution lies within one standard deviation d of the mean, what percent of the distribution is less than $(m + d)$?

(A) 18%
(B) 32%
(C) 48%
(D) 84%
(E) 94%

70. A badminton club has 21 members. What is the ratio of number of 6-member committees that can be formed from the members of the club to the number of 5-member committees that can be formed from the members of the club?

(A) 17 to 16
(B) 15 to 1
(C) 8 to 3
(D) 17 to 6
(E) 16 to 5

71. A courier company can assign its employees to its offices in such a way that one or more of the offices can be assigned no employee to all the employees. In how many ways can the company assign four employees to two different offices?

(A) 6
(B) 8
(C) 10
(D) 12
(E) 16

72. A transport company employs 5 male and 3 female officers. If a core group of 3 male and 2 female officers is to be created, how many different core groups are possible?

(A) 10
(B) 16
(C) 24
(D) 30
(E) 60

73. Company A expects earnings of $0.90 per share of stock, two-third of which will be distributed as dividends to shareholders and the rest will be used as contingency budget. If earnings are greater than $0.90 per share of stock, shareholders will receive an additional $0.05 per share for each additional $0.15 of per share earnings. If the earnings are $1.20 per share, what will be the dividend paid to a shareholder who owns 500 shares?

(A) $204

(B) \$240

(C) \$350

(D) \$360

(E) \$390

74. A marketing class of a college has a total strength of 30. The class formed three groups: G1, G2, and G3, which have 10, 10, and 6 students, respectively. If no student of G1 is in either of the other two groups, what is the greatest possible number of students who are in none of the groups?

(A) 6

(B) 7

(C) 8

(D) 10

(E) 14

75. In a consignment of toys, one-fourth of the toys are white and three-fourth of the toys are blue. Half the toys are for boys and half are for girls. If 100 out of a lot of 1,000 toys are white and for boys, how many of the toys are blue and for girls?

(A) 150

(B) 250

(C) 300

(D) 350

(E) 400

76. In an isosceles triangle PQR, if the measure of angle P is 80°, which of the following could be the measure of angle R ?

I. 20°

II. 50°

III. 80°

(A) Only I

(B) Only III

(C) Only I and II

(D) Only II and III

(E) I, II and III

77. According to a survey, 7 percent of teenagers have not used a computer to play games, 11 percent have not used a computer to write reports, and 95 percent have used a computer for at least one of the above purposes. What percent of the teenagers in the survey have used a computer both to play games and to write reports?

(A) 13%

(B) 56%

(C) 77%

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(D) 87%

(E) 91%

78. In a company survey, 1,000 employees were each asked whether they take tea or coffee. As per the survey, 60 percent of the employees take tea, 62 percent take coffee, and 35 percent take both tea and coffee. How many employees surveyed take neither tea nor coffee?

(A) 110

(B) 130

(C) 170

(D) 220

(E) 350

79. In a school, the number of students who play soccer is twice the number of students who play basketball. The number of students who play both the sports is twice the number of students who play only basketball. If 5,000 students play both the sports, how many students play only soccer?

(A) 5,500

(B) 8,500

(C) 10,000

(D) 12,000

(E) 18,500

80. $15, 20, 25, x$: (not in order)

Which of the following could be the median of the four integers listed above (not in order)?

I. 17.5

II. 21.5

III. 23.5

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) All of them

81. $46, 54, 58, 67, 77, 79, 97, 98, 99$

The list above shows the scores of 9 students obtained on a scheduled test. If the standard deviation of the 9 scores is 19.51, how many of the scores are greater than one standard deviation above the mean of the 9 scores?

(A) One

(B) Two

(C) Three

(D) Four

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(E) Five

82. A set consists of 11 different numbers. If x is a number in the list and is three times the average (arithmetic mean) of the other 10 numbers in the list, then x is what fraction of the sum of the 11 numbers in the list?

(A) $\frac{1}{10}$

(B) $\frac{3}{13}$

(C) $\frac{4}{11}$

(D) $\frac{2}{11}$

(E) $\frac{5}{11}$

83. If the average (arithmetic mean) of 3, 8 and w is greater than or equal to w and smaller than or equal to $3w$, how many integer values of w exist?

(A) Five

(B) Four

(C) Three

(D) Two

(E) One

84. If the average (arithmetic mean) of five distinct positive integers is 10, what is the least possible value of the greatest of the five integers?

(A) 11

(B) 12

(C) 24

(D) 40

(E) 46

85. If the average (arithmetic mean) of p, q and 15 is equal to the average of p, q, 15 and 35, what is the average of p and q?

(A) 40

(B) 45

(C) 50

(D) 65

(E) 75

86. A shopkeeper mentally reversed the digits of a customer's correct amount of change and thus gave the customer an incorrect amount of change. If his collection showed 54 cents less than it should have, which of the following could be the correct amount of change in cents?

(A) 24

(B) 35

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(C) 64

(D) 75

(E) 93

87. A stationery shop sold pencils for \$0.70 each and erasers for \$0.50 each. If a girl purchased both pencils and erasers from the shop for a total of \$6.30, what is the total number of pencils and erasers the girl purchased? The girl purchased at least one of both the items.

(A) 9

(B) 10

(C) 11

(D) 12

(E) 13

88. If $x + y + z = 2$, and $x + 2y + 3z = 6$ and $y \neq 0$, then what is the value of $\left(\frac{x}{y}\right)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

(C) $-\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

89. A furniture shop sells a table for \$150 per piece and a chair for \$85 per piece. Last fortnight it sold 5 more tables than chairs. If its total revenue from the sale of only tables and chairs last fortnight was \$1,925, what was the total number of tables and chairs that the shop sold last fortnight?

(A) 12

(B) 14

(C) 15

(D) 16

(E) 18

90. If $a \geq 0$ and $a = \sqrt{8ab - 16b^2}$, then in terms of b, $a =$

(A) $-4b$

(B) $\frac{b}{4}$

(C) b

(D) $4b$

(E) $4b^2$

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91. What is the difference between the maximum and the minimum value of $\left(\frac{x}{y}\right)$ for which $(x-2)^2 = 9$ and $(y-3)^2 = 25$?

(A) $-\frac{15}{8}$

(B) $\frac{3}{4}$

(C) $\frac{9}{8}$

(D) $\frac{19}{8}$

(E) $\frac{25}{8}$

92. If x and y are positive integers and $2x + 3y + xy = 12$, what is the value of $(x + y)$?

(A) 2

(B) 4

(C) 5

(D) 6

(E) 8

93. A ball, t seconds after it was thrown up, is at a height of h feet, where $h = -2(t-5)^2 + 100$. What is the height of the ball now once it reached its maximum height and then descended for 5 seconds?

(A) 36 feet

(B) 50 feet

(C) 64 feet

(D) 72 feet

(E) 96 feet

94. As per an estimate, the depth $D(t)$, in centimeters, of the water in a tank at t hours past 12:00 a.m. is given by $D(t) = -10(t-7)^2 + 100$, for $0 \le t \le 12$. At what time does the depth of the water in the tank becomes the maximum?

(A) 5:30 a.m.

(B) 7:00 a.m.

(C) 7:30 a.m.

(D) 8:00 a.m.

(E) 9:00 a.m.

95. Kabir's college is 12 miles from his hostel. He travels 6 miles from the college to basketball practice, and then 4 miles to a computer class. If he is then D miles from home, what is the range of possible values for D?

(A) $2 \le D \le 10$

(B) $2 \le D \le 16$

(C) $2 \le D \le 12$

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(D) $2 \leq D \leq 22$

(E) $4 \leq D \leq 20$

96. $2a + b = 12$, and $|b| \leq 12$

How many ordered pairs (a, b) are solutions of the above system such that a and b both are integers?

(A) 9

(B) 10

(C) 11

(D) 12

(E) 13

97. If the cost of 12 eggs varies between \$0.90 and \$1.20, exclusive and the cost of 5 sandwiches varies between \$10 and \$15, exclusive, the cost of 4 eggs and 3 sandwiches varies between

(A) \$2.15 and \$3.20

(B) \$2.30 and \$3.40

(C) \$6.40 and \$9.30

(D) \$6.30 and \$9.40

(E) \$9.30 and \$12.40

98. Given that x is a negative number and $0 < y < 1$, which of the following has the greatest value?

(A) x^2

(B) $(xy)^2$

(C) $\left(\frac{x}{y}\right)^2$

(D) $\frac{x^2}{y}$

(E) x^2y

99. Jack traveled from Town X to Town Y in 3 hours with speed between 30 and 45 miles per hour. Brian also traveled from Town X to Town Y along the same route in 2 hours with speed between 35 and 55 miles per hour. Which of the following could be the distance, in miles, from Town X to Town Y?

(A) 85

(B) 105

(C) 115

(D) 120

(E) 135

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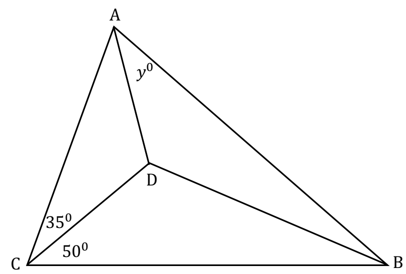
100. In the figure below, AD = BD = CD. What is the value of y? (Figure not to sclae.)



(A) 5°

(B) 10°

(C) 15°

(D) 20°

(E) 25°

101. John's home is at the same distance from his two friends Brian's home and Andy's home. The distance between Brian's home and Andy's home is 8 miles, which of the following could be the distance between the John's home and either of his friend's home?

I. 3 miles

II. 10 miles

III. 12 miles

(A) I only

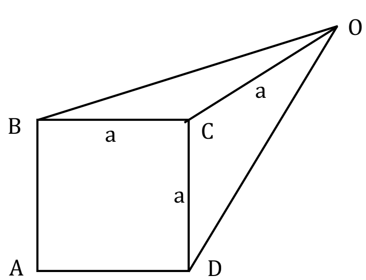
(B) II only

(C) III only

(D) I and II

(E) II and III

102. In the figure given below, ABCD is a square with side of length a unit. The length of line segment CO is also a unit, and the length of line segment BO is equal to the length of line segment DO. Note that all the points are in a plane. What is the area of the triangular region BCO?



(A) $\frac{a^2}{3}$

(B) $\frac{a^2}{2}$

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(C) $\frac{3a^2}{4}$

(D) $\frac{a^2\sqrt{2}}{4}$

(E) $\frac{a^2\sqrt{2}}{2}$

103. A right triangle has sides of length x, y and z, where $x < y < z$. If the area of this triangular region is 1, which of the following indicates all of the possible values of z?

(A) $z > 2$

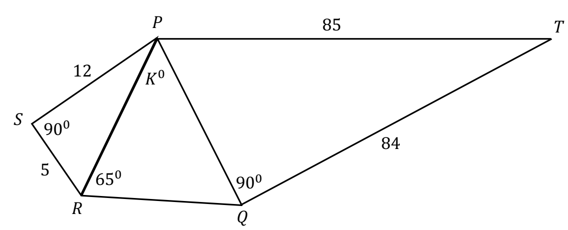
(B) $\sqrt{2} < z < 2$

(C) $\sqrt{2} < z < \sqrt{3}$

(D) $1 < z < \sqrt{2}$

(E) $z < 1$

104. In the figure shown below, what is the value of $K°$?



(A) 45

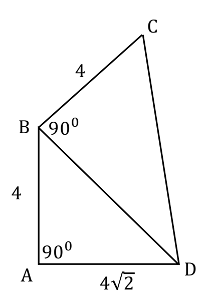
(B) 50

(C) 55

(D) 65

(E) 70

105. In the figure below, what is the perimeter of triangle BCD?



(A) $4 + 4\sqrt{3}$

(B) 12

(C) $12 + 4\sqrt{3}$

(D) $8 + 8\sqrt{3}$

(E) $16\sqrt{2}$

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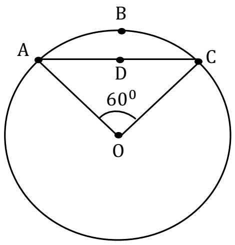
106. If the circle below has centre O and length of the arc ABC is 24π, what is the perimeter of the region ABCD?



(A) $12\,(\pi + 2)$

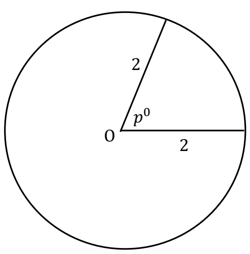
(B) $12\,(\pi + 3)$

(C) $24\,(\pi + 2)$

(D) $24\,(\pi + 3)$

(E) $24\,(\pi + 4)$

107. In the figure below, O is the center of the circle that has a radius of 2 units. If the area of the sector containing the angle p° is $\frac{\pi}{2}$, what is the value of p in degrees?



(A) 15°

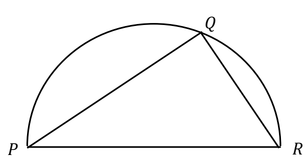
(B) 30°

(C) 45°

(D) 60°

(E) 75°

108. In the figure shown below, the triangle PQR is inscribed in a semicircle. If the length of line segment PQ is 5 and the length of line segment QR is 12, what is the length of arc PQR?



(A) 5π

(B) 12π

(C) $\frac{5\pi}{4}$

(D) $\frac{5\pi}{2}$

(E) $\frac{13\pi}{2}$

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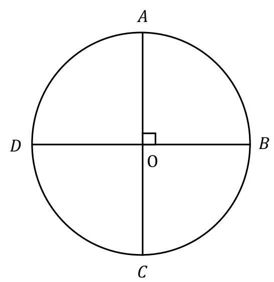
109. There are two co-centric circles of unequal diameters. The area between the two circles is shaded. If the area of the shaded region is 3 times the area of the smaller circle, what is the ratio of the radius of the larger circle to the radius of the smaller circle?

- **(A)** 3 : 1
- **(B)** 5 : 2
- **(C)** 2 : 1
- **(D)** $\sqrt{3} : 1$
- **(E)** $\sqrt{2} : 1$

110. In the figure shown below, O is the center of the circle and angle AOB is 90 degrees. If the distance between A and D is $\frac{10}{\sqrt{2}}$, what is the area of the circle?



- **(A)** 4π
- **(B)** 5π
- **(C)** 25π
- **(D)** 50π
- **(E)** 100π

111. A circular-shaped cloth with radius 10 inches is rested on a square tabletop that has its sides equal to 24 inches. Which of the following is closest to the fraction of the tabletop NOT covered by the cloth?

- **(A)** $\frac{1}{2}$
- **(B)** $\frac{3}{5}$
- **(C)** $\frac{2}{3}$
- **(D)** $\frac{1}{4}$
- **(E)** $\frac{9}{20}$

112. A rectangular floor having perimeter of 16 meters is to be covered with square carpets that measure 1 meter by 1 meter each and cost $6 apiece. What is the maximum possible cost for the number of square carpets needed to cover the rectangular floor if the sides of the floor are integers?

- **(A)** $42
- **(B)** $72

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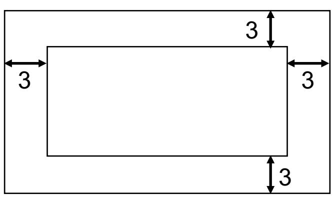
(C) \$90
(D) \$96
(E) \$120

113. A photograph rectangular in shape is surrounded by a border that is 2 centimeters wide on each side. The combined area of the photograph and the border is a square inches. Had the border been 4 centimeters wide on each side, the total area would have been $(a + 100)$ square centimeters. What is the perimeter, in centimeters, of the photograph?

(A) 18
(B) 20
(C) 24
(D) 26
(E) 52

114. A photograph, rectangular in shape, is surrounded by a border of 3 centimeters, as shown in the figure below. Without the border, the length of the photograph is twice its width. If the area of the border is 216 square centimeters, what is the width, in centimeters, of the photograph, excluding the border?



(A) 10
(B) 20
(C) 30
(D) 40
(E) 50

115. A right circular cylinder has a diameter of 10 inches. Water fills the cylinder to a height of 9 inches. The water from this cylinder is poured into a second right circular cylinder; the water fills the second cylinder to a height of 4 inches. What is the diameter of the second cylinder, in inches?

(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

116. In a certain duration, the distance covered by a smaller circular rim 24 inches in diameter and the distance covered by a larger circular rim 36 inches in diameter are equal. If the smaller rim makes r rotations per minute, how many rotations per minute does the larger rim make in terms of r?

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(A) $\frac{3r}{2}$

(B) $\frac{4r}{9}$

(C) $\frac{2r}{3}$

(D) $\frac{9r}{4}$

(E) $\frac{r}{3}$

117. A rectangular solid (cuboid) has three faces, having areas 12, 45, and 60. What is the volume of the solid?

(A) 180

(B) 200

(C) 240

(D) 600

(E) 900

118. In the coordinate plane, a diameter of a circle has the end points $(-3, -6)$ and $(5, 0)$. What is the area of the circle?

(A) 5π

(B) $10\sqrt{2}\pi$

(C) 25π

(D) 50π

(E) 100π

119. A straight line in the XY-plane has a slope of 3 and a Y-intercept of 4. On this line, what is the X-coordinate of the point whose Y-coordinate is 10?

(A) 2

(B) 4

(C) 6

(D) 7

(E) 7.5

120. In the XY-plane, a line l passes through the origin and has a slope 3. If points $(1,\ a)$ and $(b,\ 2)$ are on the line l, what is the value of $\frac{a}{b}$?

(A) 2

(B) 3

(C) $\frac{2}{3}$

(D) $\frac{2}{9}$

(E) $\frac{9}{2}$

121. In the XY-plane, the point $(3, 2)$ is the center of a circle. The point $(-1, 2)$ lies inside the circle and the point $(3, -4)$ lies outside the circle. Which of the following could be the value of r?

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

122. In the Cartesian XY-plane, the three vertices of a square are represented by points (a, b), $(a, -b)$ and $(-a, -b)$. If $a < 0$ and $b > 0$, which of the following points is in the same quadrant as the fourth vertex point of the square?

(A) $(-2, -6)$
(B) $(-2, 6)$
(C) $(2, -6)$
(D) $(6, -2)$
(E) $(6, 2)$

123. In the XY-plane, the vertices of a triangle have coordinates $(0, 0)$, $(5, 5)$ and $(10, 0)$. What is the perimeter of the triangle?

(A) 12
(B) 13
(C) $5 + 10\sqrt{2}$
(D) $10 + 5\sqrt{2}$
(E) $10 + 10\sqrt{2}$

124. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, what is the value of a in terms of b?

(A) $\frac{b-1}{b}$
(B) $\frac{b}{b+1}$
(C) $\frac{b}{b-1}$
(D) $\frac{b+1}{b}$
(E) $\frac{1}{b-1}$

125. In the XY-plane, what is the area of the triangle formed by the line $3y - 4x = 24$ and the X and Y axes?

(A) 6
(B) 14
(C) 24
(D) 36
(E) 48

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2.2 Select One or Many Questions

126. Which of the following options gives the correct values of x, for which the expression $\sqrt{\frac{2}{(x-1)(x+2)}}$ is defined?

Indicate all possible options.

(A) $-2 \le x \le 1$
(B) $-2 < x < 1$
(C) $x > 1$
(D) $x \ge 1$
(E) $x \le -2$
(F) $x < -2$
(G) $x > -2$

127. The ratio of the marbles with Jack and Dave was 13 : 9. Now, they each have 10 more marbles with them. Which of the following cannot be the ratio of their marbles now?

Indicate two such ratios.

(A) 7 : 4
(B) 7 : 5
(C) 15 : 11
(D) 31 : 23
(E) 23 : 8

128. In a scientific experiment, the population of a strain of bacteria was measured to be 64×10^4 at 12:00 noon. If the population of the colony doubles every 2 hours, the product of the populations at which of the following two times would be equal to the square of the bacteria population at 8:00 pm?

Indicate all the correct pairs of times.

(A) 4:00 pm and 6:00 pm
(B) 6:00 pm and 10:00 pm
(C) 4:00 pm and 12:00 midnight

129. Four workers worked on a project. The total time that the four workers worked on the project was in the ratio 2 : 3 : 5 : 6. If one of the four workers worked on the project for 60 hours, which of the following can be the total number of hours that the four workers worked on the project?

Indicate all possible hours.

(A) 160

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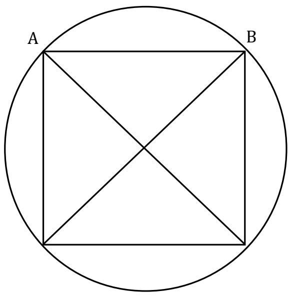
(B) 192
(C) 320
(D) 360
(E) 480

130.



In the figure above, a square is inscribed in a circle. If the length of arc AB, bounded by adjacent vertices of the square is between 4π and 8π long, exclusive, which of the following could be the diameter of the circle?

Indicate all possible hours.

(A) 16
(B) 20
(C) 25
(D) 32

131. The circumference of a circle is 4π. Which of the following values can be the area of a rectangle inscribed in the circle?

Indicate all possible values.

(A) 3
(B) 5
(C) 8
(D) 10
(E) 12

132. What could be the number of bits of computer memory that will be required to store a positive integer x, where $\sqrt{6,400} \leq x \leq \sqrt{810,000}$, if each digit requires 4 bits of memory?

Indicate all possible numbers.

(A) 6
(B) 8
(C) 10
(D) 12

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(E) 14

133. What could be the value of n, if the number of different codes that consist of 2 A's and n B's is less than 66?

Indicate all possible values.

(A) 2
(B) 3
(C) 5
(D) 9
(E) 10
(F) 12

134. What could be the number of 5-letter sequences that can be generated using 5 of the 6 letters that consist of 1 A, 2 Bs, and 3 Cs?

Indicate all possible numbers.

(A) 6
(B) 10
(C) 15
(D) 20
(E) 28
(F) 30
(G) 36

135. What could be the values of integers from 80 to 150, inclusive, that leave the remainder 1 when divided by 12 and by 8?

Indicate all possible integers.

(A) 97
(B) 109
(C) 121
(D) 129
(E) 145
(F) 149

136. How many liters of pure alcohol must be added to a 90-liter solution that is 20 percent alcohol in order to produce a solution that is at least 25 percent alcohol?

Indicate all possible values.

(A) 4.5
(B) 5.0
(C) 5.5
(D) 6.0
(E) 6.5

137. A bag contains 3 white and n red marbles, $n > 3$. Two marbles are drawn from the bag. If the probability that one marble is white and the other is red is greater than the probability that both the marbles are red, what could be value of n?

Indicate all possible values.

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

138. A number is represented as $2.151515\ldots$, where the decimal part has the digits 15 repeating indefinitely. By what positive integer should the above number be multiplied so that a positive integer is obtained?

Indicate all such integers.

(A) 11
(B) 33
(C) 99
(D) 450
(E) 990

139. For which of the integers n, where $10 < n < 20$, $(n-1)!$ is not divisible by n.

Indicate all such values.

(A) 11
(B) 12
(C) 13
(D) 15
(E) 16
(F) 17
(G) 19

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140. If the number of three-element subsets of (1, 2, 3, 4, 5, . . . n), where n is a positive integer, that do not contain both the elements 2 and 4 simultaneously is less than 35, what could be the value of n?

Indicate all possible values.

(A) 4
(B) 6
(C) 7

141. Jane lives x floors above the ground floor of a high-rise building. It takes her 30 seconds per floor to walk down and 2 seconds per floor to ride the elevator. If it takes Jane more amount of time to walk down to the ground floor as to wait for the elevator for 7 minutes and ride down, then what could be the value of x?

Indicate all possible values.

(A) 10
(B) 15
(C) 16
(D) 18
(E) 20

142. Andy's income comes from his wages and from tips. For a particular month, if Andy's wages was between 150 percent and 166.67 percent, inclusive, of the amount of tips, what percent of his income could have come from tips?

Indicate all possible percent values.

(A) 25%
(B) 33%
(C) 38%
(D) 39%
(E) 42%

143. A bowl costs at least \$60. If a trader marks it up 25% on its selling price, what could be the selling price of the bowl?

Indicate all possible values.

(A) \$65
(B) \$75
(C) \$80
(D) \$90
(E) \$125

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144. Suzy makes premium cakes and sells each cake at a price that consists of cost of making the cake and a mark-up amount, which is minimum 20 percent of the selling price. What could be the selling price of a cake that cost Suzy at least $80 to make?

Indicate all possible values.

(A) $100
(B) $85
(C) $75
(D) $60
(E) $50

145. Andy, Brian, and Suzy are three numismatics. The number of stamps that Andy collected was two-third the number of stamps that Brian collected and three-fourth the number of stamps that Suzy collected. Which could be the difference between the numbers of stamps collected by any two among the three friends as a percent of the total number of stamps collected?

Indicate all possible values.

(A) 3.50%
(B) 4.35%
(C) 6.25%
(D) 7.20%
(E) 8.70%
(F) 13.00%
(G) 15.25%

146. Kris and David work with n other workers. Two of the workers are to be chosen to work on a new project. What is the expression that gives the probability that both Kris and David will be chosen?

Indicate all possible expressions.

(A) $\frac{1}{n^2}$
(B) $\frac{2}{(n+1)(n+2)}$
(C) $\frac{1}{n(n+1)}$
(D) $\frac{2}{n(n+3)+2}$
(E) $\frac{1}{n^2+n}$

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147. In a polygon of n sides, the difference between the number of diagonals and the number of sides is equal to 3. What could be the value of n?

Indicate all possible values.

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7
(F) 8

148. A cake shop sold only two types of cakes: regular cake, each is sold for \$1 per cake, and premium cake, each is sold for \$1.25 per cake. If r percent of the shop's revenue from cake sales was from regular cake and if m percent of the cakes that the shop sold were regular cakes, which of the following expresses r in terms of m?

Indicate both the expressions.

(A) $\dfrac{4m}{5 - \dfrac{m}{100}}$
(B) $\dfrac{150m}{250 - m}$
(C) $\dfrac{300m}{500 - 2m}$
(D) $\dfrac{400m}{500 - m}$
(E) $\dfrac{500m}{625 - m}$
(F) $\dfrac{20m}{25 - \dfrac{m}{25}}$

149. Previous year, the selling price of TV increased by p percent and the earning per TV sold increased by q percent ($p > q$). By what percent did the ratio of selling price per TV to earning per TV increase, in terms of p and q?

Indicate all possible expressions.

(A) $\dfrac{p}{q}\%$
(B) $\dfrac{p - q}{\left(1 + \dfrac{q}{100}\right)}\%$
(C) $\dfrac{100\,(p - q)}{100 + p}\%$
(D) $\dfrac{100\,(p - q)}{100 + q}\%$

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(E) $\dfrac{100\left(\dfrac{p}{q}-1\right)}{\left(\dfrac{100}{q}+1\right)}\%$

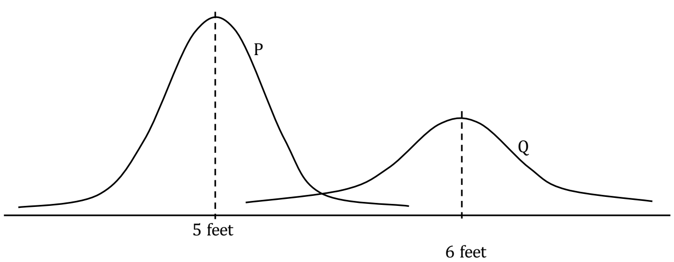
(F) $\dfrac{100\,(p-q)}{100+p+q}\%$

150. In company X, 36 employees were given pay increments and 48 employees were given premium training courses. If the number of employees that were given pay increments but not premium training courses was between 15 and 20, inclusive, what is the number of employees that were given premium training courses but did not pay increments.

Indicate all possible numbers.

(A) 15
(B) 21
(C) 27
(D) 30
(E) 32
(F) 35

151.



The diagram above shows normally distributed data for variables P and Q, where P represents the height of the students in the 7^{th} grade, while Q represents the height of the students in the 10^{th} grade of a public school. Which of the following must be correct?

Indicate all such statements.

(A) Q has a larger standard deviation than P.
(B) P has a larger mean than Q.
(C) Q has a larger median than P.
(D) The percent of students within one standard deviation of the mean in P is less than that in Q.
(E) No student of the 7^{th} grade is taller than any student of the 10^{th} grade.

152. A water sample has 10 percent of impurities present. If after each purification process, the impurity level reduces by 60 percent of the present value, what could be the number of times that the water sample needs to be purified in order to reduce the impurities present to at most

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1 percent?

Indicate all possible answers.

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

153. M = $\{-6, -5, -4, -3, -2\}$
T = $\{-2, -1, 0, 1, 2, 3, 4, \ldots n\}$, n is a positive integer

An integer is to be randomly selected from set M above and another integer is to be randomly selected from set T above; set T is contains consecutive integers. If the probability that the product of the two integers is negative is more than $\frac{3}{5}$, what could be the value of n?

Indicate all possible values.

(A) 4
(B) 5
(C) 7
(D) 8
(E) 9

154. Machine P produces parts twice as fast as machine Q does. Machine Q produces 100 parts of product K in n minutes. If each machine produces parts at a constant rate, how many parts of product R does machine P produce in t minutes, if each part of product R takes $\frac{3}{2}$ times of the time taken to produce each part of product K?

Indicate all possible values.

(A) $\dfrac{400t}{3n}$
(B) $\dfrac{300t}{4n}$
(C) $\dfrac{300t}{5n}$
(D) $\dfrac{t}{0.016n}$
(E) $\dfrac{t}{0.0075n}$

155. Set X consists of eight terms; 5 of them are equal and the remaining 3 of them are equal. If each of the 5 equal terms are equal to thrice the value of each of the remaining 3 terms, which of the following would provide sufficient additional information to determine the median of the set?

Indicate all such statements.

(A) The lowest possible average of any four terms of the set is 12.

(B) The range of all the terms in the set is twice the value of the least term.

(C) The difference between the highest and the lowest term is 16.

156. Mike rented a car for \$18 plus $\$x$ per mile driven. Tom rented a car for \$25 plus $\$y$ per mile driven. If each drove d miles and each was charged exactly the same amount for the rental then what was the charge, in dollars, each had to pay?

Indicate all possible answers.

(A) $25 + \frac{7x}{x-y}$

(B) $\frac{18x}{x-y}$

(C) $18 + \frac{7x}{x-y}$

(D) $\frac{32x - 25y}{x-y}$

(E) $\frac{25x - 18y}{x-y}$

157. Smith bought a set of six bottles of different sizes at a total price of \$75. The price of each bottle is $\$d$ more than the next one below it in size. If the price of each bottle is an integer, what could be the price, in dollars, of the smallest bottle?

Indicate all possible values.

(A) \$1

(B) \$2

(C) \$3

(D) \$4

(E) \$5

(F) \$8

(G) \$10

158. Katie passed a milestone on a highway while traveling back to her home at a constant speed of 50 miles per hour. Fifteen minutes later, Jack passed the same milestone traveling at a constant speed of 60 miles per hour while going in the same direction that Katie is going. If both Katie and Jack maintained their speeds and both remained on the highway, how long, in minutes, after Jack passed the milestone did the distance between Jack and Katie equal 10 miles?

Indicate all possible answers.

(A) 10

(B) 15

(C) 60

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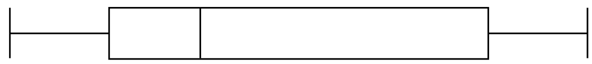
(D) 135
(E) 150

159.



Which of the following is correct about the data described by the box-and-whisker plot above?

Indicate all such statements.

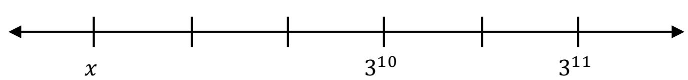
(A) The median of the whole set is closer to the median of the lower half of the data than it is to the median of the upper half of the data.
(B) The set has a standard deviation greater than zero.
(C) The mean of the whole set is greater than the median.

160. On a particular test whose scores are distributed normally, the 2nd percentile is 300, while the 84th percentile is 600. What scores lie within one standard deviation of the mean?

Indicate all possible scores.

(A) 250
(B) 390
(C) 420
(D) 540
(E) 580
(F) 640

161.



If the tick marks on the number line above are equally spaced, which of the following is correct about the value of x?

Indicate all correct options.

(A) $x = -3^{11}$
(B) $x = -2\left(3^{10}\right)$
(C) $x^3 < x$
(D) $x^5 > x^3$

162. If $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$ for all $n \geq 1$, which of the following is correct?

Indicate all correct options.

(A) $a_4 > a_3$

(B) $a_5 > a_4$

(C) $a_5 = \frac{8}{5}$

163. A man travels 720 miles in 8 hours, partly by air and partly by train. If he had travelled all the way by air, he would have saved $\frac{4}{5}$ of the time he was in train and arrived at his destination 4 hours early. Which of the following is true regarding the distance, in miles, travelled by the man by air?

Indicate all correct options.

(A) Distance travelled by air is 480 miles.

(B) Distance travelled by air is 540 miles.

(C) Distance travelled by air is thrice the distance travelled by train.

(D) Distance travelled by air is $\frac{1}{3}$ of the total distance travelled by the man.

164. If $xy \neq 0$ and $x^2y^2 - xy = 6$, which of the following could be y in terms of x?

Indicate all correct options.

A $\frac{6}{x}$

B $-\frac{2}{x}$

C $\frac{3}{x}$

165. A man covered $\frac{1}{3}$ of the distance to his destination at 20 miles per hour, $\frac{1}{2}$ of the remaining distance at 12 miles per hour and remaining distance at 40 miles per hour. Which of the following statements is true regarding the entire journey?

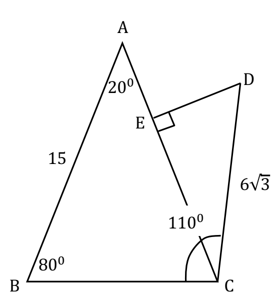
Indicate all possible statements.

(A) Time taken to travel the part of the distance at 40 miles per hour was $\frac{1}{2}$ of the time taken to cover the part of the distance at 20 miles per hour.

(B) Total time taken to complete the entire journey was 18 hours.

(C) Average speed for the entire journey is 20 miles per hour.

166.

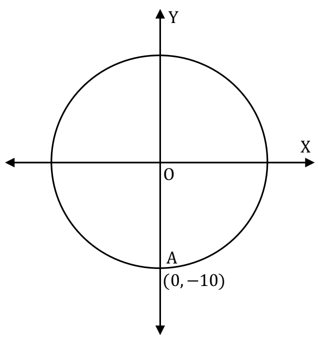


In the triangle above, which of following is correct about the length of AE?

Indicate all correct options.

(A) $3\sqrt{2} < \text{AE} < 4\sqrt{3}$
(B) $\text{AE} > CE$
(C) $\text{AE} = \text{DE}$
(D) $\text{AE}^2 + \text{CD}^2 = 12^2$

167.



The circle in the figure above is centered at the origin, O $(0, 0)$. What is the slope of the line tangent to the circle at a point P $(8, k)$?

Indicate all possible values.

(A) $\frac{5}{3}$
(B) $\frac{4}{3}$
(C) $\frac{3}{4}$
(D) $-\frac{3}{4}$
(E) $-\frac{4}{3}$

168. $$y = \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\left(2 + \cfrac{1}{\ldots\infty}\right)}}}$$

The expression of y above extends to infinity. Which of the following is correct regarding y?

Indicate all correct options.

(A) $y^2 = 2y - 1$
(B) $y^2 = 1 - 2y$
(C) $y = \sqrt{2} - 1$

(D) $y = \frac{1}{\sqrt{2}}$

169. Three friends, Andy, Bob, and Chad enter a quiz competition. The probability of Bob winning the competition is double that of Andy's and the probability of Chad winning the competition is thrice that of Andy's. If the probability of Andy winning the competition is $\frac{1}{a}$, which of the following could be the probability of winning the competition by any of the three friends?

Indicate all correct options.

(A) $\frac{(a-2)(a-3)}{a^3}$

(B) $\frac{2(a-1)(a-3)}{a^3}$

(C) $\frac{3(a-1)(a-2)}{a^3}$

170. If $f(x) = \frac{1}{x}$ and $g(x) = \frac{x}{x^2+1}$, for all $x > 0$, which of the following statements are correct?

Indicate all correct statements.

(A) $f(g(2)) = \frac{5}{2}$

(B) $g(f(x)) = x$

(C) $g(f(x)) = g(x)$

171. If $P(r) = \frac{8r}{1-r}$, for what value of r does $P(r^2) = \frac{1}{2}P\left(\frac{1}{2}\right)$?

Indicate two possible values.

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{3}$

(C) 0

(D) $-\frac{1}{3}$

(E) $-\frac{1}{\sqrt{3}}$

172. If $f(x) = ax^2 + b$ and $3f(x) + 2f(-x) = 5x^2 - 10$ for all x, for what value of x is $f(x) = 7$?

Indicate all possible values.

(A) −9

(B) −3

(C) 0

(D) 3

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(E) 4

173. If $\sqrt{x} = 25$, $N = x^3 - x^2$ is divisible by

Indicate two such values.

(A) 5^{12}
(B) 5^{10}
(C) $12(5^8)$
(D) $31\left(5^7\right)$
(E) $39(5^7)$

174. If $f(x) = x^3 - kx^2 + 2x$, and $f(-x) = -f(x)$, for what value of x is $f(x) = 3$?

Indicate all possible values.

(A) -3
(B) 1
(C) $k - 1$
(D) $1 + k$

175. The function f is defined as $f(x) = ax^2 + bx + c$ and $f(2) = 8$. If a, b and c are positive integers, what could be the value of the sum of any two among a, b and c?

Indicate all possible values.

(A) 2
(B) 3
(C) 4
(D) 5

176. If $f(x - 1) = 2x^2 - 3x + 3$, for what value of x is $f(x) = 2f(-x) - 1$?

Indicate two such values.

(A) $\frac{1}{2}$
(B) 1
(C) 2
(D) 4

177. If $K = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}$, the value of K lies between

Indicate three such ranges.

(A) 0 and $\frac{7}{9}$

(B) $\frac{1}{10}$ and $\frac{12}{17}$

(C) $\frac{1}{4}$ and $\frac{17}{18}$

(D) $\frac{3}{10}$ and $\frac{23}{21}$

(E) $\frac{11}{12}$ and $\frac{23}{21}$

178. Set X consists of four terms; of which three are equal. Each of the three equal terms is five times the fourth term. To find put the value of the median of the set, which of the following information would suffice?

Indicate all possible information.

(A) Arithmetic mean (Average) of the set

(B) Range of the set

(C) Value of any one of the terms

179. $h(x) = 2^{px+1}$ and $g(x) = 2^{px} + 1$. If $g(k) = 2h(k) - 2$, which of the following statements is correct?

Indicate all correct statements.

(A) $2^p = 1$

(B) $k = 0$, if $p \neq 0$

(C) $kp = 0$

180. If $f(x) = ax^2 + bx$, and $f(x+1) = f(x) + x + 1$, which of the following statements is correct?

Indicate all correct statements.

(A) $a + b = 1$

(B) $a = \frac{1}{3}$

(C) $a = \frac{1}{2}$

(D) $b = \frac{2}{3}$

(E) $b = \frac{1}{2}$

181. If x is an integer, which of the following ranges of x satisfies $||x-1| - |x-5|| < 4$?

Indicate all correct statements.

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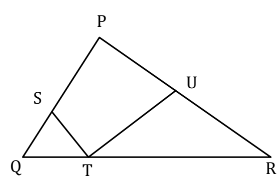
(A) $x > 1$
(B) $1 < x < 4$
(C) $0 < x < 4$
(D) $2 < x < 5$

182. If $n = \frac{18!}{15!}$, then which of the following numbers is a factor of n?

Indicate three such numbers.

(A) 51
(B) 136
(C) 153
(D) 216

183.



In the figure above, ∠PQR = ∠PRQ + 30°, ∠SPU + ∠STU = 220°, SQ = ST and TU = UR. Which angles have a measure of 50°?

Indicate two such angles.

(A) ∠SQT
(B) ∠PUT
(C) ∠UTR
(D) ∠STQ
(E) ∠PST

184. If $3^{6x} = 8100$ and $\frac{10}{3} \le \left(3^{(x-n)}\right)^3 \le 10$, what could be the value of n?

Indicate four such values.

(A) $\frac{2}{3}$
(B) $\frac{5}{6}$
(C) $\frac{9}{10}$
(D) 1
(E) $\frac{3}{2}$

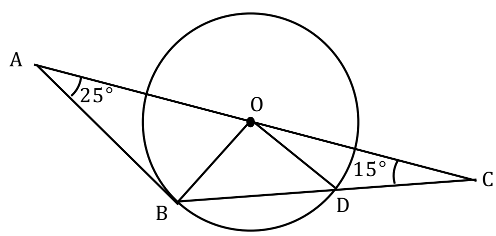
(F) $\frac{4}{3}$

185. If $P = (4^3)\left(25^2\right) = 10^x + n$, where x and n are positive integers, what could be the value of n?

Indicate all possible values.

(A) 39999
(B) 39990
(C) 39900
(D) 39800
(E) 39000
(F) 30000
(G) 38000

186.

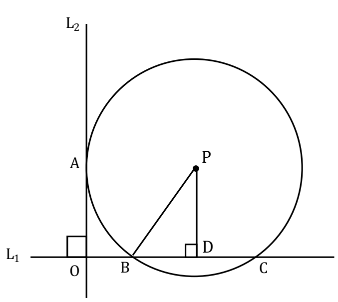


In the figure above, AB is a tangent to the circle with center O at the point B. AC is a straight line passing through the center of the circle. If $\angle BAO = 25^\circ$ and $\angle DCO = 15^\circ$, which of the following angles is less than 60°?

Indicate all possible angles.

(A) $\angle BOC$
(B) $\angle OBC$
(C) $\angle ODB$
(D) $\angle BOD$
(E) $\angle COD$

187.



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In the figure above, the straight lines L_1 and L_2 intersect each other at right angles. The circle, with center P, touches the line L_2 at A and intersects the line L_1 at B and C. If OA = 6 and BC = 16, which of the following statements is correct?

Indicate all statements.

(A) PB = 10
(B) OB = 2
(C) PD = 6
(D) OP = $2\sqrt{34}$

188. The lengths of the sides of a triangle are x, 5 and 15. If x is an integer such that $5 < x < 15$, what could be the value of x?

Indicate all possible values.

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11
(F) 12
(G) 13
(H) 14

189. If $-1 < h < 0$, which of the following statements is correct?

Indicate all possible statements.

(A) $h + h^2 > 0$
(B) $\frac{1}{h} < h$
(C) $\frac{1}{h^2} > \frac{1}{h}$

190. A solid cube is placed in a cylindrical container. Which of the following percent values could represent the ratio of the volume of the cylinder not occupied by the cube to the volume of the cylinder? (Assume the value of π to be 3.)

Indicate all possible values.

(A) 16%
(B) 25%
(C) 32%
(D) 36%

(E) 42%

191. If $0 < p < 1 < q < 2$, which of the following MUST be less than 1?

Indicate all possible options.

(A) $\frac{p}{q}$
(B) pq
(C) $q - p$

192. If $N = (9,999 \times 74) - \left(10^5 - 26\right)$, has its sum of digits d, which of the following are factors of d?

Indicate all possible factors.

(A) 2
(B) 3
(C) 4
(D) 11
(E) 17
(F) 22
(G) 34

193. In a polygon of n sides, the number of diagonals is a prime number. What is a possible value of n?

Indicate two possible values.

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

194. There are three distinct numbers such that the greatest common divisor of each pair of numbers is 7. If the product of the numbers is 8,232, what are the values of the three numbers?

Indicate all three numbers.

(A) 7
(B) 14
(C) 21
(D) 28

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(E) 35
(F) 42
(G) 56

195. If $4x < x < x^3 < x^2$, which of the following is a possible value of x?

Indicate three possible values.

(A) $-\sqrt{3}$
(B) $-\frac{3}{2}$
(C) -1
(D) $-\frac{2}{3}$
(E) $-\frac{1}{\sqrt{3}}$
(F) $-\frac{1}{2}$

196. The greatest common divisor of a pair of numbers is 15 and their least common multiple is 180. Which of the following is a possible pair of such numbers?

Indicate all possible pairs.

(A) 30, 45
(B) 45, 60
(C) 45, 75
(D) 60, 90
(E) 75, 120
(F) 15, 180
(G) 30, 180

197. The numbers a, b, c and d represent four consecutive positive integers, not necessarily in an ascending or a descending order. What is the value of the positive difference between any two of the possible values for $(ab + cd)$?

Indicate all possible values.

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5
(G) 6

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198. If p and q are positive integers such that $(p - q)$ and $\frac{p}{q}$ are both even integers, which of the following can be an odd integer?

Indicate all possible options.

(A) $\frac{p}{2}$

(B) $\frac{q}{2}$

(C) $\frac{p+q}{2}$

(D) $\frac{q+2}{2}$

(E) $\frac{p+2}{2}$

199. If an unbiased coin is tossed, the probability that the coin will land heads is $\frac{1}{2}$. If the coin is tossed 5 times, and the probability that it will land heads up on only 3 times is represented by $\frac{a}{b}$, where a and b are positive integers less than 50, what is a possible value of $(a + b)$?

Indicate all possible values.

(A) 5

(B) 16

(C) 21

(D) 32

(E) 42

(F) 48

(G) 63

(H) 84

200. If a code word is defined to be a sequence of different letters chosen from the six letters A, B, C, D, E and F, what could be the number of 4-letter codes that can be formed (a) consisting of only one of the vowels and (b) consisting of both the vowels?

Indicate both the numbers.

(A) 144

(B) 165

(C) 192

(D) 240

(E) 288

201. John makes a necklace with 150 beads in a repeating pattern of red, blue, blue, blue, yellow and blue, in that order. Which of the beads in the following positions are blue?

Indicate all possible positions.

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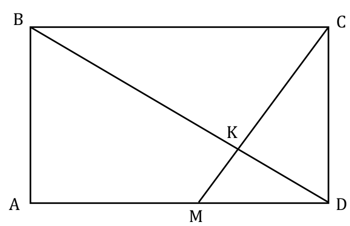
(A) 38^{th}
(B) 82^{nd}
(C) 102^{nd}
(D) 119^{th}
(E) 125^{th}

202. In a sequence of positive integers, every term after the first two terms is the sum of the previous two terms of the sequence. If the fifth term of the sequence is 18, what is a possible value of the second term?

Indicate both possible values.

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
(F) 6

203.



In the figure above, ABCD is a rectangle with AB = 6 and $\angle DBC = 30^\circ$. If M is a point on AD such that CM and BD are perpendicular to one another, which of the following are correct?

Indicate all possible values.

(A) $DM = 2\sqrt{3}$
(B) $DM = 3\sqrt{2}$
(C) $MK = \sqrt{3}$
(D) $MK = 2\sqrt{3}$

204. What is the value of the sum of digits of the 4-digit integer, having distinct digits, such that the product of the digits is 126?

Indicate all possible values.

(A) 15
(B) 17
(C) 18

(D) 19

(E) 21

205. If $f(x) = x^2 + 3$ and $g(x) = 3f(x)$, what is a possible value of $f(x-1)$ if $g(x) = 84$?

Indicate both possible values.

(A) 11

(B) 19

(C) 29

(D) 39

(E) 41

206. If a number between 0 and $\frac{2}{3}$ is selected at random, the probability of the number falling between which of the following is the greatest?

Indicate all possible options.

(A) 0 and $\frac{7}{20}$

(B) $\frac{1}{5}$ and $\frac{1}{4}$

(C) $\frac{3}{20}$ and $\frac{1}{5}$

(D) $\frac{1}{4}$ and $\frac{3}{10}$

(E) $\frac{3}{10}$ and $\frac{9}{20}$

(F) $\frac{3}{10}$ and $\frac{13}{20}$

207. If $x \neq 0$, what could be the expression of the quantity that can be added to $\left(\frac{x+1}{x}\right)$ or multiplied with $\left(\frac{x}{x-1}\right)$ to obtain the same result?

Indicate all possible expressions.

(A) $1 + \frac{1}{x-1}$

(B) $\frac{(x-1)(x+1)}{x}$

(C) $\frac{x}{x-1}$

(D) $-(x+1)$

(E) $x^2 - \frac{1}{x^2}$

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208. The sum of the squares of two numbers x and y is 20. If the sum of their reciprocals is 2, what is the product of the numbers?

Indicate two possible values.

(A) -2

(B) $-\frac{1}{2}$

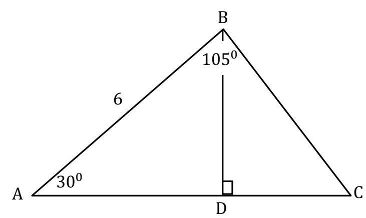
(C) 2

(D) $\frac{5}{2}$

(E) 3

(F) 4

209.



In the figure above, the length of AB is 6. If $\angle$BAC is 30°, $\angle$ABC is 105° and BD is perpendicular to AC, which of the options is correct?

Indicate three possible options.

(A) BD = 3

(B) BD = $3\sqrt{3}$

(C) CD = 3

(D) CD = $3\sqrt{2}$

(E) BC = $2\sqrt{3}$

(F) BC = $3\sqrt{2}$

210. If the number of ways in which $(n + 2)$ IDENTICAL gifts can be given to n children, with each child receiving at least one gift and each gift being given to exactly one child is 15, what could be the value of n?

Indicate all possible values.

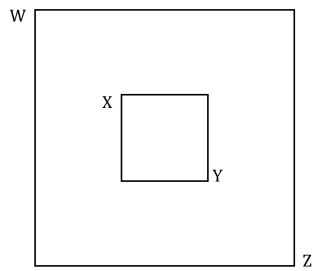
(A) 3

(B) 4

(C) 5

(D) 6

211.



The figure above shows two squares. The corners of the squares, points W, X, Y, and Z lie on a straight line. If the distance between W and X is equal to the distance between X and Y and also to the distance between Y and Z, then which of the following options are correct?

Indicate two correct options.

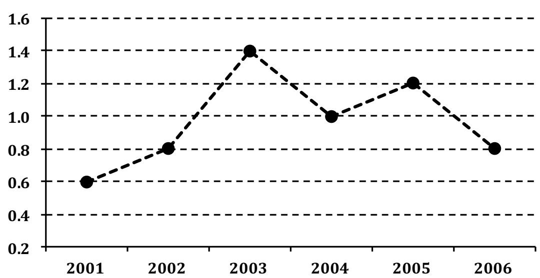
(A) Ratio of the area of the smaller square to that of the larger square is 1 : 9

(B) Ratio of the area of the smaller square to that of the larger square is 1 : 6

(C) Ratio of the perimeter of the smaller square to that of the larger square is 1 : 6

(D) Ratio of the perimeter of the smaller square to that of the larger square is 1 : 3

212.



Year-wise ratios of import to export of a particular company X is shown in the graph above with ratios presented on the Y-axis and years presented on the X-axis. If the exports were $60 million in 2001 and the exports increased by $2 million every year, what was the percent increase in imports (i) in 2005 over 2001 and (ii) in 2006 over 2001?

Indicate both the percent values.

(A) 45.5%

(B) 55.5%

(C) 60.0%

(D) 75.0%

(E) 83.3%

(F) 126.7%

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213. During a political poll, some people were asked about their preference to vote. M percent said that they would vote for candidate X. Of those who said they would vote for X, N percent actually voted for X, and of those who did not say they would vote for X, P percent actually voted for X. Which of the following expressions correctly denotes the percent of the people who voted for X?

Indicate <u>two</u> possible expressions.

(A) $\left(\frac{NM + P\,(100 - M)}{100}\right)\%$

(B) $(NM + P\,(100 - M))\,\%$

(C) $\frac{M\,(N - P)}{100}\%$

(D) $\left(\frac{M\,(N - P)}{100} + P\right)\%$

(E) $(M\,(N - P) + 100P)\,\%$

214.

	Bank X	
Attribute	**2011**	**2012**
Loan Sanctions	650	750
Total Revenue	1,600	1,800

The table above shows the performance of Bank X (all values are in million dollars) for the years 2011 and 2012. If the Loan Sanctions increases by at least 20% from 2012 to 2013 and the Total Revenue increases by at most 50% in the same time span, what could be a possible value of the ratio of Loan Sanctions and Total Revenue in 2013?

Indicate <u>all</u> possible values.

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

215. Of the 10 employees in a company 4 are women. If the number of possible groups of $(3 + m)$ employees consisting of 3 women and m men is 60, what could be the value of m?

Indicate <u>all</u> possible values.

(A) 1

(B) 2

(C) 3

(D) 4

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216. Of the 5,500 tons of coal mined daily at a coal-mine, a particular pure metal makes up for only 0.5 percent. If x percent of the pure metal is lost during mining, and in n days of mining, the total amount of pure metal mined at the coal-mine is equal to the daily amount of coal mined, what are the possible consistent values of x and n, respectively?

Indicate all possible values.

(A) 10; 200
(B) 20; 250
(C) 40; $333\frac{1}{3}$
(D) 50; 400

217.

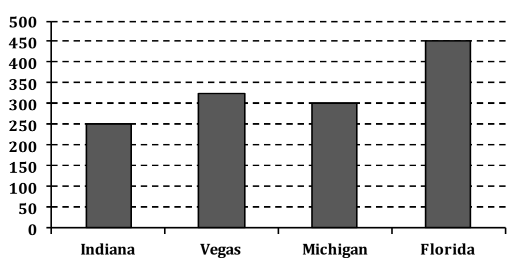
Price of balloons	
Packet size	**Price per packet**
1 balloon	$1
10 balloons	$9
100 balloons	$75
1,000 balloons	$600

Balloons are priced according to the chart above. If Charles wants to buy 2,000 balloons, what different sums of money can he spend if he can buy all the balloons only at a single rate at a time?

Indicate all different values.

(A) $1,000
(B) $1,200
(C) $1,500
(D) $1,600
(E) $1,800
(F) $2,000

218.



The bar-chart above shows the number of high speed internet customers (all values are in thousands) for a particular company in four states in 2011. If the number of customers in Indiana and Michigan together was 20 percent of the total number of customers of the company in 2011 (the company serves in more than four states), the number of customers in any single state represents what percent of the total customers of the company in 2011?

Indicate four possible percent values.

(A) 16.4%

(B) 15.3%

(C) 12.4%

(D) 11.8%

(E) 10.9%

(F) 9.8%

(G) 9.1%

Answer the following TWO questions based on the chart shown below:

	Finances of company X		**Finances of company Y**	
Attribute	**2010**	**2011**	**2010**	**2011**
Total Income	250	450	400	650
Net Profit	125	150	170	250
Cash Profit	80	100	128	215

The above table shows the finances of Company X and Y (all values are in million dollars) for the years 2010 and 2011. For either company, the percent increase in Net Profits from 2010 to 2011 was the same as that from 2011 to 2012, while the Total Income of each company doubled from 2011 to 2012.

219. Among the following options, fpr which of the following companies and in which year, the Cash Profit to Total Income is the highest?

Indicate all possible values.

(A) Company X in 2010

(B) Company X in 2011

(C) Company Y in 2010

(D) Company Y in 2011

220. Which of the following differences is greater than $650 million?

Indicate all possible values.

(A) Positive difference between Net Profits and Total Income in 2011 for Company X

(B) Positive difference between Net Profits and Total Income in 2012 for Company X

(C) Positive difference between Net Profits and Total Income in 2011 for Company Y

(D) Positive difference between Net Profits and Total Income in 2012 for Company Y

221. A bookshelf holds fewer than 50 books. If the books in the bookshelf are divided into 3 stacks each having an equal number of books, 2 books are left over. However, if the books in the bookshelf are divided into a number of stacks of 7 books each, again, 2 books are left over. What is the number of books in the bookshelf?

Indicate two correct options.

(A) 15
(B) 23
(C) 31
(D) 44
(E) 47

222. For a week Raymond is paid at the rate of x dollars per hour for the first t hours ($t > 4$) he works and 2 dollars per hour for the hours worked in excess of t hours. For working $(t - 3)$ hours in one week, Raymond would earn 14 dollars. If x and t are integers, what could be the values of t?

Indicate all possible values.

(A) 5
(B) 6
(C) 9
(D) 10
(E) 15
(F) 17

223. For each positive integer n, the integer $n^{\#}$ is defined by $n^{\#} = n^2 + 1$. If $17 \leq k^{\#} \leq 37$, what is the value of the positive integer k?

Indicate all possible values.

(A) -5
(B) -4
(C) 3
(D) 6
(E) 8

224. If $x > 0$ and two sides of a triangle have lengths $2x$ and $(5x + 1)$, which of the following COULD be the length of the third side of the triangle?

Indicate all possible lengths.

(A) $2x + 5$
(B) $3x$
(C) $7x + 3$

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(D) $8x$

225. Of the following values of n, the value of $\left(-\frac{1}{5}\right)^{|n|}$ will be greatest for $n =$

Indicate all possible values.

(A) -3
(B) -2
(C) 2
(D) 3

226. If $\left|x^2 - 10\right| < 6$, and x is an integer, the value of $x =$

Indicate all possible values.

(A) -4
(B) -3
(C) -2
(D) 2
(E) 3
(F) 4

227. If $u = x^2 + 2|x| + 1$, and x being an integer, which of the following statements are true?

Indicate all correct statements.

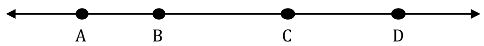
(A) The value of u is always positive
(B) The minimum value of $u = 1$
(C) The value of u is always a perfect square

228. AAB is a three-digit number. If $B = 2A$, which of the following prime numbers are common factors of all such three-digit numbers?

Indicate two possible values.

(A) 2
(B) 3
(C) 5
(D) 7
(E) 11

229.



On the line segment AD shown above, $AB = \frac{1}{2}CD$ and $BD = \frac{3}{2}AC$. If BC = 24, which of the options below has length 48?

Indicate two possible values.

(A) AB
(B) AC
(C) BD
(D) CD

230. On a test, 9 students each took a test having 100 questions. Of them, the average (arithmetic mean) number of the correct answers was 50, and the median number of correct answers was 40. Which of the following statements must be true?

Indicate all correct statements.

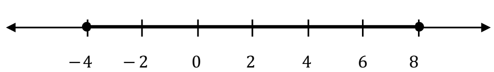
(A) At least one student had more than 60 correct answers.
(B) At least one student had more than 40 and less than 50 correct answers.
(C) At least one student had exactly 40 correct answers.
(D) At least one student had less than 40 correct answers.

231. On the morning of day 1, Suzy began her tracking tour. She plans to return home at the end of the first day on which it rains. If the probability of rain on each day is the same and the probability that Suzy will return home at the end of the day 2 is 0.24, what is the probability of rain on each day?

Indicate two possible values.

(A) 0.3
(B) 0.4
(C) 0.5
(D) 0.6
(E) 0.8

232.



Which of the following inequalities represent the highlighted region on the number line shown above?

Indicate all correct answers.

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(A) $|x| \le 2$

(B) $|x| \le 5$

(C) $|x| \le 9$

(D) $|x-2| \le 4$

(E) $|x-2| \le 6$

(F) $(x-2)^2 - 6 \le 30$

233. Ten grams of a food supplement contains 9 percent of the minimum daily requirement of protein and 11 percent of the minimum daily requirement of vitamin C. If protein and vitamin C are to be obtained from no other source, how many grams of the food supplement may be consumed daily to provide the minimum daily requirement of protein and vitamin C?

Indicate all possible values.

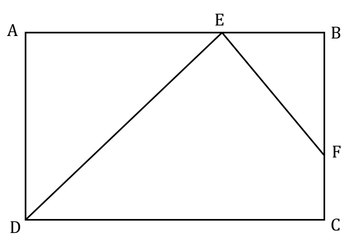
(A) 92

(B) 100

(C) 112

(D) 123

234.



In the rectangle above, E and F are two points on AB and BC, respectively. Area of triangle ADE is 20 and area of BEF is 8. If BF is greater than FC, what could be the area of the rectangle ABCD?

Indicate all possible values.

(A) 50

(B) 54

(C) 60

(D) 70

(E) 80

235. Pumping alone, one inlet pipe fills an empty tank to $\frac{1}{2}$ of capacity in 3 hours. A second inlet pipe fills the tank to capacity, when it is less than half full, in 6 hours. How many hours will it take both pipes, pumping simultaneously at their respective constant rates, to fill the empty tank to capacity?

Indicate all possible values.

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(A) 2.0 hours

(B) 2.5 hours

(C) 3.5 hours

(D) 4.5 hours

(E) 5.0 hours

236. John, a student of the ninth grade, scored 75 in an exam in which the mean score of all students was 70 and standard deviation of the scores was 2. Bob, a student of the tenth grade, scored 72 in an exam in which the mean score of all students was 65 and standard deviation of the scores was 3. Which of the following statements is correct?

Indicate all correct options.

(A) John's score in his exam was farther away from the mean score of the students in the ninth grade than Bob's score was from the mean score of the students in the tenth grade.

(B) There was at least one student who scored more than John.

(C) There was at least one student who scored less than Bob.

237. If the average (arithmetic mean) of five positive distinct integers is 16, and and greatest of them is 40, what is a possible value of the median of the five integers?

Indicate all correct options.

(A) 2

(B) 3

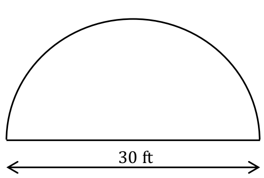
(C) 6

(D) 14

(E) 18

(F) 19

238. The figure given below shows a semicircular cross section of a one-way tunnel and is 30 feet wide. The one-way traffic lane is 20 feet wide and is equidistant from the sides of the tunnel. If vehicles must clear the top of the tunnel by at least 1 feet when they are inside the tunnel, what could be the height of vehicles that may pass the the tunnel?



Indicate all possible values.

(A) $5\sqrt{3} + 1$ ft

(B) $5\sqrt{3}$ ft

(C) $5\sqrt{3} - 1$ ft

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(D) $5\sqrt{3} - 2$ ft

(E) $5\sqrt{3} - 3$ ft

(F) $5\sqrt{3} - 4$ ft

239. The function f is defined by $f(x) = -\frac{1}{x}$ for all non-zero numbers x. If $f(p) = -\frac{1}{2}$ and $f(pq^2) = -\frac{1}{18}$, then $q =$

Indicate both the possible values.

(A) 3

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) -3

(E) -12

240. A function f is defined by $f(x) = \sqrt{x} - 10$ for all positive numbers x. If $p = f(q)$ for some positive numbers q and p, what is q in terms of p?

Indicate two possible expressions.

(A) $\sqrt{p + 10}$

(B) $(p + 10)^2$

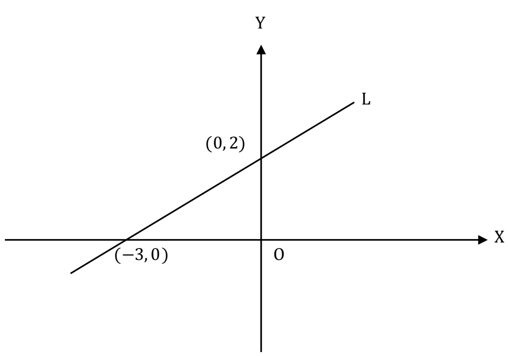
(C) $\sqrt{p^2 + 10}$

(D) $(p - 10)^2 + 40p$

(E) $p^2 + 10$

(F) $(p - \sqrt{10})(p + \sqrt{10}) + 20$

241.



The graph of which of the following equations is a straight line that is parallel to line L in the figure above and intersects the positive direction of X-axis?

Indicate all correct answers.

(A) $3y + 2x = 0$

(B) $3y - 2x = -3$

(C) $3y - 2x = 3$

(D) $3y + 2x = -3$

(E) $3y - 2x = -4$

(F) $3y - 2x = 0$

242. The numbers of defects in the first five cars to come through a new production line are 9, 7, 10, 4, and 6, respectively. If the sixth car through the production line has n defects, for which of these values of n does the mean number of defects per car for the six cars equal the median?

Indicate all correct answers.

(A) 3

(B) 7

(C) 12

243. Two three-digit positive integers p and q are such that $(p + q)$ is a four-digit positive integer. The tens digit of p is 7 and the tens digit of q is 5. If $p < q$, which of the following must be correct?

Indicate all correct answers.

(A) The unit digit of $(p + q)$ is greater than the unit digit of either p or q.

(B) The tens digit of $(p + q)$ is either 2 or 3.

(C) The hundreds digit of q is at '6' or more.

244. The operation '%' is defined for all non-zero numbers p and q by $p\%q = \frac{p}{q} - \frac{q}{p}$. If m and n are non-zero numbers, which of the following statements must be correct?

Indicate all correct answers.

(A) $m\%mn = m\,(1\%n)$

(B) $m\%n = -\,(n\%m)$

(C) $\frac{1}{m}\%\frac{1}{n} = n\%m$

245. The operation '~' is defined by $a \sim b = \frac{1}{a} + \frac{1}{b}$ for all non-zero numbers a and b. If $c > 1$, which of the following must be correct?

Indicate all correct answers.

(A) $c \sim (-c) = 0$

(B) $c \sim \left(\frac{c}{c-1}\right) = 1$

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(C) $\frac{2}{c} \sim \frac{2}{c} = c$

246. If $kx + 15 = 3k$ and x a positive integer ($x \neq 3$), what are the possible integer values of k?

Indicate all possible values.

(A) −30
(B) −15
(C) −5
(D) −3
(E) −1
(F) 3
(G) 5
(H) 15

247. Three numbers, a, b and c are obtained by rounding 22.567 to its tenth place, 225.78 to its unit place and 2.45 to its unit place, respectively. Which of the following are correct?

Indicate all possible values.

(A) $b = 10a$
(B) $10c > a$
(C) $b - 10a < c$

248. The sum of three integers is 40. The largest integer is 3 times the middle integer, and the smallest integer is 23 less than the largest integer. What are the three integers?

Indicate all three numbers.

(A) 2
(B) 4
(C) 6
(D) 9
(E) 18
(F) 24
(G) 27

249.

Sector	**Net Income in the second quarter, 2016 (billion $)**	**Percent change from first quarter, 2016**
Telecommunications	4.80	−20%
Power	7.20	+20%
Agriculture	5.00	−10%
Services	9.00	+200%
Information Technologies	2.50	−50%

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The table above represents the combined net income of all German companies in five sectors for the second quarter of 2016 and the percent change from the first quarter. Which sector had the greatest net income during the first quarter of 2016?

Indicate all possible sectors.

(A) Telecommunications
(B) Power
(C) Agriculture
(D) Services
(E) Information Technologies

250.

Year	Consumption (million kilograms)
2011	540
2012	648
2013	810
2014	972
2015	1,296

The table above gives the coffee consumption for a country X for five years. In which of the above years was the percent growth in consumption over the previous year the lowest?

Indicate all possible years.

(A) 2012
(B) 2013
(C) 2014
(D) 2015

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2.3 Numeric Entry Questions

251. In 2001, Joe paid 5.1 percent of his income in taxes. If from 2001 to 2002, Joe's income increased by 10 percent, and taxes paid in 2002 are equivalent to 3.4 percent of Joe's income in 2001, what percent of his income did Joe pay in taxes, in 2002?

[ ] %

252. In 2001, there were 350 students at a school. The number of students increased from 1981 to 1991 by 50 percent. If, from 1981 to 2001, the number of students increased by 250 percent, by what percent, in nearest integer value, did the number of students increase from 1991 to 2001?

[ ] %

253. Mr. John bought a total of n shares of stock X and Mrs. John bought m shares of stock X. The couple held all of their respective shares throughout 2005. If Mr. John's dividends on his n shares totaled \$150 and Mrs. John's dividends on her m shares totaled \$ $\left(\frac{4{,}500}{n}\right)$, how many shares did Mrs. John buy in 2005?

[ ]

254. In a school election, each of the 900 voters voted for either Edith or Jose or both. 60 percent of the female voters voted for Edith. If 20 percent of the female voters voted for both candidates, what percent of the female voters in this election voted only for Jose?

[ ] %

255. In a survey of 200 college graduates, 30 percent said that they had received student loans during their college careers, and 40 percent said that they had received scholarships. If 25 percent of those surveyed said that they had received scholarships but no loans, what percent of those surveyed said that they had received neither student loans nor scholarships during their college careers?

[ ] %

256. In June, salesperson X made 50 percent more sales transactions than salesperson Y did in May. In June, salesperson Y made 25 percent more sales transactions than he made in May. What was the ratio of the number of sales transactions made by salesperson X in June to the number of sales transactions made by salesperson Y in June?

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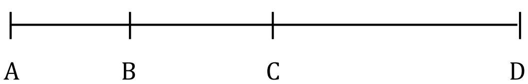
Give your answer as a fraction.

257.

$$\begin{array}{r} \blacksquare \\ + \ \triangle \\ \hline 14 \\ \hline \end{array}$$

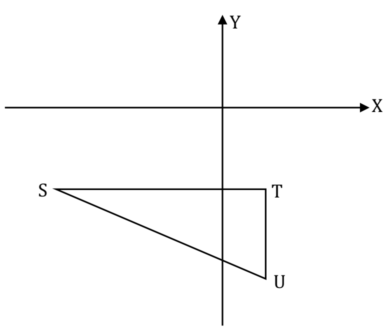
In the addition problem above, each of the symbols ■, △ and ∀ represents a positive digit. If ■ < △, and △ is an odd number, what is the value of △?

258.



If AC = 10, BD = 15 and $\frac{AB}{BC} = \frac{BC}{CD}$, what is the length of the line AD?

259.



In the figure above, ST and TU are parallel to the X-axis and Y-axis, respectively. If the X-coordinate of point U is 1, and the Y-coordinate of point S is −5, what is the sum of the coordinates of point T?

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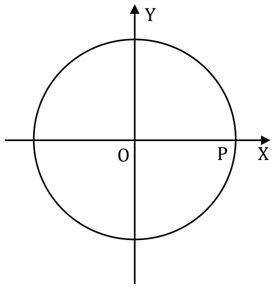
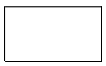
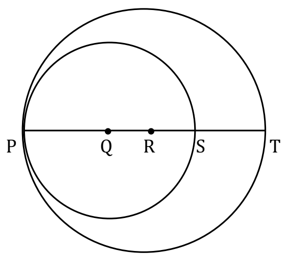
260.



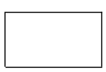
In the figure above, the circle has center O, and point P has coordinates $(50, 0)$. If point Q (not shown) is on the circle, having its X-coordinate -30, what is the length of line segment PQ?



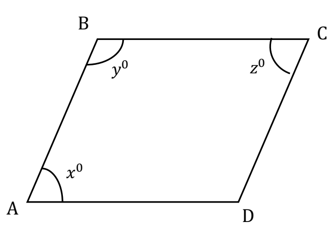
261.



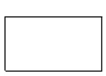
In the figure above, points P, Q, R, S, and T lie on a line. Q is the center of the smaller circle and R is the center of the larger circle. P is the point of contact of the two circles, S is a point on the smaller circle, and T is a point on the larger circle. If RS = 1 and ST = 4, what is the area of the region inside the larger circle but outside the smaller circle?



262.



In the parallelogram ABCD shown above, if $y = 3x$, what is the value of x?

 degrees

263. In the sequence of non-zero numbers $t_1,\ t_2,\ t_3, \ldots t_n, \ldots$, the value of $t_{(n+1)} = \frac{t_n}{2}$ for all positive integers n. If $t_1 - t_5 = \frac{15}{16}$, what is the value of t_5?

Give your answer as a fraction.

264. In the XY-plane, lines l and k intersect at the point $\left(\frac{16}{5}, -\frac{12}{5}\right)$. The product of the slopes of lines l and k is -1. If line k passes through the origin, what is the slope of line l?

Give your answer as a fraction.

265. In the XY-plane, lines a and b are parallel. If both the X- intercept and the Y-intercept of line a is -1, and line b passes through the point $(10,\ 20)$, what is the Y-intercept of line b?

266. In the XY-plane, the line with equation $4x + by + c = 0$, where $bc \neq 0$, has slope $\frac{2}{3}$. What is the value of b?

267. In the XY-plane, the sides of a rectangle are parallel to the X and Y axes. If two of the vertices of the rectangle are $(-1, -2)$ and $(2,\ 3)$, what is the perimeter of the rectangle?

268. In triangle ABC, point X is the midpoint of side AC and point Y is the midpoint of side BC. The point R is the midpoint of line segment XC and point S is the midpoint of line segment YC. If the area of the triangular region ABX is 32, what is the area of the triangular region CRS?

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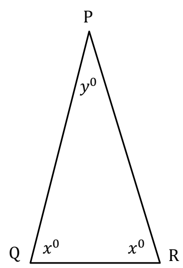
269. In triangle PQR, the measure of angle P is 30° greater than twice the measure of angle Q. If PQ = QR, what is the measure of angle R?

[ ] degrees

270.



If in triangle PQR above, $x + y = 100°$, what is the value of y?

[ ] degrees

271. If $2^{\sqrt{x}} = 8$, what is the value of 2^x?

[ ]

272. If $|x + 1| = 2\,|x - 1|$, what is the value of $\left|x - \frac{5}{3}\right|$?

Give your answer as a fraction.

[ ] / [ ]

273. Two containers contain milk and water solutions such that the volume of solution in one container is double the volume of solution in the other. What would be the minimum concentration of milk in either container so that when the entire contents of both containers are mixed, 30 liters of 80 percent milk solution is obtained?

[ ] %

274. 20 liters of milk is drawn out from a cask containing 100 liters of milk and then 10 liters of water is added to it. This process is repeated one more time. What is the fraction of milk finally present in the mixture in the cask?

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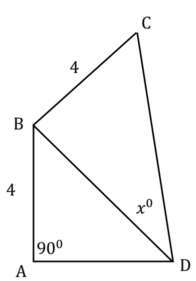
Give your answer as a fraction.

275. A person mixes three varieties of wine named P, Q and R, containing 80 percent, 70 percent and 50 percent alcohol concentration, respectively, so that the resultant mixture has 60 percent alcohol concentration. If the ratio of quantity of P and Q = 1 : 1, what is the ratio of quantity of Q and R used?

Give your answer as a fraction.

276. If $0 \le x \le 1$, what is the value of $|x| + |x - 1|$?

277.



In the figure above, if the lengths of AD and CD are $4\sqrt{2}$ and 8, respectively, what is the value of x?

degrees

278. Among three people P, Q and R, P is the youngest and R is the oldest. The difference between the ages of P and Q age is 6 years and R is 6 years elder to Q. If the average (arithmetic mean) age of P, Q, and R is 24 years, what is the age of P?

years

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279. A, B and C have a total of \$60 in cash with them. A and B together have 40 percent more cash than what C has. If B has \$6 more in cash than what A and C together have, what is the amount in cash present with A?

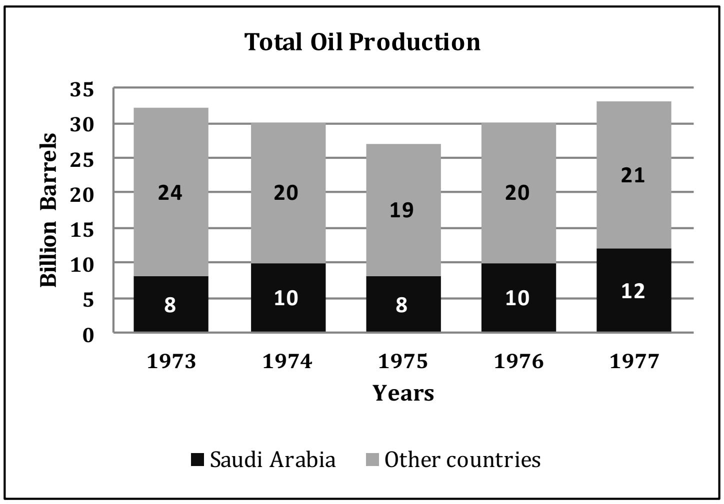
\$ [ ]

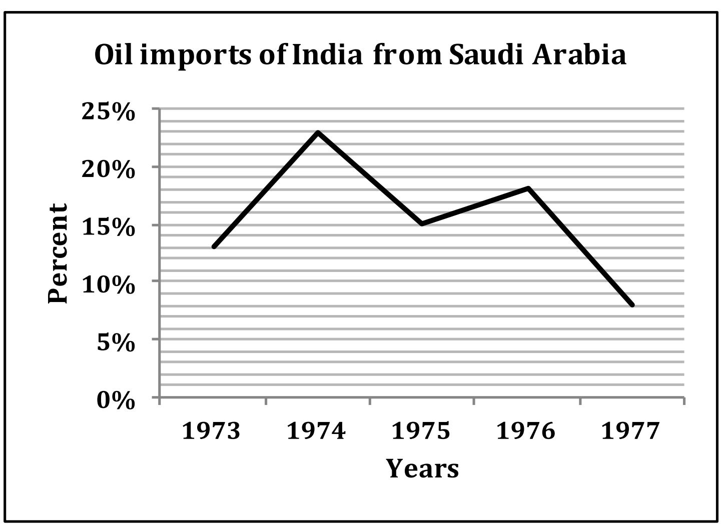
280. If $z = w - x + 3$, $y + z = x - 4$, and $y - w = z - 7$, what is the value of z?

[ ]

Following four questions are based on the following graphics.

The bar graph below shows the total oil production by Saudi Arabia and other countries for five years. The line graph shows the oil imports of India from Saudi Arabia as a percent of the oil production of Saudi Arabia in those years.





Answer the four questions that follow.

281. Estimated oil imports by India from Saudi Arabia in 1975 are what percent of oil production of other countries in that year?

[ ] %

282. If the oil productions by Saudi Arabia in the given five years is used to form a pie-chart, what will be the angle for 1974?

[ ] degrees

283. What is the share of oil production by Saudi Arabia as a percent of the total oil production by all countries (including Saudi Arabia) in for the given five years combined?

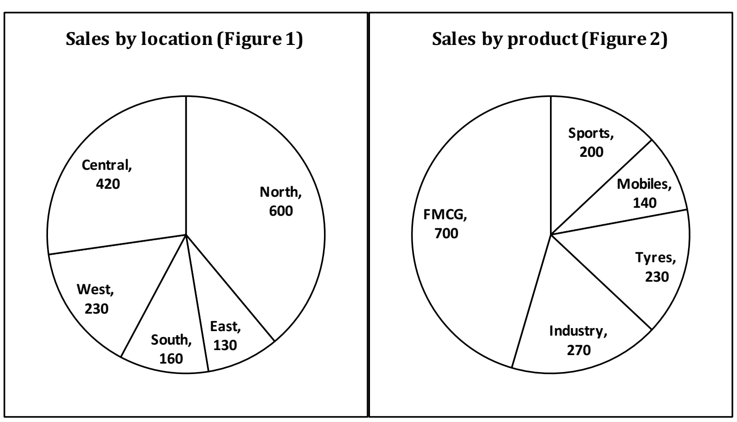
[ ] %

284. By what percent oil imports by India from Saudi Arabia decreased from 1975 to 1977?

[ ] %

Following three questions are based on the following graphics.

For the year 2010-2011, figure (1) shows sales by location of company XYZ Ltd. and figure (2) shows sales by product for XYZ Ltd. All figures are in millions of dollars.



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Answer the three questions that follow.

285. If 20 percent of the tyre sales were in the East, what was the value of the sales of the products, other than tyre, in the East in millions of dollars?

[ ] million dollars

286. If in the next year, the sales of sports goods were expected to double, then assuming that the sales of other products do not change, what would be the percentage share of sports goods in the total sales, rounded off to the nearest integer?

[ ] %

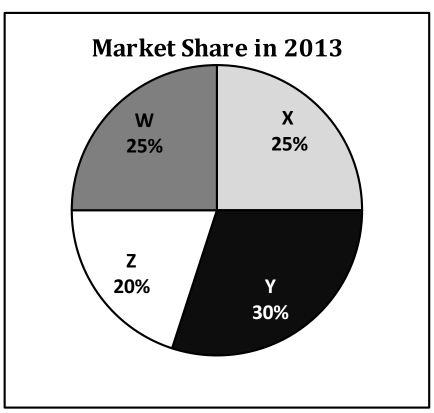
287. The sales from FMCG goods form what percent of the total sales from all products in 2010-11?

[ ] %

The following three questions are based on the following graphics.

The table below shows the data for sales of printers by Company X across four years. The pie-chart shows the market revenue share for the companies X, Y, Z and W, for the sale of printers, in the year 2013. The price per printer is same for all the four companies.

	Units of printers sold by Company X	**Price per unit ($)**
2010	14,000	50
2011	16,000	55
2012	20,000	58
2013	23,000	63



Answer the three questions that follow.

288. What is the total revenue, in thousand dollars, from printers sold by all four companies in 2013?

[ ] thousand dollars

289. What is the percent increase in revenue for Company X through the sales of printers from 2010 to 2011?

[ ] %

290. If, in 2014, the revenue from the sales of printers of Company W grew by 20 percent over the sales of printers in 2013, what is the increase in revenue, in thousand dollars, from printers sold by company W in 2014, rounded off to the nearest integer?

[ ] thousand dollars

291. In a class, overall grade average is computed by taking the weighted average of the scores of quizzes and tests; each test is assigned a weight three times that of each quiz. If Dan scored 81 and 76 on two quizzes and scored 96 on the only test, what is his overall grade average?

[ ]

292. Joe's test scores in English are 80, 82, 79 and 84. What average (arithmetic mean) score should he get on his next two English tests to raise his average score to 85?

[ ]

293. A set of four integers has a range of 2 and an average (arithmetic mean) of 3. What is the maximum possible product of the four integers?

[ ]

294. The lengths of a population of flies are normally distributed with a mean length of 3 centimeters and a standard deviation of 0.3 centimeters. One of the worms is picked at random. What is the probability that the fly is between 3 centimeters and 3.6 centimeters long?

[ ]

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295. The hourly wage paid to employees in an organization is normally distributed around a mean of \$22.50 per hour with a standard deviation of \$4.50. What percent of the employees are paid hourly wages greater than \$27?

[ ] %

296. If a set of data consists of only the first twelve positive multiples of 6, what is the inter-quartile range (Positive difference between Q1 (quartile 1) and Q3 (quartile 3)) of the set?

[ ]

297. Like 'Quartiles', which divide a set of data in four ordered groups, 'Octiles' divide a set of data in eight ordered groups, each with the same number of terms. Thus, there are seven such 'Octiles.' What is the value of the 4^{th} Octile if the set of data is composed of consecutive integers from 13 to 45, inclusive?

[ ]

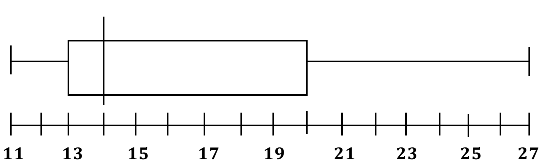
298. In a test, 15 students scored only in whole numbers from 0 to 7, inclusive. Each possible score was obtained at least once. If the average score of the 15 students was 5, what could be the minimum possible sum of scores obtained by the lowest scoring seven of the 15 students?

[ ]

299. Some rock samples are weighed, and it was found that their weights are normally distributed. One standard deviation below the mean is 250 grams, and two standard deviations above the mean is 265 grams. What is the value of the standard deviation of the weights of the rock samples?

[ ] grams

300.



The box-and-whisker plot above shows the score, out of 30, in a quiz, for 60 students. How many students have a score between 20 and 25, if a score of 25 represents the 85^{th} percentile value on the plot above?

[ ]

301. If the length of a rectangle having an area 1,200 is doubled while the width is increased by 25%, what is the new area of the rectangle?

[ ]

302. John lent one part of an amount of money at 10 percent rate of simple interest and the remaining at 22 percent rate of simple interest, both parts for one year. If the total sum lent was $2,400 and the total interest earned was $360, at what rate was the larger part lent?

[ ] %

303. What is the value of $\left(\frac{a}{b}\right)$, if $ad = bc$, and $\left(\frac{a+c}{b+d}\right) = 7$? $(a, b, c, d \neq 0)$

[ ]

304. Imran bought a few pencils, each for $0.15 and a few erasers, each for $0.29. If he bought $4.40 worth of items, how many pencils did he buy?

[ ]

305. David bought some diaries that cost $8 each and some notebooks that cost $25 each. He bought more than 10 diaries and the total cost of the notebooks that David bought was at least $150. If the total cost of all the items that David bought was less than $260, how many notebooks did he buy?

[ ]

306. $p,\ q,\ 12,\ 6,\ 17$

If $p < q$ and the median of the numbers in the list is 10, what is the value of q in the list above?

[ ]

307. A garment shop sold three-fourth of the shirts in its inventory last month. Each shirt was sold for $20. If all but 40 shirts in the shop's inventory were sold last month, what was the total revenue last month from the sale of these shirts?

$ [ ]

308. A manufacturer sold a total of 1,000 units of its two items X and Y. The selling prices of each unit of item X and Y were $1,500 and $2,000, respectively. If the manufacturer's revenue from

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the sale of units of items X was 3 times the revenue from the sale of units of item Y, how many units of item X did the manufacturer sell?

[ ]

309. In a group of 30 employees, 21 are unmarried and 15 work part-time. If 12 of the 30 employees are unmarried and work part-time, how many of the employees are single and do not work part-time?

[ ]

310. John spent a total of $12,000 on painting, furnishing, and interior designing his home. If the amount that he spent on furnishing was 20 percent of the total amount that he spent on painting and interior designing, how much did he spend on furnishing?

$ [ ]

311. A group of 5 equally efficient skilled workers together take 18 hours to finish a job. If an apprentice works at $\frac{2}{3}$ the rate of a skilled worker, how long will it take for a group of 4 skilled workers and 3 apprentices to do the same job, if each skilled worker works at an identical rate and each apprentice works at an identical rate?

[ ] hours

312. R is an integer between 1 and 9, exclusive, and $P - R = 2,370$. If P is divisible by 9, what is the value of $\left(\frac{P}{R}\right)$?

[ ]

313. Craig worked for the last two and half years. His average annual earnings for the first 25 months was $2,700 and the average annual earnings for the last 25 months was $3,400. If his average (arithmetic mean) annual earnings for the first 5 months was $1,500, what was his average annual earnings for the last 5 months?

$ [ ]

314. In an experiment with n bacteria, it was found that each bacteria weighed 10^{-12} grams. Each of the n bacteria gave birth to n new bacteria, each of which also weighed 10^{-12} grams. If the first n bacteria weighed $\frac{1}{16}$ of the total weight of all bacteria, what was the value of n?

[ ]

315. A total of 4,800 parliamentarians voted FOR and AGAINST a particular ordinance. Out of the 4,800 parliamentarians, 1,800 belonged to party A and 3,000 belonged to party B. Three-fourth of the party A parliamentarians and $\frac{2}{3}$ of the party B parliamentarians voted FOR the ordinance. Also, $\frac{1}{3}$ of the party A parliamentarians who voted FOR the ordinance and $\frac{1}{2}$ of the party B parliamentarians who voted FOR the ordinance were females. What was the total number of female voters who voted FOR the ordinance?

[ ]

316. Of the 600 people in a school, $\frac{2}{3}$ are students. The school has more than twice as many girls as it has boys. If the school has more than 132 boys, how many of the students are girls?

[ ]

317. Of the bottles of squash examined in a fruit juice factory, 25 percent failed to pass. All of the bottles that failed to pass were either incorrectly tagged or had broken seal. Of the bottles that failed to pass, $\frac{3}{4}$ were incorrectly tagged and the rest had broken seal. If 6,000 bottles of squash were examined, how many bottles examined had broken seal?

[ ]

318. Of the school students who participated in a survey, 11 had taken a course in math, 23 had taken a course in science, and 12 had taken neither of the two courses. If 6 had taken both the courses, how many students participated in the survey?

[ ]

319. Of the people who attended a training program, 70 percent were males and some of them were freelancers. If 300 people attended the training program and 90 of the males who attended the training program were not freelancers, what percent of the people who attended the training program were males freelancers?

[ ] %

320. Chris invested $25,000 at r percent simple annual interest and a different amount at $2r$ percent simple annual interest, both for one year. If the interest from the second investment was three-fourth of the interest from the first, what amount did Chris invest at $2r$ percent simple annual interest?

$ [ ]

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321. S is a set of five points in the plane, of which no three are collinear. How many distinct triangles can be drawn that have three of the points in S as vertices?

[ ]

322. Seven consecutive numbers are selected from the integers 1 to 100, and each number is divided by 7. What is the sum of the remainders obtained?

[ ]

323. Some TVs at a showroom are LCD and the rest are LED. If the ratio of the number of LED TVs to the number of LCD TVs at the showroom is 7 : 6, and there are a total of 260 TVs at the showroom, how many of the TVs are LED?

[ ]

324. A circle is inscribed in a square such that the circle touches all four sides of the square. The area of the regions of the square outside the circle is $16(4-\pi)$. What is the area of square?

[ ]

325. Jack's monthly remuneration consist of a monthly salary and a 4 percent commission on the his monthly sales that exceeds \$2,000. Jack gets a fixed monthly salary. If Jack's monthly remunerations were \$3,620 in January and \$3,580 in February, how much greater were his sales in January than in February?

\$ [ ]

326.

+	x	y	z
d	p	-3	m
e	q	n	12
f			

The figures above represent an addition table where four entries, p, q, m and n are shown; for example, $d + x = p$. If $d + y = -3$ and $e + z = 12$, what is the value of $(m + n)$?

[ ]

327. The number of units n of its product that Company X is scheduled to produce in month t of its next fiscal year is given by the formula $n = \frac{900}{1+2^{-t}c}$, where c is a constant and t is a positive integer between 1 and 6, inclusive. If Company X is scheduled to produce 300 units of its product in month 2 of its next fiscal year, what is the value of c?

328. The operation '#' represents one among addition, subtraction, and multiplication of integers. If 2#0 = 2, what is the value of 1#0?

329. The product of the unit digit, the tens digit, and the hundreds digit of a positive integer m is 96. If m is odd and hundreds digit of m is 8, what is the unit digit of m?

330. The ratio of the number of women to the number of men to the number of children in a room is 5 : 2 : 7, respectively. If the total number of women and children in the room is a perfect square and there are fewer than 8 men in the room, what is the number of women in the room?

331. The selling price of an article is equal to the cost of the article plus the markup. If the markup on the television set is 25 percent of the cost, the markup is what percent of the selling price?

%

332. The sum of positive integers x and y is 47. If $x = y + 1$, what is the value of x?

333. The sum of positive integers x and y is 77. If x and y have the same tens digit, what is the value of xy?

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334.

	Number of students interested in arts	Number of students not interested in arts	Total
Number of boys			36
Number of girls			
Total	52		

The table above shows the number of boys and girls interested in arts and not interested in arts in a particular class of a school. If the number of girls is twice the number of students who are not interested in arts, what is the total number of students in the class?

[ ]

335.

	Like	Dislike	Not sure
Brand X	40	20	40
Brand Y	30	35	35

The table above shows the results of a survey of 100 respondents, each of whom responded “Like’,’ “Dislike” or “Not sure”, when asked about their liking for Brand X and for Brand Y. If the number of respondents who did not respond “Like” for either brand was 40, what was the number of respondents who responded “Like” for both the brands?

[ ]

336. The total bill of a party was shared equally by x persons and each of them paid \$23. Had the total bill of the party been shared equally by $(x + 1)$ persons, each of them would have paid \$22. What was the total bill of the party?

\$ [ ]

337. An auditorium has 35 rows with 30 seats in each row. During a program, there was an average (arithmetic mean) of 13 UNOCCUPIED seats per row for the front 12 rows and an average of 10 OCCUPIED seats per row for the back 23 rows. How many of the seats were occupied during the program?

[ ]

338. A bag had two pencils and five erasers. After additional pencils and erasers were placed in the bag, the ratio of the number of pencils to the number of erasers was $\frac{1}{2}$. If the number of pencils added was $\frac{2}{3}$ the number of erasers added, how many pencils were added?

[ ]

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339. In University P, there are 105 more male teachers than female teachers. If the university were to hire 14 more female teachers, the ratio of the number of male teachers to the number of female teachers would then be 16 : 9. What is the number of female teachers in University P?

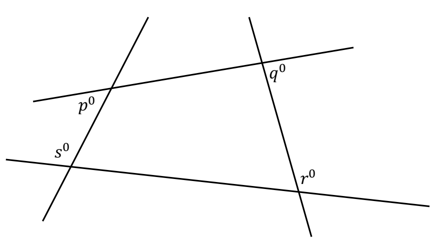
340. The remainder when a two-digit positive integer x is divided by 9 is 5. What is the remainder when x is divided by 3?

341. If the average (arithmetic mean) of w, x, y and z is n, what is the value of $(n - w) + (n - x) + (n - y) + (n - z)$?

342. If $(|x| + 1)(x + 2) = 0$, what is the sum of all possible values of x?

343. If $|x + 2| \leq 4$, where x is an integer, what is the median value of all possible values of x?

344.



In the figure above, if $s = 105^\circ$ and $r = 145^\circ$, what is the value of $(p + q)$?

345. If the length of the longest possible diagonal of a cube is $10\sqrt{3}$, what is the volume of the cube?

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346. A merchant sold an appliance for \$50 more than his cost of the appliance. If the merchant's gross profit on the appliance was 20 percent of the price at which he sold the appliance, what was the selling price of the appliance?

\$ [ ]

347. When 200 gallons of Chemical X was removed from a storage tank, the volume of the chemical left in the storage tank was $\frac{3}{7}$ of the storage tank's capacity, which was 1,600 gallons less than the storage tank's capacity. What was the storage tank's capacity?

[ ] gallons

348. A piece of chalk, 8 cm long is broken into three pieces whose lengths, in cm, are distinct integers. The length of the longest piece is equal to the sum of the lengths of the other two pieces. What is the product of the length of the three pieces?

[ ]

349. If a, b and c are three numbers such that $a + b + c = 5$, and $ab = ac = 2$, what is the greatest possible value of any of the three numbers?

[ ]

350. The pages of a book were numbered with the first page numbered 1. If total of 97 digits were used in numbering the pages, how many pages did the book have?

[ ]

351. If $x = 2$ is a root of the quadratic equation $x^2 + (k^2 - 2)\,x = 0$, what is the value of k?

[ ]

352. A student was asked to calculate the average (arithmetic mean) of n positive consecutive integers, starting from 1. He, however, missed the largest number and calculated the average of $(n - 1)$ numbers instead and got the average equal to 3. What was the number that the student missed?

[ ]

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353. The students in a class are made to stand in a square formation having few rows and columns. If the position of the students ois changed to form a rectangle, the number of columns increases by 3. How many students were there in the class?

[ ]

354. In a grocery shop, the total cost of 5 apples, 4 oranges and 3 lemons is \$13 and the total cost of 3 apples, 4 oranges and 5 lemons is \$11. What is the total cost of 1 apple, 1 orange and 1 lemon?

\$ [ ]

355. The sum of the ages of A and B is 70 years. A, at present, is 2 times as old as B was x years ago. B, at present, has the same age as A had x years ago. What is the present age of A?

[ ] years

356. The quadratic equation $x^2 + qx + p = 0$ has the same roots as the quadratic equation $x^2 + px + q = 0$. If $p^2 + q^2 = 9$, what is the value of p^2?

Give your answer as a fraction.

$\frac{[\ ]}{[\ ]}$

357. If a, b & c are non-negative integers and $a^3 + b^3 + c^3 = 3$, what is the value of $(a + b)(b + c)(c + a)$?

[ ]

358. x, y & z, are positive, and $xy = r$, $xz = r^2$, and $yz = r^3$. If $x^2 + y^2 + z^2 = 91$, what is the value of r?

[ ]

359. Joe has a bag containing two varieties of beans – a cheaper variety, costing \$6 each, and a costlier variety. The price of each bean of the costlier variety is thrice that of each bean of the cheaper variety. If one-third of the beans cost \$6 each, the price of the costlier beans forms what percent of the total price of all the beans?

[ ] %

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360. In a class of 20 students, 60% of the students have a lower GPA than Joe has and 10% of the students have a GPA equal to that of Joe, excluding Joe. In another class of 10 students, 80% of the students have a higher GPA than Joe has. If the scores of the students of both classed are considered, how many students have a higher GPA than that of Joe?

[ ]

361. In a marathon race of 1,000 meters between two competitors, A and B, A ran with a speed of 10 meters per second. Had A allowed B to start the race from a point 100 meters ahead of him, A would have still managed to beat B by 20 seconds. What was the speed of B?

[ ] meters per second

362. m is a non-prime integer. If $(m - 2)$ and the quotient when m is divided by 2 have the same integer value, what is the value of m?

[ ]

363. If $3 < x < 4$ and $x + 0.005 > 4$, what is the hundredth digit of the decimal representation of x?

[ ]

364. If $9^x = 4(3^x) - 3$, where x is a positive integer, what is the value of x?

[ ]

365. If p, q & r are positive integers and $pq + qr + rp = 3$, what is the value of $\left(\frac{1}{p} + \frac{1}{q} + \frac{1}{r}\right)$?

[ ]

366. A two-digit number ab is 11 more than the product of the digits a and b. What is the units digit of the two-digit number ab?

[ ]

367. If n is a positive integer such that $(n + 3)^n = 216$, what is the value of n?

[ ]

368. If $f(x) = 3x + 2$ and $f(\sqrt{c}) = 8$, what is the value of $f(c)$?

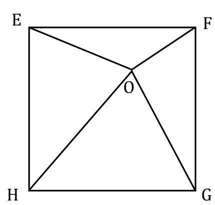
369. If $f(x) = x^{\frac{3}{4}} - 7$ and $f(k^2) = 57$, what is the value of k?

370. If $f(p^2) = p^4 - p^2 + 1$ and $f(5) = p$, what is the value of p?

371. EFGH is a rhombus. If the measures of $\angle$GEF and $\angle$EFH are $2x$ and $(x + 3)$, respectively, what is the measure of $\angle$GFE?

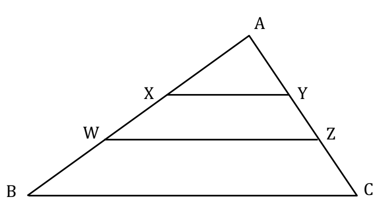
degrees

372.



In the square EFGH shown above, O is any random point inside the square. If the area of the square EFGH is 16, what is the sum of areas of triangle EOF and triangle HOG?

373.



In the figure shown above, area of triangle ABC is 25. If AX : XW : WB = 2 : 1 : 2 and XY, WZ are lines parallel to BC, what is the area of triangle AXY?

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374. A, B and C play a few rounds of a game of marbles. A lost 3 rounds, B lost 4 rounds and C lost 5 rounds. If there is only one winner in each round, how many rounds of the game were played?

375. In the three-digit number xyz, the digits x, y and z represent different integers. If $x^2 = y$ and $z = y + 1$, what is the value of $(x + y + z)$?

2.4 Quantitative Comparison Questions

Quantitative Comparison: Options

Directions: Compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given, and select one of the following four answer choices:

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

A symbol that appears more than once in a question has the same meaning throughout the question.

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376. The ratio of the number of male and female workers in a company in 2002 was 3 : 4. The ratio of the number of male and female workers in 2003 was 10 : 7.

Quantity A	**Quantity B**
Percent increase in the number of males from 2002 to 2003	Percent increase in the number of females from 2002 to 2003

377. In a particular professional club, exactly 75 percent of the female members are engineers, and, of them, $\frac{1}{3}$ are mechanical engineers. Exactly 30 percent of the male members are engineers. It is known that, only those who are engineers can be mechanical engineers.

Quantity A	**Quantity B**
Fraction of the members who are mechanical engineers in the club	$\frac{1}{3}$

378. For an election rally, the party president anticipated a gathering of 100,000 to 120,000 persons, and an expense of \$50,000 to \$75,000.

Quantity A	**Quantity B**
Dollar expense per person	\$0.60

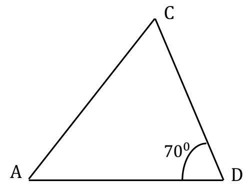
379. Steve estimated the distance, in kilometers, and the average speed, in kilometers per hour for a trip. Steve's estimate for the distance was within 4 kilometers of the actual distance, and his estimate for his average speed was within 8 kilometers per hour of his actual average speed. Steve's estimate of the time was within t hours of the actual time that he took for the trip.

Quantity A	**Quantity B**
t	1

380. $8x$ is an integer, where $0 < x < 1$.

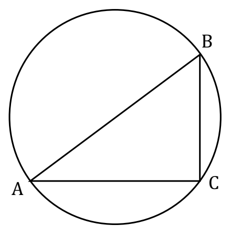
Quantity A	**Quantity B**
The tenth digit in the decimal representation of x	0

381.



Quantity A	**Quantity B**
Length of AC	Length of AD

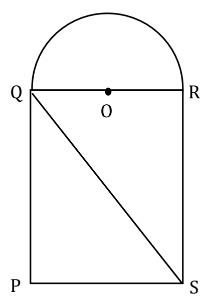
382.



In the figure above, the triangle ABC is inscribed in the circle; the ratio of the sides BC, AC, and AB is 3 : 4 : 5 and the perimeter of the triangle is 48.

Quantity A	**Quantity B**
Circumference of the circle	20π

383.



In the figure above, PQRS is a rectangle. The length of QS is 13 and QR : PQ :: 5 : 12.

Quantity A	**Quantity B**
Area of the semi-circular region with center O and diameter QR	3π

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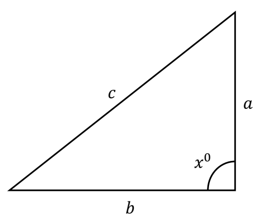
384. Quadrilateral ABCD is inscribed in a circle with AC as a diameter of the circle, such that ∠DAC is 30° and ∠BAC is 45°. The radius of the circle is 1. The area of ABCD to can be expressed as a fraction in simplest radical form as $\left(\frac{a+\sqrt{b}}{c}\right)$.

Quantity A	Quantity B
$a + b + c$	7

385. The numbers a, b and c form a sequence such that b is 3 more than a, and c is 9 more than b. Also, $\frac{a}{b} = \frac{b}{c}$.

Quantity A	Quantity B
b	$\frac{3}{2}$

386.



In the triangle shown above, $a^2 + b^2 < 15$ and $c = 4$

Quantity A	Quantity B
$x°$	90°

387. In the rectangular coordinate system, (r, s) and (u, v) are two points. It is known that $r = v = 1 - s$ and $u = 1 - r$.

Quantity A	Quantity B
Distance of the point (r, s) from the origin	Distance of the point (u, v) from the origin

388. Line k makes a positive intercept on the Y-axis.

Quantity A	Quantity B
Slope of the line k	0

389. In the XY-plane, the line k, having a negative slope, passes through the origin and the point (a, b), where $ab \neq 0$ and $a < b$.

Quantity A	**Quantity B**
b	0

390. A group of 20 friends went out for a lunch. Five of them spent \$21 each and the rest spent \$3 less than the average (arithmetic mean) of all of them.

Quantity A	**Quantity B**
The average amount spent by all the friends	\$12

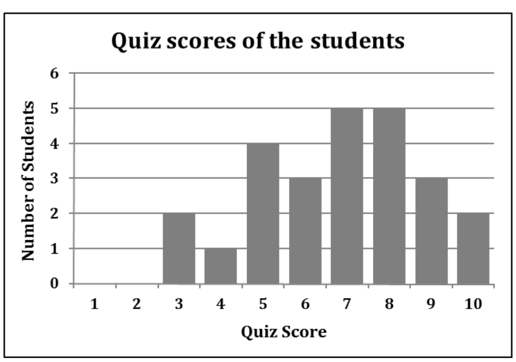
391. Jack's sister, Suzy, is 5 years elder to Jack. In 2010, Suzy turned 24 years old.

Quantity A	**Quantity B**
The year in which Jack was born	1990

392. $2x + 3y < 6$ and $3x + 2y = 6$

Quantity A	**Quantity B**
x	y

393. The graph below shows few students' distribution of quiz scores a teacher recorded for a quiz consisting of 10 questions. Each question was worth one point.



After recording the scores and knowing that question number '2' was incorrect, the teacher decided to give full credit for question number '2' to every student.

Quantity A	Quantity B
The new mean quiz score of the students, considering score for question number '2'	7

394. The side of a square ABCD measures 1 unit. Point E on side AB is such that, when the square is folded along the line DE, side AD coincides with diagonal BD.

Quantity A	Quantity B
The length of AE	0.5

395. A store offers a discount on carpets which are priced on a 'per square foot' basis. Joe claimed that the sales price he paid for a piece of carpet measuring 10-feet by 12-feet was the same as the sales price, before discount, for a piece of the same type of carpet measuring 6-feet by 8-feet.

Quantity A	Quantity B
Percent discount offered by the store	60%

396. x is such that $-x\,|x| > 0$

Quantity A	Quantity B
$\sqrt{(x-5)^2}$	$(x-5)$

397. $xy > 0$ and $y < -1$

Quantity A	Quantity B
$\frac{x}{y}$	xy

398. $|y| > |z|$

Quantity A	Quantity B
$\|x-y\|$	$\|x-z\|$

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399. $m^2 + n < 0, mn \neq 0$

Quantity A	Quantity B
$\frac{m^2}{n}$	-1

400. The average weekly salary per head of the entire staff of a factory consisting only of clerks and supervisors is \$120. The average weekly salary per head of the supervisors in the factory is \$120 more than the average salary per head of the clerks in the factory. The total number of clerks and supervisors is 60.

Quantity A	Quantity B
Twice the number of clerks working in the factory	The numerical value of the average weekly salary, in dollars, of the clerks

401. Thirty-five percent of all employees of a company are men. 20 percent of the men and 40 percent of the women in the company attended a meeting.

Quantity A	Quantity B
Percent of the total employees who attended the meeting	33%

402. Three friends, R, Y and M, each have a different number of toffees. M has $\frac{1}{4}$ of the total number of toffees all three friends together have and the difference between the number of toffees of Y and R is $\frac{1}{10}$ of the total number of toffees.

Quantity A	Quantity B
Percent of total toffees with the two friends who have the least number of toffees	50%

403. On a scale drawing showing the floor plan of an apartment, a rectangular kitchen measures 6 cm by 2.7 cm, where, 1 centimeter represents 1.5 meters.

Quantity A	Quantity B
The area of the floor of the actual kitchen, in square meters, expressed as an integer to the nearest unit	37 square meters

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404. The diagonal of a particular square is 7 inches. The diameter of a particular circle is also 7 inches.

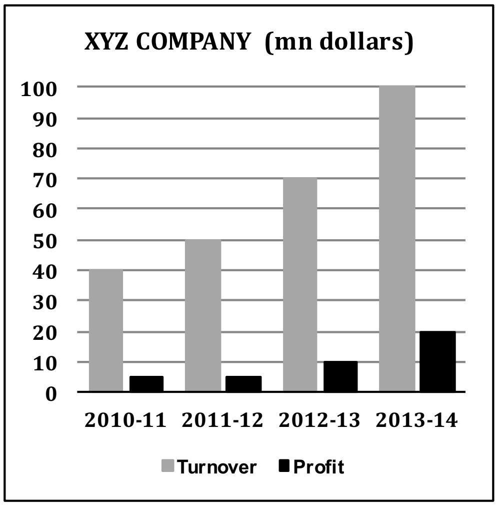
Quantity A	**Quantity B**
The positive difference between the area of the square and that of the circle, expressed as a decimal to the nearest tenth	14

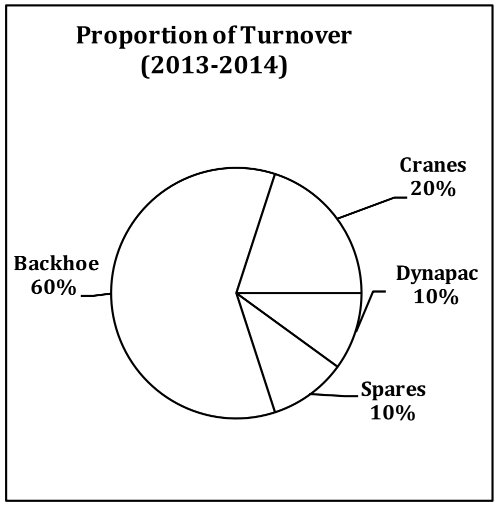
405. Joe always runs around the track at a rate of 30 laps per 75 minutes, and Carl always runs around the track at a rate of 20 laps per 40 minutes. They start running at the same time.

Quantity A	**Quantity B**
Time taken, in minutes, for Joe and Carl to run a combined distance of 99 laps	222.75 minutes

The following four questions are based on the following graphics.

XYZ is a company involved in the manufacture and selling of three products – Backhoe, Cranes and Dynapac. It also sells the spares for these implements. The following charts give the data on the turnover and profits of XYZ in the period 2010-2014, and the profile of the customer segments in the year 2013-2014.





Profile of Customer Segments – 2013-2014					
BACKHOE		**CRANES**		**DYNAPAC**	
Mining	10%	Engineering Industries	40%	Contractors	60%
Process Industries	10%	Steel Industries	10%	Government	20%
Plant Hirers	10%	Plant Hirers	10%	Plant Hirers	10%
Government	15%	Granite Quarries	30%	Mining	10%
Steel Industries	55%	Mining	10%		

Answer the four questions that follow.

406.

Quantity A	**Quantity B**
Contribution to XYZ's turnover in 2013-2014 from the purchases of the three products by the government	$10 mn

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407. In 2013-2014, the profit from the sale of Cranes was twice the combined profit from the sale of Backhoe, Dynapac and spares.

Quantity A	Quantity B
The ratio of the profits from Cranes to the sale of Cranes	$\frac{2}{3}$

408.

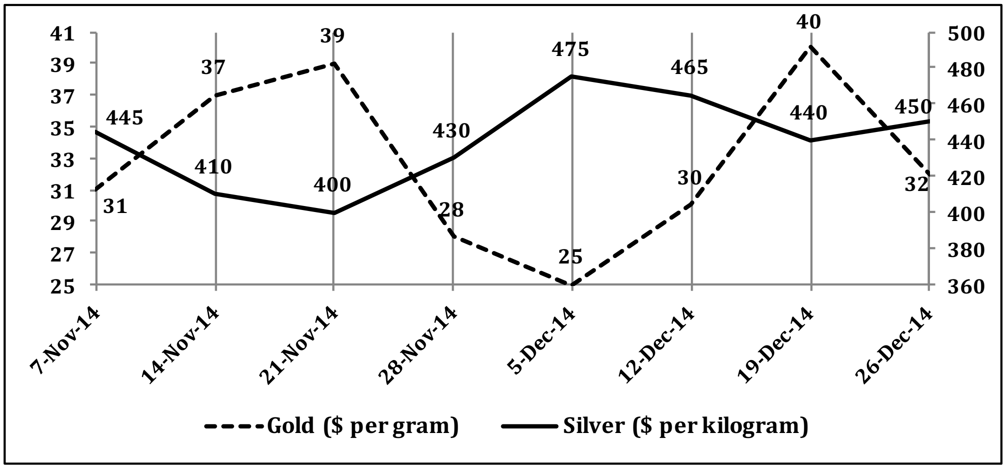
Quantity A	Quantity B
The greatest ratio of XYZ's profit to turnover across the four year periods	$\frac{1}{4}$

409. "XYZ's total turnover in 2013-2014 from the three products from purchase made by the government = a" and "XYZ's total turnover in 2013-2014 from the three products from purchase made by the contractors = b"

Quantity A	Quantity B
$\frac{a}{b}$	$\frac{2}{1}$

The following three questions are based on the following graphics.

The prices of gold in dollars per gram (refer left-hand side of X-Axis) and silver in dollars per kilogram (refer right-hand side of X-Axis) are shown below for a period of 8 days, from 7th November 2014 to 26th December 2014.



Answer the three questions that follow.

410.

Quantity A	Quantity B
Positive difference of the combined price of 40 gram gold and 1.5 kilogram silver on 21st November and their combined price on 12th December	\$300

411. For the period shown in the graph, the difference between the highest and the lowest price of gold per gram is \$$A$ & that of silver per kilogram is \$$B$.

Quantity A	Quantity B
$\frac{A}{B}$	0.10

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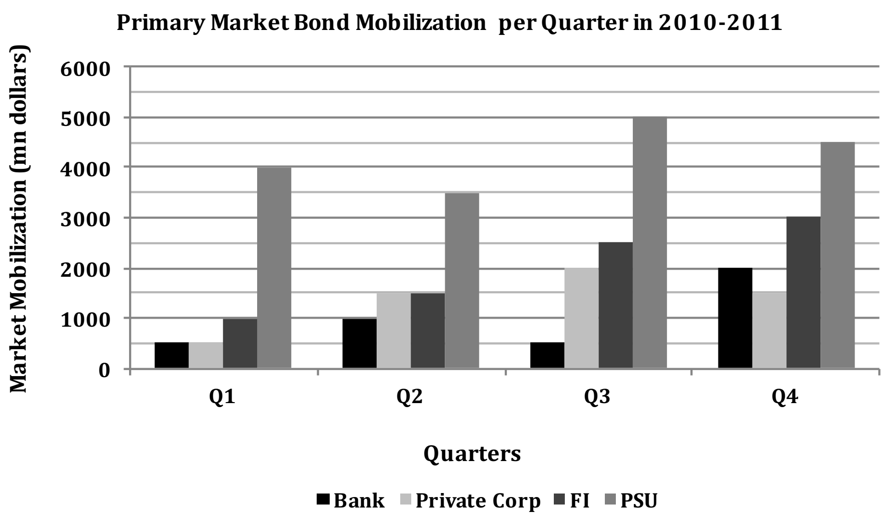
412.

Quantity A	**Quantity B**
The ratio of the median price of gold per gram and the median price of silver per kilogram on the 8-day period	$\frac{1}{20}$

The following three questions are based on the following graphics.

The bar-graph shows the breakup of the primary market mobilization by different sectors for the year 2010-2011 in four quarters (Q1 to Q4).



Answer the three questions that follow.

413.

Quantity A	**Quantity B**
The difference between the total values, in million dollars, of bonds mobilized in primary market by the four sectors in Q1 and in Q2	\$1500 mn

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414.

Quantity A	Quantity B
The percent decrease in the value of bonds mobilized in primary market by PSU in Q4 over Q3 in 2010-2011	11.11%

415.

Quantity A	Quantity B
The simple quarterly growth rate in the value of bonds mobilized in primary market by FI from Q1 to Q4 in 2010-2011	50%

416. A sum of $600 was divided among some boys and girls so that on average, each received $12. The number of girls is one less than half the number of boys.

Quantity A	Quantity B
The number of boys	36

417. Wheat varieties A and B are to be mixed to form a mixture worth $16 per pound. Variety A of wheat is priced at $18 per pound. One of the varieties is priced at $3 per pound greater than the other.

Quantity A	Quantity B
The ratio in which wheat varieties A and B are mixed	$\frac{1}{2}$

418. If a group of students having an average age of 17 years join a class, the average age of all the students in the class reduces from 20 years to 19 years.

Quantity A	Quantity B
Twice the number of students who joined the class	The number of students initially present in the class

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419. A discount of 5 percent on the listed price of an article was offered, reducing the profit by \$120 that could have been earned if it were sold at the listed price.

Quantity A	Quantity B
The actual price at which the article was sold	\$2,000

420. Two interior angles of a convex quadrilateral ABCD are right angles (all interior angles are less than 180° and the two diagonals both lie inside the quadrilateral). The degree measure of $\angle ABC$ is twice the degree measure of $\angle BCD$.

Quantity A	Quantity B
Measure of the largest interior angle of quadrilateral ABCD	120°

421. Of the patrons of Club A, 25 percent are also patrons of Club B. Of the patrons of Club B, 40 percent are also patrons of Club A.

Quantity A	Quantity B
Number of patrons of Club A	Number of patrons of Club B

422. 1 mile equals 5,280 feet and 1 yard equals 3 feet.

Quantity A	Quantity B
Number of seconds required to travel d miles at r yards per second	Number of seconds required to travel d kilometers at r meters per second

423. Bob's average (arithmetic mean) marks after over a number of tests were 85. After taking two more tests, his average marks for all tests decreased to 81.

Quantity A	Quantity B
Bob's average marks in the last two tests	80

424. The average (arithmetic mean) score of Class Q's students is greater than the average (arithmetic mean) score of Class P's students. Also, the median score of Class Q's students is greater than the median score of Class P's students.

Quantity A	Quantity B
Standard deviation of the scores of Class P's students	Standard deviation of the scores of Class Q's students

425. A positive integer n is such that if $3n$ is divided by 15, the remainder is 6.

Quantity A	Quantity B
The remainder if n is divided by 5	2

426. If 4 people with average age 40 years join a group, the average age of the group doubles in value.

Quantity A	Quantity B
Average age of the group of people initially	20

427. Four years ago, B's age was double the age of A.

Quantity A	Quantity B
Present age of B	Double the present age of A

428. $(2^x)(2^k) = 4$ and $(9^x)(3^k) = 81$

Quantity A	Quantity B
x	k

429. $x - y^2 > 0$ and $xy < 0$

Quantity A	Quantity B
x	y

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430. $9 < a < 15$ and $5b > 12$

Quantity A	**Quantity B**
ab	27

431. $y > 0$ and $x = 1 - y$

Quantity A	**Quantity B**
$y - x$	0

432. Mac can buy a TV for p dollars from Showroom X, where the sales tax is t percent, or he can buy the same TV for P dollars from Showroom Y, where the sales tax is T percent. It is known that $PT > pt$.

Quantity A	**Quantity B**
Total cost of the TV from Showroom X	Total cost of the TV from Showroom Y

433. Suzy purchased three books from a shop. The average (arithmetic mean) price of the books was $1.50, which was also the price of one of the books.

Quantity A	**Quantity B**
The median price of the three books	$1.50

434. Joe decided to buy some toffees. On visiting a shop, he found that toffees were sold in boxes. The boxes had different number of toffees and the prices of the boxes were also different. The prices are shown in the table below:

Number of toffees per box	**Price per box**
5	$4.50
10	$6.50
15	$8.50

Quantity A	**Quantity B**
The amount spent by Joe if he buys 90 toffees in boxes of 5 toffees	Twice the amount spent by Joe if he buys 90 toffees in boxes of 15 toffees

435. The price of coffee increased by 20% following the shortage in availability in the market. As a result, Toby decided to reduce his coffee consumption so that there is no increase in the expense on coffee.

Quantity A	**Quantity B**
The percent reduction in coffee consumption	20%

436. A group of 5 equally efficient men together take 20 hours to finish a job. 8 men and 3 women take 10 hours to complete the same job. Each man works at an identical rate and each woman works at an identical rate.

Quantity A	**Quantity B**
Efficiency of doing work of a man	Efficiency of doing work of a woman

437. Sets X and Y consist of the same number of positive integers. The integers in X are consecutive even integers, and the integers in Y are consecutive odd integers, such that the sum of the integers in X is greater than the sum of the integers in Y.

Quantity A	**Quantity B**
Median of the integers in X	Average (arithmetic mean) of the integers in Y

438. Dave bought a few pencils and a few erasers. Each pencil costs 12 cents and each eraser costs 20 cents. He bought at least one of both the items and spent a total of 108 cents.

Quantity A	**Quantity B**
Number of pencils	Number of erasers

439. Aluminum costs \$2 per kilogram, and copper costs \$4 per kilogram. 10 kilograms of alloy K consists of x kilograms of aluminum and y kilograms of copper, and costs less than \$30.

Quantity A	**Quantity B**
x	y

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440. Kevin has \$1,250 worth of garments consisting of jackets each worth \$200 and shirts each worth \$50. Kevin has fewer than 4 shirts.

Quantity A	Quantity B
Number of shirts	2

441. Of the students in a cookery class, 15 percent joined Chinese cuisine course, while 10 percent joined Indian cuisine course. Two-third of the students who joined Chinese cuisine course also joined Indian cuisine course.

Quantity A	Quantity B
Percent of the students in the class who joined neither of the two courses	85%

442. On an excursion trip of 5 hours, Mary averaged p miles per hour for 2 hours and q miles per hour for the rest of the journey. It is known that $p + \frac{3}{2}q = 140$.

Quantity A	Quantity B
Mary's average speed for the entire trip	56

443. On a test day, 100 candidates took a psychometric test. The first group of 60 candidates scored on an average (arithmetic mean) 68 points. The second group of 40 candidates scored on an average half of the average score of the first group.

Quantity A	Quantity B
The average score on the test for the 100 candidates	55 points

444. On an adventure trip, Steve drove 50 miles. He drove 30 miles at an average speed of 60 miles per hour and then drove the remaining distance at a different constant speed. Had he driven entirely at the speed at which he drove the remaining distance of the trip, he would have driven for 1 hour.

Quantity A	Quantity B
Steve's average speed for the entire 50 miles	56 miles per hour

445. On a particular day, in a shop, at the beginning of a sale, there were a total of 90 fruit cans of brand X and 60 fruit cans of brand Y in stock. On the day, half of the total number of cans were sold and only two-third of the total number of cans of brand Y in stock were sold.

Quantity A	**Quantity B**
The number of cans of brand X sold	The number of cans of brand Y sold

446. One kilogram of a tea mixture consists of p kilogram of type I tea and q kilogram of type II tea. The cost of the mixture is C dollars per kilogram, where $C = 65p + 85q$ and $C \geq 73$.

Quantity A	**Quantity B**
p	0.8

447. A professor gave a quiz, scored out of 100, to two classes, A and B, each having 25 students. In each class, no two students received the same score and all scores were positive integers. The lowest scores in classes A and B were 66 and 76, respectively.

Quantity A	**Quantity B**
Maximum score for class A	Maximum score for class B

448. Machines A, B, and C each manufacture parts at their respective constant rates. The ratio of Machine A's constant rate to Machine B's constant rate is r_p and the ratio of Machine B's constant rate to Machine C's constant rate is r_q. It is known that $r_p < r_q < 1$.

Quantity A	**Quantity B**
Rate at which Machine C manufactures parts	Rate at which Machine A manufactures parts

449. Two buses A and B travelled towards each other's station on two separate, straight, parallel rail tracks from their station that are 420 miles away. They passed each other after traveling for 3 hours. When the two buses passed, Bus A had averaged a speed of 75 miles per hour.

Quantity A	**Quantity B**
Distance left for Bus A to cover to reach its destination	Distance left for Bus B to cover to reach its destination

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450. The yearly rent collected by a firm from a property was p percent more in 2008 than that in 2007 and q percent less in 2009 than that in 2008. It is known that $p - q > \frac{pq}{100}$.

Quantity A	**Quantity B**
Annual rent collected by the firm from the property in 2009	Annual rent collected by the firm from the property in 2007

451. The area of a square is equal to the area of a rectangle, which has one of its sides equal to thrice the length of a side of the square.

Quantity A	**Quantity B**
Twice the perimeter of the square	Perimeter of the rectangle

452.

Quantity A	**Quantity B**
The average (arithmetic mean) of a list of 8 numbers as a percent of the sum of the numbers	12.5%

453. A deposit of \$1,000 is made in a bank account, compounded annually, at a rate of r percent. The amount accumulated after two years is greater than \$1,210.

Quantity A	**Quantity B**
r	10%

454. $\lceil x \rceil$ denotes the least integer greater than or equal to x, and $0 < x + \lceil x \rceil < 2$.

Quantity A	**Quantity B**
x	1

455. $f(x)$ is a function such that $f(x) = 2x + f(x-1)$ for all $x \geq 2$. It is known that $f(1) = 1$.

Quantity A	**Quantity B**
$f(3)$	11

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456. The tens digit of positive integer A is 4. The tens digit of $2A$ is 9.

Quantity A	**Quantity B**
Unit digit of A	4

457. The total amount to rent a taxi for one day from Taxi X consists of a fixed amount of \$10 and a amount of \$0.25 per mile driven. The total amount to rent a taxi for one day from Taxi Y consists of a fixed amount of \$20 and a amount of \$0.10 per mile driven. The total amount to rent a taxi from Taxi Y for one day and drive it for m miles is less than \$25.

Quantity A	**Quantity B**
Total amount to rent a taxi from Taxi X for one day and drive it for m miles	\$22.5

458. The total weight of six identical balls of type P and five identical balls of type Q is less than the total weight of five balls of type P and six balls of type Q.

Quantity A	**Quantity B**
Weight of one ball of type P	Weight of one ball of type Q

459. David, Suzy, and Mary each purchased a new car. The average (arithmetic mean) price of the three cars was \$30,000. The price of Suzy's car was \$30,000.

Quantity A	**Quantity B**
The median price of the three cars	\$30,000

460. Two trucks, X and Y, each traveled the same distance at the speeds of 60 miles per hour and 40 miles per hour, respectively. For the entire distance, truck X traveled 25 miles per gallon of gas and truck Y traveled 35 miles per gallon of gas.

Quantity A	**Quantity B**
Amount of gas used by truck X in an hour	Amount of gas used by truck Y in an hour

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461. Smith's yearly remuneration is $20,000 more than Jack's yearly remuneration, but is less than $40,000.

Quantity A	**Quantity B**
Smith's yearly remuneration	Twice of Jack's yearly remuneration

462. Total weight of all mangoes in a box was greater than 2,020 grams. Fewer than 21 mangoes are in the box.

Quantity A	**Quantity B**
Average (arithmetic mean) weight of all mangoes in the box	100 grams

463. The positive integers j and k satisfy the relation: $k = j + 1$

Quantity A	**Quantity B**
Greatest common divisor of j and k	2

464. At a residential society, 30 percent of the families have 5 or more vehicles per family and 40 percent of the families have 3 or fewer vehicles per family.

Quantity A	**Quantity B**
The median number of vehicles per family in the residential society	4

465. When a positive integer x is divided by 12, the remainder is 5.

Quantity A	**Quantity B**
The remainder when x is divided by 8	7

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466. A and B sold articles such that the ratio of selling price of each article of A and that of B was in the ratio 4 : 5. Sales revenue of B was less than $\frac{5}{14}$ of the total sales revenue of A and B combined.

Quantity A	**Quantity B**
Number of articles sold by A	Twice the number of articles sold by B

467. Of a cylinder, the base radius is increased by 10 percent and the height is increased by 20 percent.

Quantity A	**Quantity B**
Percent increase in volume of the cylinder	40%

468.

Quantity A	**Quantity B**
$\sqrt[6]{6!}$	$\sqrt[4]{4!}$

469. A bowl contains few mangoes and few oranges in the ratio 3 : 1. Another bowl contains four fruits, two of which are mangoes and the rest are oranges. One of the fruits from the first bowl, selected at random, is replaced by one of the fruits from the second bowl, also selected at random.

Quantity A	**Quantity B**
Probability that the number of mangoes in the first bowl will increase	$\frac{1}{8}$

470. There are 30 fruits, few apples and few oranges, in a box and both the number of apples and the number of oranges are prime numbers. One fruit is randomly picked from the box, the probability that it will be an orange is less than $\frac{1}{3}$.

Quantity A	**Quantity B**
Number of apples	22

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471. The perimeter of a triangle is 24.

Quantity A	**Quantity B**
Area of the triangle	32

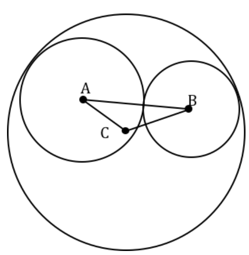
472. A two-digit number is equal to the sum of 7 times the tens digit and 6 times the units digit.

Quantity A	**Quantity B**
Product of the digits of the two-digit number	15

473. A two-digit number is equal to the sum of the digits and thrice the product of the digits.

Quantity A	**Quantity B**
The two-digit number	83

474.



A, B, C are the centers of the three circles, each of which touches the other. The radius of circle with center C is 10 units.

Quantity A	**Quantity B**
Perimeter of triangle ABC	20

475. $(k-1)^2 - 5 < 4$

Quantity A	**Quantity B**
k	-3

476. On comparing the salaries of five friends A, B, C, D and E, it was found that A's salary is greater than C's salary by the same amount as C's salary is greater than B's salary. Also, E's salary is greater than the individual salaries of A and D.

Quantity A	**Quantity B**
Greatest salary among the five friends	Salary of E

477. The total number of garments sold at a shop during the week starting with Monday and ending with Sunday is 30, with the number of garments sold on any day is greater than the number of garments sold on the previous day. At least one piece of garment was sold on each day.

Quantity A	**Quantity B**
The number of garments sold on Thursday	4

478. AB represents a two-digit number. $(AB)^2 = DCB$, where DCB is a three digit number.

Quantity A	**Quantity B**
B	4

479. $|x - 1| = 3\,|x - 3|$

Quantity A	**Quantity B**
$\|x\|$	2

480. $a < -b$ and $a < 0$

Quantity A	**Quantity B**
a^2	b^2

481. $x > 3^{20}$

Quantity A	**Quantity B**
x	6^{12}

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482. The price of a ticket in a multiplex A is 20 percent higher than that in multiplex B. Multiplex A reduces its ticket price by 20 percent and adds \$5 to the reduced price, while multiplex B increases its ticket price by 10 percent and adds \$10 to the increased price.

Quantity A	**Quantity B**
Price of a ticket in multiplex A after price revision	Price of a ticket in multiplex B after price revision

483. The price of a stock decreased by 20 percent and then increased by x percent. $x < 25$.

Quantity A	**Quantity B**
Final price of the stock after the increase	Initial price of the stock before the decrease

484. A father divided his savings of \$15,000 among his three sons A, B and C. A received more than B the same amount that C received less than B.

Quantity A	**Quantity B**
The amount B received	\$5,000

485. Some equally efficient people are employed to complete a piece of work in 18 days. One person alone can complete the work in 270 days.

Quantity A	**Quantity B**
The number of additional people required to complete the work 3 days before time	18

486. $xy < 0$ and $xz < 0$

Quantity A	**Quantity B**
xyz	0

487. $a\,(m - n)$ is non-negative as well as non-positive.

Quantity A	**Quantity B**
m	n

488. $(|x| - 1)(|x| + 2) = 0$

Quantity A	**Quantity B**
x	2

489. a and b are prime numbers such that $a \times b = 6$.

Quantity A	**Quantity B**
$\sqrt{(a-b)^2}$	1

490. x and y are positive integers such that $2^{x+2y} = 8$.

Quantity A	**Quantity B**
xy	1

491. $f(x) = x^2 + 2^x$ for all integers x. $f(k) = \frac{3}{2}$

Quantity A	**Quantity B**
k	0

492. $f(x) = 2x + k$ and $f(3) = 2k + 1$

Quantity A	**Quantity B**
k	5

493. The graph of a quadratic function $f(x) = x^2 + ax + b$ intersects X-axis at two points: $x = 2$ and $x = 8$.

Quantity A	**Quantity B**
$a + b$	5

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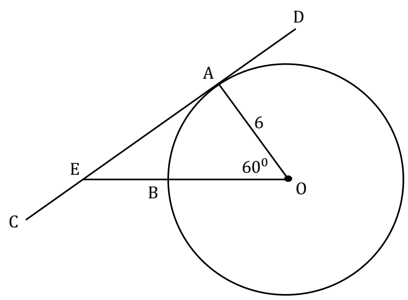
494. $a, b, x, y, \ldots$

Each term of the sequence above is 9 more than $\frac{1}{3}$ the previous term, and $y = \frac{5}{6}x$.

Quantity A	**Quantity B**
a	54

495.



In the diagram above CD is a tangent to the circle with center O at A.

Quantity A	**Quantity B**
Length of EO	$6\sqrt{3}$

496. A 60 ml mixture of juice concentrate and water contained 40 percent juice. A part of the mixture was removed and replaced with an equal quantity of water. The resulting mixture contained 10 percent juice.

Quantity A	**Quantity B**
Volume of the mixture that was removed	45 ml

497. Joe purchased a number of cakes priced at $13 each and a number of biscuit packets priced at $7 each. The total price of items purchased is $33.

Quantity A	**Quantity B**
The total number of cakes and biscuit packets purchased by Joe	3

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498. $(a-2)^2+|b-2|=0$

Quantity A	**Quantity B**
$\frac{a}{b}$	2

499. $\frac{ab}{a+b}=\frac{1}{6}$

Quantity A	**Quantity B**
$\frac{1}{a}+\frac{1}{b}$	6

500. $f(x)=(a-x^n)^{\left(\frac{1}{n}\right)}$ and $f(1)=p$.

Quantity A	**Quantity B**
$f(p)$	0

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Chapter 3

Answer-key

3.1 Multiple Choice Questions

(1) E	(23) E	(45) B	(67) B
(2) D	(24) C	(46) B	(68) D
(3) C	(25) D	(47) D	(69) D
(4) A	(26) C	(48) C	(70) C
(5) B	(27) B	(49) B	(71) E
(6) A	(28) D	(50) C	(72) D
(7) E	(29) C	(51) C	(73) C
(8) C	(30) C	(52) C	(74) D
(9) D	(31) C	(53) C	(75) D
(10) D	(32) C	(54) D	(76) E
(11) C	(33) E	(55) D	(77) D
(12) C	(34) C	(56) D	(78) B
(13) C	(35) C	(57) B	(79) C
(14) B	(36) C	(58) D	(80) C
(15) A	(37) D	(59) D	(81) C
(16) B	(38) E	(60) A	(82) B
(17) A	(39) B	(61) E	(83) B
(18) D	(40) B	(62) C	(84) B
(19) C	(41) D	(63) D	(85) B
(20) D	(42) B	(64) C	(86) E
(21) C	(43) E	(65) E	(87) C
(22) C	(44) C	(66) E	(88) B
			(89) C

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(90) D

(91) E

(92) B

(93) B

(94) B

(95) D

(96) E

(97) D

(98) C

(99) B

(100) A

(101) E

(102) B

(103) A

(104) B

(105) C

(106) D

(107) C

(108) E

(109) C

(110) C

(111) E

(112) D

(113) D

(114) A

(115) D

(116) C

(117) A

(118) C

(119) A

(120) E

(121) A

(122) E

(123) E

(124) C

(125) C

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3.2 Select One or Many

(126) C & F

(127) A & E

(128) B & C

(129) A, B, C & E

(130) B & C

(131) A, B & C

(132) B & D

(133) A, B, C & D

(134) B, D & F

(135) A, C & E

(136) D & E

(137) C, D & E

(138) B, C & E

(139) A, C, F & G

(140) A, B & C

(141) C, D & E

(142) C & D

(143) C, D & E

(144) A

(145) B, E & F

(146) B & D

(147) A & D

(148) A & D

(149) B, D & E

(150) C, D & E

(151) A & C

(152) B, C, D & E

(153) B, C, D & E

(154) A & E

(155) A & C

(156) C & E

(157) E & G

(158) B & D

(159) A & B

(160) C, D & E

(161) B & C

(162) A & C

(163) B & C

(164) B & C

(165) A

(166) A & D

(167) B & E

(168) B & C

(169) A, B & C

(170) A & C

(171) A & E

(172) B & D

(173) C & E

(174) B & D

(175) A & B

(176) A & B

(177) C, D & E

(178) A, B & C

(179) B & C

(180) A, C & E

(181) B & D

(182) A, B & C

(183) A & D

(184) A, B, C & D

(185) B, C, E & F

(186) B, C & E

(187) A, B, C & D

(188) E, F, G & H

(189) B & C

(190) D & E

(191) A

(192) A, E & G

(193) B & C

(194) A, C & G

(195) D, E & F

(196) B & F

(197) B, D & E

(198) B, C, D & E

(199) C, E & G

(200) A & C

(201) A, B & C

(202) B & D

(203) A & C

(204) B & D

(205) B & D

(206) A & F

(207) B

(208) A & D

(209) A, C & F

(210) C

(211) A & D

(212) B & F

(213) A & D

(214) B, C & D

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(215) B & D

(216) B, C & D

(217) B, C, E & F

(218) A, D, E & G

(219) D

(220) B & D

(221) B & D

(222) A, D & F

(223) D

(224) A & D

(225) B & C

(226) B & E

(227) A, B & C

(228) A & D

(229) B & D

(230) A & C

(231) B & D

(232) E & F

(233) C & D

(234) C & D

(235) C

(236) A & C

(237) B, C, D & E

(238) C, D, E & F

(239) A & D

(240) B & D

(241) B & E

(242) A & C

(243) B

(244) B & C

(245) A, B & C

(246) B, C, D, E & H

(247) A & C

(248) B, D & G

(249) A & B

(250) A & C

3.3 Numeric Entry Questions

(251) 3.09

(252) 133

(253) 30

(254) 40

(255) 45

(256) $\frac{6}{5}$

(257) 9

(258) 19

(259) −4

(260) 89.44

(261) 50.24

(262) 45

(263) $\frac{1}{16}$

(264) $\frac{4}{3}$

(265) 30

(266) −6

(267) 16

(268) 4

(269) 78

(270) 20

(271) 512

(272) $\frac{4}{3}$

(273) 40

(274) $\frac{7}{9}$

(275) $\frac{1}{3}$

(276) 1

(277) 30

(278) 18

(279) 2

(280) 2

(281) 6.32

(282) 75

(283) 31.58

(284) 20

(285) 84

(286) 23

(287) 45.45

(288) 5796

(289) 25.71

(290) 290

(291) 89

(292) 92.5

(293) 72

(294) 0.48

(295) 16

(296) 36

(297) 29

(298) 20

(299) 5

(300) 6

(301) 3000

(302) 10

(303) 7

(304) 10

(305) 6

(306) 10

(307) 2400

(308) 800

(309) 6

(310) 2000

(311) 15

(312) 396

(313) 5000

(314) 15

(315) 1450

(316) 267

(317) 375

(318) 40

(319) 40

(320) 9375

(321) 10

(322) 21

(323) 140

(324) 64

(325) 1000

(326) 9

(327) 8

(328) 1

(329) 3

(330) 15

(331) 20

(332) 24

(333) 1482

(334) 68

(335) 10

(336) 506

(337) 434

(338) 2

(339) 103

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(340) 2

(341) 0

(342) −2

(343) −2

(344) 110

(345) 1000

(346) 250

(347) 2800

(348) 12

(349) 4

(350) 53

(351) 0

(352) 6

(353) 36

(354) 3

(355) 40

(356) $\frac{9}{2}$

(357) 8

(358) 3

(359) 85.71

(360) 13

(361) 7.5

(362) 4

(363) 9

(364) 1

(365) 3

(366) 9

(367) 3

(368) 14

(369) 16

(370) 21

(371) 64

(372) 8

(373) 4

(374) 6

(375) 11

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3.4 Quantitative Comparison Questions

(376) A
(377) B
(378) D
(379) D
(380) A
(381) D
(382) C
(383) A
(384) C
(385) A
(386) A
(387) C
(388) D
(389) A
(390) C
(391) A
(392) A
(393) A
(394) B
(395) C
(396) A
(397) B
(398) D
(399) A
(400) C
(401) C
(402) A
(403) B
(404) C
(405) B
(406) A
(407) C
(408) B
(409) B
(410) B
(411) A
(412) A
(413) C
(414) B
(415) A
(416) B
(417) C
(418) C
(419) A
(420) D
(421) A
(422) A
(423) B
(424) D
(425) C
(426) B
(427) B
(428) A
(429) A
(430) D
(431) D
(432) D
(433) C
(434) B
(435) B
(436) A
(437) A
(438) A
(439) A
(440) B
(441) C
(442) C
(443) B
(444) B
(445) B
(446) B
(447) D
(448) A
(449) B
(450) A
(451) A
(452) C
(453) A
(454) B
(455) C
(456) A
(457) B
(458) B
(459) C
(460) A
(461) A
(462) A
(463) B
(464) C

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(465) B

(466) A

(467) A

(468) A

(469) C

(470) A

(471) B

(472) C

(473) D

(474) C

(475) A

(476) C

(477) C

(478) D

(479) A

(480) D

(481) A

(482) B

(483) B

(484) C

(485) C

(486) D

(487) D

(488) B

(489) C

(490) C

(491) D

(492) C

(493) A

(494) C

(495) A

(496) C

(497) C

(498) B

(499) C

(500) A

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Chapter 4

Solutions

4.1 Multiple Choice Questions

1. Here given expression is in the form of $a^2 - b^2 = (a + b)(a - b)$

=> $99,998^2 - 2^2 = (99,998 + 2)(99,998 - 2)$

=> $100,000 \times (100,000 - 4) = 10^5 \times (10^5 - 4)$

The correct answer is Option E.

2. $\left(3\sqrt{80} + \frac{3}{9 + 4\sqrt{5}}\right)^{\frac{1}{2}}$

=> $\left(3\sqrt{16 \times 5} + \frac{3}{9 + 4\sqrt{5}}\right)^{\frac{1}{2}}$

By applying rationalization:

=> $\left[12\sqrt{5} + \frac{3(9 - 4\sqrt{5})}{[(9 + 4\sqrt{5})(9 - 4\sqrt{5})]}\right]^{\frac{1}{2}}$

=> $\left[12\sqrt{5} + \frac{27 - 12\sqrt{5}}{(9)^2 - (4\sqrt{5})^2}\right]^{\frac{1}{2}}$

=> $\left[12\sqrt{5} + \frac{27 - 12\sqrt{5}}{81 - 80}\right]^{\frac{1}{2}}$

=> $(12\sqrt{5} + 27 - 12\sqrt{5})^{\frac{1}{2}}$

=> $\sqrt{27}$

=> $3\sqrt{3}$

The correct answer is Option D.

Alternate approach:

$\left(3\sqrt{80} + \frac{3}{9 + 4\sqrt{5}}\right)^{\frac{1}{2}}$

$\left(3 \times 8.94 + \frac{3}{9 + 4 \times 2.24}\right)^{\frac{1}{2}}$; use calculator

$\left(26.82 + \frac{3}{17.944}\right)^{\frac{1}{2}}$

$(26.82 + 0.1672)^{\frac{1}{2}}$

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$(26.99)^{\frac{1}{2}}$

=> 5.19

Option D: $3\sqrt{3} = 3 \times 1.732 = 5.19$: correct answer.

3. $\left(\frac{1}{4-\sqrt{15}}\right)^2 = ?$

By rationalization,

$$=> \left(\frac{(4+\sqrt{15}) \times 1}{(4-\sqrt{15})\,(4+\sqrt{15})}\right)^2$$

$$=> \left(\frac{4+\sqrt{15}}{(4)^2-(\sqrt{15})^2}\right)^2$$

$$=> \left(\frac{4+\sqrt{15}}{16-15}\right)^2$$

$$=> (4+\sqrt{15})^2$$

$$=> 16+15+8\sqrt{15}$$

$$=> 31+8\sqrt{15}$$

The correct answer is Option C.

Alternate approach:

$$\left(\frac{1}{4-\sqrt{15}}\right)^2$$

$$= \left(\frac{1}{4-3.87}\right)^2\text{; use calculator}$$

$$= \left(\frac{1}{0.13}\right)^2$$

$$= (7.69)^2$$

$$= 59.14$$

We see that Option C: $31 + 8\sqrt{15} = 59.14$, the correct answer.

4. $5^3 + 5^3 + 5^3 + 5^3 + 5^3 = ?$

=> $5^3 (1 + 1 + 1 + 1 + 1)$

=> $5^3 \times 5$

=> 5^4

The correct answer is Option A.

5. $\dfrac{\left(\frac{1}{3}\right)^{-2}}{3^{-2}} = ?$

Here we know that $a^{-m} = \dfrac{1}{a^m}$

=> $\dfrac{3^2}{\left(\frac{1}{3}\right)^2}$

=> $\dfrac{9}{\frac{1}{9}}$

=> 81

The correct answer is Option B.

6. The length of the smallest sized shirt = 20 inches.

The length of the largest (15^{th}) size shirt = 40 inches = 20×2 = 2 times length of the smallest size shirt.

Let the ratio of length of a particular size shirt to that of its next larger size shirt = r.

For the terms in Geometric Progression, n^{th} term is given by $a \times r^{(n-1)}$, where a = first term

Thus, the length of the 15^{th} size shirt = $20 \times r^{14}$ inches.

Thus, we have

$20 \times r^{14} = 40$

=> $r^{14} = 2$

=> $r = \sqrt[14]{2}$

Thus, the length of the 8^{th} size shirt = $20 \times r^7$

$= 20 \times \left(\sqrt[14]{2}\right)^7 = 20 \times 2^{\left(\frac{7}{14}\right)} = 20\sqrt{2}$ inches.

The correct answer is Option A.

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7. We have:

$\sqrt{2k+3} = k+2$

$=> 2k + 3 = (k+2)^2$; squaring both sides of the equation

$=> 2k + 3 = k^2 + 4k + 4$

$=> k^2 + 2k + 1 = 0$

$=> (k+1)^2 = 0$

$=> k + 1 = 0$

$=> k = -1$

Checking the statements:

I. $k^k = (-1)^{-1} = \frac{1}{(-1)} = -1 = k$; True
II. $|k| = |-1| = 1 = -(-1) = -k$; True
III. $k^0 = (-1)^0 = 1 = -(-1) = -k$; True

The correct answer is Option E.

8. We know that k is a two-digit positive integer with tens digit x and units digit y.

Thus, we have

$k = 10x + y$

$=> k^2 = (10x + y)^2$

$= 100x^2 + 20xy + y^2$

Thus, we have

$k^2 - (x+y)^2$

$= 100x^2 + 20xy + y^2 - (x^2 + 2xy + y^2)$

$= 99x^2 + 18xy$

$= 9x(11x + 2y)$

Thus, we see that $k^2 - (x+y)^2$ is definitely divisible by $9x$.

The correct answer is Option C.

9. Let us simplify $0.0025 \times 0.025 \times 0.00025$

$= \left(25 \times 10^{-4}\right) \times \left(25 \times 10^{-3}\right) \times \left(25 \times 10^{-5}\right)$

$= 25^3 \times 10^{-4-3-5}$

$= 5^6 \times 10^{-12}$

$= 5^6 \times (2 \times 5)^{-12}$

$= 5^6 \times 2^{-12} \times 5^{-12}$

$= 2^{-12} \times 5^{-6}$

We know that $0.0025 \times 0.025 \times 0.00025 \times 2^k \times 5^l$ is an integer

=> $2^{-12} \times 5^{-6} \times 2^k \times 5^l$ is an integer

=> $2^{k-12} \times 5^{l-6}$ is an integer

=> If $k = 12$ & $l = 6$, the integer value is $2^0 \times 5^0 = 1$ (the smallest positive integer)

Thus, $k = 12$ & $l = 6$ are the minimum possible values.

Thus, the minimum value of $(k + l) = 12 + 6 = 18$.

The correct answer is Option D.

10. Working with the options one at a time:

Option A: $k^2 - 6k + 8 < 0$

=> $(k - 2)(k - 4) < 0$ => $2 < k < 8$

Since we have $k < 1$, it does not satisfy the above condition. – Incorrect

Option B: $k^2 - 2k + 1 < 0$

=> $(k - 1)^2 < 0$

This is never possible for any given value of x since the square of any number can never be negative. – Incorrect

Option C: $|k| - k^2 < 0$

For a fractional value of k, say $k = \frac{1}{2}$, we have

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$k^2 = \frac{1}{4}$ and $|k| = \frac{1}{2}$

$=> |k| - k^2 \nless 0$ – Incorrect

Option D: $2^k - 3 < 0$

$=> 2^k < 3$

Since $k < 1$, possible values of k are either fractions between 0 and 1 or negative numbers.

For fractional values of k, we definitely have $2^k < 3$

For negative values of k, for example:

If $k = -\frac{1}{2} => 2^k = 2^{(-\frac{1}{2})} = \frac{1}{\sqrt{2}} < 3$

If $k = -2 => 2^k = 2^{-2} = \frac{1}{4} < 3$

Thus, $2^k < 3$ – Correct

Option E: Since even number index of any integer is always positive, k^4 can never be negative. – Incorrect

The correct answer is Option D.

11. The cost of repairing the current machine = \$1,200

The cost of new machine = \$2,800

Since the new machine lasts for two years, the average cost per year = $\$\frac{2,800}{2}$ = \$1,400.

Thus, the required percentage = $\frac{1,400 - 1,200}{1,200} \times 100\%$ = 16.67%.

The correct answer is Option C.

12. Let the price of the item be $\$x$.

So, the tax is applicable on $\$(x - 200)$

Thus, tax paid = 10% of $\$\ (x - 200)$

$=> \$\left(\frac{10}{100} \times (x - 200)\right)$

$=> \$\left(\frac{x - 200}{10}\right)$, which is equals to \$10.

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Thus, we have

$$\frac{x - 200}{10} = 10$$

=> x = \$300

The correct answer is Option C.

Alternate approach:

We see that tax paid = 10% of excess amount = \$10

=> Excess amount = \$100

Thus, the price = \$(200 + 100) = \$300

13. Tax paid on \$50 = \$0.82

Thus, a tax which thrice as much as the above, would be \$$(0.82 \times 3)$ = \$2.46 per \$50.

Thus, the new tax per \$100 = 2 $\times$ Tax paid per \$50

=> \$$(2.46 \times 2)$ = \$4.92

Thus, this tax, expressed as a percentage = $\frac{4.92}{100} \times 100 = 4.92\%$.

The correct answer is Option C.

14. Cyclist P increased his speed from 10 mph to 25 mph.

Total increase in speed of Cyclist P = 25 − 10 = 15 mph

Thus, the percentage increase in speed of Cyclist P = $\frac{15}{10} \times 100 = 150\%$.

Cyclist Q increased his speed from 8 mph to 24 mph.

Total increase in speed of Cyclist Q = 24 − 8 = 16 mph

Thus, the percentage increase in speed of Cyclist Q = $\frac{16}{8} \times 100 = 200\%$.

Apparently it seems that the required answer is simply 200% − 150% = 50%.

However, it is not so since we are here asked to find percent change not percent point change.

Here, the absolute change is 200% − 150% = 50%, which we are going to compare with percent change in the speed of Cyclist P, which is 150%.

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The required percent = $\frac{50}{150} \times 100 = 33.33\%$.

The correct answer is Option A.

15. Population of Country X = 120,108,000 =~120,000,000.

Land area of Country X = 2,998,000 square kilometers =~3,000,000 square kilometers.

Thus, population density of Country X = $\frac{120,000,000}{3,000,000} = 40$.

Population of Country Y = 200,323,000 =~200,000,000.

Land area of Country Y = 7,899,000 square kilometers =~8,000,000 square kilometers.

Thus, population density of Country Y = $\frac{200,000,000}{8,000,000} = 25$.

Thus, the percent by which population density of Country X is greater than that of Country Y $= \left(\frac{40-25}{25}\right) \times 100$

$= 60\%$.

Note: An exact calculation would lead to:

Population density of Country X = $\frac{120,108,000}{2,998,000} = 40.06$.

Thus, population density of Country Y = $\frac{200,323,000}{7,899,000} = 25.36$.

Thus, required percent difference = $\frac{40.06-25.36}{25.36} \times 100 = 57.96\% =\sim 60\%$.

Thus, the above approximations are justified. A close option (50%) to 57.96% is too far compared to 60%; thus Option B cannot be the answer.

The correct answer is Option A.

16. Since $\left(\frac{4}{5}\right)^{\text{th}}$ of the stock was sold, the remaining $\left(1-\frac{4}{5}\right) = \left(\frac{1}{5}\right)^{\text{th}}$ of the stock was not sold.

Thus, $\left(\frac{1}{5}\right)^{\text{th}}$ of the total stock = 100.

Thus, $\left(\frac{4}{5}\right)^{\text{th}}$ of the total stock = $4 \times 100 = 400$.

Thus, 400 items were sold each at \$3.

Thus, total amount received = $\$\,(400 \times 3) = \$1,200$.

The correct answer is Option B.

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17. We know that the shopkeeper procured 1,600 boxes a cost of \$10 per box.

Thus, the total cost of the boxes = $\$ \left(10 \times 1,600\right) = \$16,000$.

Selling price of $\left(\frac{3}{4} \text{ of } 1,600\right)$ or 1200 boxes = $\$ \left(1.50 \times 10\right) = \$15$ per box.

Thus, selling price of 1,200 boxes = $\$ \left(15 \times 1,200\right) = \$18,000 \ldots$(i)

Selling price of $(1600 - 1,200)$ or 400 boxes = $\$ \left(100 - 25\right) \%$ of 10 = $\$ \left(\frac{75}{100} \times 10\right) = \$7.50$ per box.

Thus, selling price of 400 boxes = $\$ \left(7.50 \times 400\right) = \$3,000 \ldots$(ii)

Thus, from (i) and (ii):

Total selling price = $\$ \left(18,000 + 3,000\right) = \$21,000$.

Thus, gross profit = Total selling price − Total procurement price

$= \$ \left(21,000 - 16,000\right) = \$5,000$.

The correct answer is Option A.

18. Selling price of a brand X cake = \$20.

Thus, 40% of the selling price of a brand Y cake = \$20.

Thus, selling price of a brand Y cake = $\$ \left(20 \times \frac{100}{40}\right) = \$50$.

Total number of cakes sold by the merchant = 900.

Thus, the number of brand Y cakes sold = $\frac{2}{3} \times 900 = 600$.

Thus, the number of brand X cakes sold = 900 − 600 = 300.

Thus, total revenue from the sale of all cakes

= Revenue from brand X cakes + Revenue from brand Y cakes

$= \$ \left(20 \times 300\right) + \$ \left(50 \times 600\right)$

$= \$ \left(6,000 + 30,000\right)$

$= \$36,000$

The correct answer is Option D.

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19. Purchase price of the consignment = \$800.

Percent profit made on the Purchase price = 30%.

Thus, selling price of the consignment

$= (100 + 30)\,\%$ of \$800

$= \$\left(\frac{130}{100} \times 800\right)$

$= \$1,040$

This selling price is 20% less than the marked price.

Thus, $(100 - 20)\,\%$ = 80% of the marked price is equal to the selling price of \$1,040.

Thus, the marked price

$= \$\left(1,040 \times \frac{100}{80}\right)$

$= \$1,300$

The correct answer is Option C.

20. Cost of production of each unit = \$2.50

Selling price of each unit = \$4.50

Thus, margin on each unit = $\$\,(4.50 - 2.50) = \$2.00$

Investment made on the machines = \$10,000.

Thus, number of units required to be sold to recover the investment on machines

$= \$\frac{10,000}{2}$

$= \$5,000$

The correct answer is Option D.

Alternate approach:

Let the number of required units = n

Thus, total cost = $\$\,(10,000 + 2.5n)$

Total selling price = $\$4.5n$

Thus, we have

$10,000 + 2.5n = 4.5n$

$=> n = 5,000$

21. Initial selling price of the commodity

= (100 + 30) % of the cost of the commodity

= 130% of the cost of the commodity

New selling price of the commodity

= (100 + 40) % of the cost of the commodity

= 140% of the cost of the commodity

Thus, difference between the above two selling prices

= (140% – 130%) of the cost of the commodity

= 10% of the cost of the commodity

Since the difference between the two selling prices is $100, we have

10% of the cost of the commodity = $100

=> Cost of the commodity = $\$\left(100 \times \frac{100}{10}\right) = \$1,000$

Thus, the initial selling price of the commodity

= 130% of the cost of the commodity

$= \$\left(1,000 \times \frac{130}{100}\right)$

= $1,300

The correct answer is Option C.

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22. Total pens produced = 1,000.

Price of each of the first 200 pens = \$6.00

Thus, total cost of the first 200 pens = $\$\,(6 \times 200) = \$1,200$.

Price of each of the remaining 800 pens = \$4.50

Thus, total cost of the remaining 800 pens = $\$\,(4.50 \times 800) = \$3,600$.

Thus, total cost of 1,000 pens = $\$\,(1,200 + 3,600) = \$4,800$.

Total selling price of the 1,000 pens sold at \$9.00 each = $\$\,(9 \times 1,000) = \$9,000$.

Thus, gross profit = Total selling price − Total cost price

= \$9,000 − \$4,800

= \$4,200

The correct answer is Option C.

23. Initial selling price of the item = \$60.

Since the initial percent profit was 20% of the cost, we have

$(100 + 20)\,\%$ of the cost of the item = \$60

$=>$ Cost of the item = $\$\left(\frac{60}{120} \times 100\right) = \$50$

Increased selling price of the item = \$65.

Thus, the required percent profit

$$= \left(\frac{\text{Selling price} - \text{Cost price}}{\text{Cost price}}\right) \times 100$$

$$= \left(\frac{65 - 50}{50}\right) \times 100$$

$$= \frac{15}{50} \times 100$$

= 30%

The correct answer is Option E.

24. Number of teachers = $\frac{\text{Total annual salaries}}{\text{Average annual salaries}} = \frac{3,780,000}{42,000} = 90$

=> Number of students = $\frac{25}{2} \times 90 = 1,125$

The correct answer is Option C.

25. Let the number of boys and girls be b and g, respectively.

Total score of boys = $65b$.

Total score of girls = $80g$.

Thus, total score of the class = $(65b + 80g)$.

Thus, average score of the class

$$= \left(\frac{\text{Total score of the class}}{\text{Total number of students}}\right) = \left(\frac{65b + 80g}{b + g}\right).$$

Thus, we have

$$\left(\frac{65b + 80g}{b + g}\right) = 70$$

=> $65b + 80g = 70b + 70g$

=> $10g = 5b$

=> $\frac{b}{g} = \frac{2}{1}$

We cannot get the absolute number of boys and girls. There are infinite number of possibilities.

Among the options, the values that bears the ratio of 2 : 1 would be the correct answer. We see that only Option D (18; 9) bears that ratio of 2 : 1.

The correct answer is Option D.

26. It is given that the total number of trainees = 30.

Let the number of trainees in group X = n.

Thus, the number of trainees in group Y = $(30 - n)$.

Total score of all trainees in the two groups combined = $76n + 70(30 - n)$.

Thus, average score considering all trainees

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$= \dfrac{\text{Total Score}}{\text{Total number of trainees}} = \dfrac{76n + 70\,(30 - n)}{30}$

Thus, we have

$\dfrac{76n + 70\,(30 - n)}{30} = 74$

$=> 6n + 2,100 = 2,220$

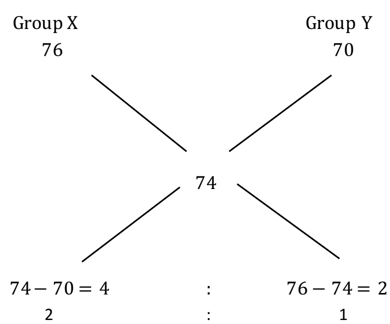
$=> 6n = 120$

$=> n = 20$

The correct answer is Option C.

Alternate approach:

We can solve this using the method of alligation as shown below:



Thus, ratio of the number of trainees in group X to that in group Y = 2 : 1.

Thus, number of trainees in group X = $\left(\dfrac{2}{2+1}\right) \times 30 = 20$.

27. The store has $(500 + 200) = 700$ kgs of tea.

Amount of dust in the first stock = 30% of 500 = 150 kgs.

Amount of dust in the second stock = 40% of 200 = 80 kgs.

Thus, the total amount of dust = 150 + 80 = 230 kgs.

Thus, the percent of dust in the total stock

$= \dfrac{230}{700} \times 100\%$

$= 32.85\% =\approx 33\%$

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The correct answer is Option B.

Alternate approach:

The required percent is the weighted average of the percentages of the above two stocks

$$= \left(\frac{\left(500 \times \frac{30}{100}\right) + \left(200 \times \frac{40}{100}\right)}{500 + 200}\right) \times 100\%$$

$$= \left(\frac{150 + 80}{700}\right) \times 100\%$$

$$= \left(\frac{230}{700}\right) \times 100\%$$

$$= \approx 33\%$$

28. Let the number of students in the sections P, Q, R and S be p, q, r and s, respectively.

Thus, the average weight of all students together in the four sections

$$= \left(\frac{\text{Total weight of all the students combined}}{\text{Total number of students}}\right)$$

$$= \left(\frac{45 \times p + 50 \times q + 55 \times r + 65 \times s}{p + q + r + s}\right) \text{lb.}$$

Thus, we have

$$\left(\frac{45 \times p + 50 \times q + 55 \times r + 65 \times s}{p + q + r + s}\right) = 55$$

$=> 45p + 50q + 55r + 65s = 55p + 55q + 55r + 55s$

$=> 10p + 5q = 10s$

$=> 2p + q = 2s$

Since we need to maximize r, we need to find the minimum possible values of p, q and s so that the above equation holds true.

Since the RHS is $2s$, it is even.
Also, in the LHS, $2p$ is even.
Thus, q must be even.

Since the smallest even number that we can consider for q is 2, as we have at least one student in each section. Thus, we have $q = 2$.

Thus, the equation gets modified to:

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$2p + 2 = 2s$

$=> p + 1 = s$

Thus, we use the minimum possible values: $p = 1, s = 2$.

Thus, we have $p = 1, q = 2$ and $s = 2$.

Since there are a total of 40 students in all sections combined, the maximum value of students in section R = $r = 40 - (p + q + s)$

$= 40 - 5 = 35$

The correct answer is Option D.

29. Let the seven numbers be: a, b, c, d, e, f and g.

Thus, we have

$$\frac{a + b + c + d + e + f + g}{7} = 20$$

$=> a + b + c + d + e + f + g = 140 \ldots$(i)

Since the average of the first four numbers is 19, we have

$$\frac{a + b + c + d}{4} = 19$$

$=> a + b + c + d = 76 \ldots$(ii)

Since the average of the last four numbers is 24, we have

$$\frac{d + e + f + g}{4} = 24$$

$=> d + e + f + g = 96 \ldots$(iii)

Adding (ii) and (iii), we have

$a + b + c + 2d + e + f + g = 172 \ldots$(iv)

Subtracting (i) from (iv), we have

$d = 172 - 140 = 32$

Thus, the value of the fourth number is 32.

The correct answer is Option C.

30. The expense on rent = \$250.

The total expense for 10 guests = \$650.

Thus, the variable component of the total expense for 10 guests = \$ (650 − 250) = \$400.

Thus, variable expense per guest = $\$\frac{400}{10} = \$40$.

Thus, the variable component of the total expense for 20 guests = \$ (40×20) = \$800.

The total expense for 20 guests = \$ (250 + 800) = \$1,050.

The correct answer is Option C.

31. We know that the beaker was filled with 40 liters of water and liquid chemical with the components in the ratio 3 : 5, respectively.

Initial quantity of water = $\left(\frac{3}{3+5}\right) \times 40 = 15$ liters.

Initial quantity of liquid chemical = $\left(\frac{5}{3+5}\right) \times 40 = 25$ liters.

Amount of water evaporated per day = 2% of 15= 0.3 liters.

Thus, total amount of water evaporated in 10 days = $0.3 \times 10 = 3$ liters.

Amount of liquid chemical evaporated per day = 5% of 25 = 1.25 liters.

Thus, total amount of liquid chemical evaporated in 10 days = $1.25 \times 10 = 12.5$ liters.

Thus, total quantity of mixture evaporated in 10 days = 3 + 12.5 = 15.5 liters.

Thus, the percent of the original amount of mixture evaporated

$= \frac{15.5}{40} \times 100$

= 38.75%.

The correct answer is Option C.

32. Fraction of total dolls that are Barbie = $\frac{3}{5} \ldots$(i)

Thus, fraction of total dolls that are non-Barbie = $1 - \frac{3}{5} = \frac{2}{5} \ldots$(ii)

Fraction of Barbies purchased before the age of 10 = $\frac{4}{7}$

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Thus, fraction of Barbies purchased at the age of 10 or later = $1 - \frac{4}{7} = \frac{3}{7} \ldots$(iii)

Thus, from (i) and (iii), we have

Fraction of total dolls that are Barbies and purchased at the age of 10 or later

$= \frac{3}{5} \times \frac{3}{7} = \frac{9}{35}$

Thus, we have

$\frac{9}{35}$ of the total dolls = 90

=> Total dolls = $90 \times \frac{35}{9} = 10 \times 35 = 350$

Thus, from (ii):

Number of non-Barbie dolls = $350 \times \frac{2}{5} = 140$

The correct answer is Option C.

33. We know that, for a ratio $0 < \frac{x}{y} < 1$, if k is a positive number:

- $0 < \frac{x}{y} < \left(\frac{x+k}{y+k}\right) < 1$ ... (i)
- $0 < \left(\frac{x-k}{y-k}\right) < \frac{x}{y} < 1$ ... (ii)

Also, for a ratio $\frac{x}{y} > 1$, if k is a positive number:

- $1 < \left(\frac{x+k}{y+k}\right) < \frac{x}{y}$ ... (iii)
- $1 < \frac{x}{y} < \left(\frac{x-k}{y-k}\right)$ ... (iv)

In the above problem, we have

The given ratio of ages of A and B = $\frac{7}{11}$ (< 1) =~ 0.63

Thus, after 5 years, both their ages would increase by 5.

Hence, the final ratio must be greater than $\frac{7}{11}$ (from relation (i) above).

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Working with the options, we have

Option A:

$\frac{1}{3} = 0.3 < 0.63$ – Does not satisfy

Option B:

$\frac{9}{20} = 0.45 < 0.63$ – Does not satisfy

Option C:

$\frac{4}{15} = 0.26 < 0.63$ – Does not satisfy

Option D:

$\frac{3}{5} = 0.6 < 0.63$ – Does not satisfy

Option E:

$\frac{2}{3} = 0.66 > 0.63$ – Satisfies

The correct answer is Option E.

34. Since the problem asks us to find a fraction value, we can assume any suitable value of the total number of phones and the time taken to produce a feature phone since the initial value does not affect the final answer.

Let the total number of phones be 5.

Thus, the number of feature phones = $\frac{2}{5} \times 5 = 2$.

Number of smartphones = $5 - 2 = 3$.

Let the time taken to produce a feature phone = 5 hours.

Thus, the time taken to produce a smartphone = $\frac{8}{5} \times 5 = 8$ hours.

Thus, total time taken to produce smartphones = $3 \times 8 = 24$ hours.

Total time taken to produce feature phones = $2 \times 5 = 10$ hours.

Thus, total time taken to produce all the phones = $24 + 10 = 34$ hours.

Thus, the required fraction = $\frac{24}{34} = \frac{12}{17}$

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The correct answer is Option C.

35. We have:

$$\frac{\text{Number of shirts}}{\text{Number of trousers}} = \frac{4}{5} \ldots \text{(i)}$$

$$\frac{\text{Number of jackets}}{\text{Number of shirts}} = \frac{3}{8} \ldots \text{(ii)}$$

$$\frac{\text{Number of sweaters}}{\text{Number of trousers}} = \frac{6}{5}$$

Taking reciprocal on both the sides:

$$=> \frac{\text{Number of trousers}}{\text{Number of sweaters}} = \frac{5}{6} \ldots \text{(iii)}$$

Thus, from the above three equations, we have

$$\frac{\text{Number of jackets}}{\text{Number of sweaters}}$$

$$= \left(\frac{\text{Number of jackets}}{\text{Number of shirts}}\right) \times \left(\frac{\text{Number of shirts}}{\text{Number of trousers}}\right) \times \left(\frac{\text{Number of trousers}}{\text{Number of sweaters}}\right)$$

$$= \frac{3}{8} \times \frac{4}{5} \times \frac{5}{6}$$

$$= \frac{1}{4}$$

The correct answer is Option C.

36. Since the question asks about a fraction value, we can choose any suitable initial value of the total number of members for ease of calculation, since the initial value will not affect the final answer.

Let the number of members = 100.

Thus, the number of male members = $\frac{3}{5} \times 100 = 60$.

Number of female members = $100 - 60 = 40$.

Fraction of male members who attended the prayer = $\frac{3}{5}$

Thus, the fraction of male members who did not attend the prayer = $\left(1 - \frac{3}{5}\right) = \frac{2}{5}$

Thus, the number of male members who did not attend the prayer = $\frac{2}{5} \times 60 = 24$.

Fraction of female members who attended the prayer = $\frac{7}{10}$

Thus, the fraction of female members who did not attend the prayer = $\left(1 - \frac{7}{10}\right) = \frac{3}{10}$

Thus, the number of female members who did not attend the prayer = $\frac{3}{10} \times 40 = 12$.

Thus, total number of members who did not attend the prayer = 24 + 12 = 36.

Thus, the required fraction = $\frac{24}{36} = \frac{2}{3}$

The correct answer is Option C.

37. Total amount to be paid = \$135.

Amount paid by Betty = \$51.

Thus, the amount paid by Amy and Chris = \$ (135 − 51) = \$84.

We know that Amy paid $\left(\frac{3}{5}\right)^{\text{th}}$ of what Chris paid.

Thus, ratio of the amounts paid by Amy and Chris = 3 : 5.

Thus, we need to divide \$84 in the ratio 3 : 5.

Thus, amount paid by Chris

$= \$\left(\frac{5}{3+5} \times 84\right) = \$\frac{105}{2}$

Thus, fraction of the total amount paid by Chris

$= \frac{\frac{105}{2}}{135} = \frac{7}{18}$

The correct answer is Option D.

38. Let the the average speed for the trip be S.

T_1 = Time required to travel 900 miles with speed $S = \frac{900}{S}$ hours

S is increased by 10 miles; so new speed = $(S + 10)$ mph

T_2 = Time required to travel 900 miles with speed $(S + 10) = \frac{900}{S + 10}$ hours

Given that the difference between T_1 and T_2 is 1 hour

$=> \frac{900}{S} - \frac{900}{S + 10} = 1$

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$=> 900\left(\frac{1}{S} - \frac{1}{S+10}\right) = 1$

$=> \frac{S+10-S}{S(S+10)} = \frac{1}{900}$

$=> 10 \times 900 = S^2 + 10S$

$=> S^2 + 10S - 9000 = 0$

Roots of above quadratic equations are '90' and '−100'.

Since speed cannot be negative, '−100' is ignored, so $S = 90$ mph.

The correct answer is Option E.

39. Truck traveled 4 miles less per gallon on the state highway compared to on the national highway.

Let's consider if the truck travels x miles per gallon on the national highway then the truck travels $(x - 4)$ miles per gallon on the state highway.

Capacity of the full tank of diesel on the national highway = $\frac{336}{x}$ gallons

Capacity of the full tank of diesel on the state highway = $\frac{224}{x-6}$ gallons

Since the above fractions must be equal, we have

$\frac{336}{x} = \frac{224}{x-4}$

$=> \frac{3}{x} = \frac{2}{x-4}$

$=> 3x - 12 = 2x$

$=> x = 12$

As the truck travels $(x - 4)$ miles per gallon on the state highway, required answer is $12 - 4 = 8$.

The correct answer is Option B.

40. Time required to travel 10 miles at speed 60 miles per hour

$= \frac{10}{60}$ hours $= \frac{10}{60} \times 60$ minutes = 10 minutes

Now new time required = 10 + 5 = 15 minutes = $\frac{15}{60} = \frac{1}{4}$ hours.

Speed = $\frac{\text{Distance}}{\text{Time}}$

$= \dfrac{10 \text{ miles}}{15 \text{ Minutes}} = \dfrac{10}{\frac{1}{4}} = 40$ miles per hour

The correct answer is Option B.

41. Speed for the first 10-minutes interval = 30 miles per hour

Speed for the second 10-minutes interval = 40 miles per hour

Speed for the third 10-minute interval = 50 miles per hour

Speed for the fourth 10-minute interval = 60 miles per hour

Distance travelled for the fourth 10-minute interval = $\dfrac{60}{60} \times 10 = 10$ miles

The correct answer is Option D.

42. Time taken for the onward journey = $\frac{600}{400} = \frac{3}{2}$ hours.

Time taken for the return journey = $\frac{600}{500} = \frac{6}{5}$ hours.

Thus, total time taken for the round trip = $\dfrac{3}{2} + \dfrac{6}{5} = \dfrac{27}{10}$ hours.

Total distance travelled = $2 \times 600 = 1200$ miles.

Thus, average speed = $\dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{1200}{\frac{27}{10}} = 1200 \times \dfrac{10}{27} = 1000 \times \dfrac{4}{9} = 1000 \times 0.444 =\approx 444$ miles/hr.

The correct answer is Option B.

Alternate approach:

Since the distance travelled for the onward and the return journey is the same, we have

Average speed $= \dfrac{2 \times \text{Speed}_1 \times \text{Speed}_2}{\text{Speed}_1 + \text{Speed}_2} = \dfrac{2 \times 400 \times 500}{400 + 500} = \dfrac{2 \times 400 \times 500}{900} = \dfrac{4000}{9} =\approx 444$ miles/hr.

43. Number of units produced per hour by the first machine = 2,000.

Thus, number of units produced per hour by the second machine = 6,000
(Since it is thrice as efficient as the first machine)

Since the second machine works 15 hours a day, number of units produced in a day

$= 6,000 \times 15 = 90,000$

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Thus, number of units produced in 10 days = $90,000 \times 10$ = 900,000

The correct answer is Option E.

44. According to the given data, the pump filled $\left(\frac{5}{6} - \frac{1}{2}\right) = \frac{1}{3}$ of the pool in $2\frac{1}{3} = \frac{7}{3}$ hours

Thus, time taken by the pump to fill the entire pool = $\frac{\left(\frac{7}{3}\right)}{\left(\frac{1}{3}\right)} = \frac{7}{3} \times \frac{3}{1} = 7$ hours

The time taken by the pump to fill the empty pool completely = 7 hours

The correct answer is Option C.

45. Let the volume of the cistern = LCM (20, 30) = 60 liters.

The first tap can fill the cistern in 20 minutes.
Thus, rate of the first tap = $\frac{60}{20}$ = 3 liters per minute.

The second tap can fill the cistern in 30 minutes.
Thus, rate of the second tap = $\frac{60}{30}$ = 2 liters per minute.

The first tap was open for the entire 15 minutes in which it filled = $15 \times 3 = 45$ liters.

Thus, the remaining $(60 - 45) = 15$ liters was filled by the second tap.

Time taken for the second tap to fill 15 liters = $\frac{15}{2}$ = 7.5 minutes.

Thus, of the total 15 minutes, the second tap was open for 7.5 minutes.

Thus, the first tap alone was open for $(15 - 7.5) = 7.5$ minutes.

Thus, $x = 7.5$ minutes

The correct answer is Option B.

46. The emptying pipe can empty pool which is $\frac{3}{4}$ full in 9 hours.

Thus, time taken to empty the entire pool = $9 \times \frac{4}{3}$ = 12 hours.

It is given that capacity of swimming pool is 5,760 gallons.

Thus, the rate at which the emptying pipe removes water
$= \frac{5,760}{12}$ = 480 gallons per hour.

The rate at which the pool can be filled

= 12 gallons per minute

= $12 \times 60 = 720$ gallons per hour.

Thus, the effective filling rate when both filling and emptying occur simultaneously
= $720 - 480 = 240$ gallons per hour.

Since we need to fill only half the pool, the volume required to be filled
= $\frac{5,760}{2} = 2,880$ gallons.

Thus, time required = $\frac{2,880}{240} = 12$ hours.

The correct answer is Option B.

47. Amount of solution leaked out in q hours = p liters.

Thus, the amount of solution leaked out in 1 hour = $\frac{p}{q}$ liters.

Thus, the amount of solution leaked out in r hours = $\frac{pr}{q}$ liters.

Cost of 1 liter of the solution = \$10.

Thus, cost of the solution leaked out in r hours = $\$\left(10 \times \frac{pr}{q}\right) = \$\left(\frac{10pr}{q}\right)$.

The correct answer is Option D.

48. We know that the measurements on the X-scale of 10 and 30 correspond to measurements on the Y-scale of 30 and 60, respectively.

Thus, $60 - 30 = 30$ divisions of the Y-scale equals $30 - 10 = 20$ divisions of the X-scale

Thus, 1 division of the Y-scale equals $\frac{20}{30} = \frac{2}{3}$ divisions of the X-scale.

Thus, a measurement of 90 on the Y-scale corresponds to $90 - 60 = 30$ divisions above the measurement of 60 on the Y-scale.

Now, 30 divisions of the Y-scale equals $\frac{2}{3} \times 30 = 20$ divisions of the X-scale.

Thus, the corresponding measure on the X-scale is 20 divisions above the measurement of 30 (since 60 on the Y-scale corresponds to 30 on the X-scale) = $30 + 20 = 50$.

The correct answer is Option C.

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49. The cost of four pencils = \$ (1.35×4) = \$5.40

The cost of two erasers = \$ (0.30×2) = \$0.60

Thus, total cost = \$(5.40 + 0.60) = \$6.00

Thus, Suzy has one-third of the above amount = \$ $\left(\frac{6}{3}\right)$ = \$2

The correct answer is Option B.

50. Total increase in population = 378 − 300 = 18 million

Increase in population per month = 30,000

Thus, increase in population per year = $30,000 \times 12 = 360,000 = 0.36$ million

Thus, number of years required for the increase = $\frac{18}{0.36}$ = 50 years

Thus, the population would be 378 million in the year (2012 + 50) = 2062

The correct answer is Option C.

51. The restaurant uses $\frac{1}{2}$ cup milk-cream in each serving of its ice-cream.

Since each carton has $2\frac{1}{2} = \frac{5}{2}$ cups of milk-cream, number of servings of ice-cream possible using one carton = $\frac{\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)} = 5$

Thus, number of cartons required for 98 servings of the ice-cream = $\frac{98}{5} = \approx 19.3$

However, the number of cartons must be an integer.

Thus, the minimum number of cans required is 20.

The correct answer is Option C.

52. We need to minimize the total number of marbles such that each pouch has at least 1 marble.

We know that at the most 4 pouches can have the same number of marbles.

Since we need to minimize the total number of marbles, we must have as many pouches having the same number (minimum possible number, i.e. 1 marble) of marbles as possible.

Thus, for each of the 4 pouches containing an equal number of marbles, we have '1' marble.

Thus, number of marbles in the 4 pouches = $1 \times 4 = 4$.

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Since each of the remaining 4 pouches have a different number of marbles, let us use 2, 3, 4, and 5 marbles for those pouches.

Thus, the total number of marbles = 4 + (2 + 3 + 4 + 5) = 18.

The correct answer is Option C.

53. Amount after three years = $1,200

Amount after five years = $1,500

Thus, interest accumulated in two years = $\$\ (1,500 - 1,200) = \$300$

Thus, interest accumulated per year = $\$\left(\frac{300}{2}\right) = \$150$ (since under simple interest, interest accumulated every year is constant)

Thus, interest accumulated in the first three years = $\$\ (150 \times 3) = \$450$

Thus, principal amount invested = $\$\ (1,200 - 450) = \$750$

Thus, on $750 invested, interest accumulated is $150 every year.

Thus, rate of interest = $\frac{150}{750} \times 100 = 20\%$

The correct answer is Option C.

54. Simple interest accumulated after two years = $180

Thus, simple interest every year = $\$\left(\frac{180}{2}\right) = \$90$ (since under simple interest, interest accumulated every year is constant)

Thus, compound interest accumulated after the first year = $90 (equal to the simple interest accumulated after one year)

Thus, compound interest accumulated in the second year = $\$\ (90 + 18) = \$108$ (since the total compound interest accumulated in two years is $18 more than that under simple interest)

The higher interest in the second year is due to the additional interest on the interest accumulated after one year.

Thus, we can say that interest on $90 for one year = $18

Thus, rate of interest = $\frac{18}{90} \times 100 = 20\%$

The correct answer is Option D.

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55. Let the sum borrowed at 4% and 5% rate of interest be $\$x$ each.

Let the time after which he repays the second sum be t years.

Thus, the time after which he repays the first sum is $\left(t - \frac{1}{2}\right)$ years.

Since the amount to be repaid in either case is the same, the interest accumulated is also equal.

Simple Interest $= \left(\frac{PRT}{100}\right)$, where P =Principal, R = Rate of Interest, and T = Time Interval

Hence, we have

$$\frac{5 \times x \times \left(t - \frac{1}{2}\right)}{100} = \frac{4 \times x \times t}{100}$$

$$=> 5\left(t - \frac{1}{2}\right) = 4t$$

$$=> t = \frac{5}{2}$$

Since the amount to be repaid is \$1,100, we have

Principal amount + Interest accumulated in two years $= 1,100$

$$=> x + \frac{5 \times x \times 2}{100} = 1,100$$

$$=> \frac{11x}{10} = 1,100$$

$$=> x = \$1,000$$

Thus, the total sum borrowed = $x + x = 2x = \$2,000$.

The correct answer is Option D.

56. Total amount invested = $\$100,000$.

Let $\$x$ be invested at 3% and $\$(100,000 - x)$ be invested at 4%

Thus, at the end of 1 year, interest on $\$x$

$$= \$\left(\frac{x \times 3 \times 1}{100}\right) = \$\left(\frac{3x}{100}\right)$$

Also, at the end of 1 year, interest on $\$(100,000 - x)$

$$= \$\left(\frac{(100,000 - x) \times 4 \times 1}{100}\right) = \$\left(\frac{4\,(100,000 - x)}{100}\right)$$

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Since the total interest is \$3,600, we have

$$\frac{3x}{100} + \frac{4\,(100,000 - x)}{100} = 3,600$$

=> $400,000 - x = 360,000$

=> $x = 40,000$

=> $100,000 - x = 60,000$

Thus, the fraction of the total invested at 4% = $\frac{60,000}{100,000} = \frac{3}{5}$

The correct answer is Option D.

Alternate approach:

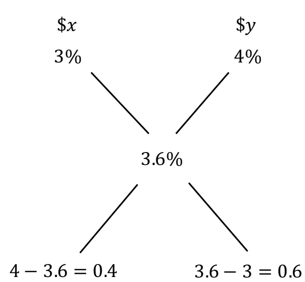
We can use the method of alligation:

Let the amounts invested at 3% and 4% be \$$x$ and \$$y$, respectively.

The total interest on \$100,000 is \$3600.

Thus, the effective rate of interest for the whole = $\frac{3,600}{100,000} \times 100 = 3.6\%$.

Thus, we have



Hence, we have

$$\frac{x}{y} = \frac{0.4}{0.6} = \frac{2}{3}$$

Hence, the required fraction = $\frac{3}{2+3} = \frac{3}{5}$

57. Number of retailers at the beginning of the year = 35% of 120 = $\frac{35}{100} \times 120 = 42$.

Number of retailers after the 24-month period = 25% of 240 = $\frac{25}{100} \times 240 = 60$.

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Thus, increase in the number of retailers = 60 − 42 = 18.

Let the simple annual percent growth rate in the number of retailers be r%.

Thus, in two years (the 24-month period), increase in the number of retailers at r% rate

$$= \left(\frac{42 \times r \times 2}{100}\right)$$

(The value is calculated on 42 since 42 is the value at the start of the year)

Thus, we have

$$\frac{42 \times r \times 2}{100} = 18$$

$$=> r = \frac{18 \times 100}{2 \times 42} = 21.43\%$$

The correct answer is Option B.

58. Number of ways of selecting any 2 apples from 5 apples = $C_2^5 = \frac{5!}{2!(5-2)!} = 10$

Number of ways of selecting 1 spoiled apple from 1 spoiled apple = 1

Number of ways of selecting 1 apple from 4 good apples = 4

So, the number of ways of selecting 1 spoiled and 1 good apple = $1 \times 4 = 4$

So, the required probability = $\frac{\text{Desired outcomes}}{\text{Total outcomes}} = \frac{4}{10} = \frac{2}{5}$

The correct answer is Option D.

59. Population at the start of the experiment = x

Increase in population at the end of the 1st month = $2x$

Thus here we can say that, rate of increase = $\left(\frac{2x}{x}\right) \times 100 = 200\%$

This 200% increase remains same for each of the next 4 months.

Thus, applying the concept of compounding, we have

$$x\left(1 + \frac{200}{100}\right)^5 > 1,000$$

$$=> x > \frac{1,000}{(3)^5}$$

$=> x > \frac{1,000}{243}$

$=> x > 4.115$

Since x must be an integer value (it represents the number of organisms), the minimum possible value of $x = 5$.

The correct answer is Option D.

Alternate approach:

Population at the start of the experiment = x.

Increase in population at the end of the 1^{st} month = $2x$.

Thus, population size at the end of the 1^{st} month = $x + 2x = 3x$.

Increase in population after the 2^{nd} month = $2 \times 3x = 6x$.

Thus, population size at the end of the 2^{nd} month = $3x + 6x = 9x$.

Thus, we observe that the population size triples after every month.

Thus, the population size at the end of the 5^{th} month

= $3\times$ (The population size at the end of the 4^{th} month)

= $3 \times 3\times$ (The population size at the end of the 3^{rd} month)

= $3 \times 3 \times 3\times$ (The population size at the end of the 2^{nd} month) = $3^3 \times 9x$

= $243x$

Thus, we have

$243x > 1,000$

$=> x > \frac{1,000}{243}$

$=> x > 4.115$

Since x must be an integer value (it represents the number of organisms), the minimum possible value of $x = 5$.

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60. We have:

$$f(p) = p^2 + \frac{1}{p^2}$$

$$=> f\left(-\frac{1}{\sqrt{p}}\right) = \left(-\frac{1}{\sqrt{p}}\right)^2 + \frac{1}{\left(-\frac{1}{\sqrt{p}}\right)^2}$$

$$= \frac{1}{p} + p$$

$$=> \left(f\left(-\frac{1}{\sqrt{p}}\right)\right)^2 = \left(\frac{1}{p} + p\right)^2$$

$$= \frac{1}{p^2} + p^2 + 2 \times \frac{1}{p} \times p$$

$$= p^2 + \frac{1}{p^2} + 2 = f(p) + 2$$

The correct answer is Option A.

61. $f(x) = \frac{1}{x}$

$$g(x) = \frac{x}{x^2 + 1}$$

$$=> f(g(x)) = \frac{1}{g(x)} = \frac{1}{\left(\frac{x}{x^2+1}\right)}$$

$$= \frac{x^2 + 1}{x}$$

$$= \frac{(x-1)^2 + 2x}{x}$$

$$= \frac{(x-1)^2}{x} + 2$$

Since $x > 0$, the minimum value of the above expression will occur when the square term becomes zero (since a square term is always non-negative, the minimum possible value occurs when it is zero).

$$=> (x-1)^2 = 0 => x = 1$$

Thus, the minimum value of $f(g(x)) = 0 + 2 = 2$

The correct answer is Option E.

62. Since $C_3^5 = C_x^5$, $x = 3$, but it is given that $x \neq 3$.

We know that $C_n^m = C_{m-n}^m$

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=> $C_3^5 = C_x^5 = C_{5-x}^5$

=> $3 = 5 - x$

=> $x = 2$

The correct answer is Option C.

63. This is a question on permutation with indistinguishable or identical objects.

We know that if there are n objects, out of which p objects are indistinguishable, then

Total number of way of arranging them = $\frac{n!}{p!}$

In this question, let's first assume that we use two BLACK and one RED dot, thus,

Total number of way of arranging them = $\frac{3!}{2!} = 3$.

Similarly, let's now assume that we use two RED and one BLACK dot, thus,

Total number of way of arranging them = $\frac{3!}{2!} = 3$.

There can be two more cases where we use all three BLACK or all three RED.

(Note: the question does not say that both colors must be used)

Total number of codes = 3 + 3 + 2 = 8.

The codes would be: RRB, RBY, RBB, BBR, BRB, BRR, BBB, & RRR.

The correct answer is Option D.

64. There are two different sizes and four different colors of mugs.

Packages having the same size and same color of mugs:

Number of ways in which the size can be chosen = $C_1^2 = \frac{2!}{(2-1)! \times 1!} = 2$ ways

Number of ways in which the color can be chosen = $C_1^4 = \frac{4!}{(4-1)! \times 1!} = 4$ ways

Thus, total number of such packages = $2 \times 4 = 8$.

Packages having the same size and different colors of mugs:

Number of ways in which the size can be chosen = $C_1^2 = \frac{2!}{(2-1)! \times 1!} = 2$ ways

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Number of ways in which the three different colors can be chosen = $C_3^4 = \frac{4!}{(4-3)! \times 3!} = 4$ ways

Thus, total number of such packages = $2 \times 4 = 8$

Thus, total number of different packages = 8 + 8 = 16

The correct answer is Option C.

65. We know that there are six kinds of toppings and two kinds of breads for pizzas.

Since there are equal number of kinds of toppings and an equal number of kinds of breads, we have the following two possibilities:

(1) If there is one kind of topping and one kind of bread:

Number of ways of selecting one kind of topping = $C_1^6 = 6$.
Number of ways of selecting one kind of bread = $C_1^2 = 2$.

Thus, number of pizzas possible = $6 \times 2 = 12$.

(2) If there are two kinds of topping and two kinds of breads:

Number of ways of selecting two kinds of topping = $C_2^6 = \frac{6!}{4!2!} = 15$.
Number of ways of selecting two kinds of bread = $C_2^2 = 1$.

Thus, number of pizzas possible = $15 \times 1 = 15$.

Thus, total possibilities = 12 + 15 = 27.

The correct answer is Option E.

66. Using one-letter code, the botanist can uniquely designate 26 plants (since there are a total of 26 letters).

Using two-letter codes:

The first position can be assigned in 26 ways.

The second position can also be assigned in 26 ways (since the letters may be repeated).

Thus, total two-letter codes possible = $26 \times 26 = 676$.

Thus, using two-letter codes, the botanist can uniquely designate 676 plants.

Using three-letter codes:

Each of the three positions can be assigned in 26 ways.

Thus, total three-letter codes possible = $26 \times 26 \times 26$ = 17,576.

Thus, using three-letter codes, the botanist can uniquely designate 17,576 plants.

Thus, total number of unique designations possible using one-, two- or three-letter codes = 26 + 676 + 17,576 = 18,278.

The correct answer is Option E.

Alternate approach:

There is a cheeky method for this question.

Using one-letter codes: 26 ≡ units digit is 6.

Using two-letter codes: 26×26 ≡ units digit is 6.

Using three-letter codes: $26 \times 26 \times 26$ ≡ units digit is 6.

Thus, the units digit of the sum = 6 + 6 + 6 ≡ 8.

Only Option E has the units digit as 8.

Note: This is not a respectable method, and it is not to be used when two or more options are with same units digit.

67. There are total 22 balls, out of which 11 are black.

Probability that both balls will be black = $\frac{\text{Number of ways of drawing both black balls}}{\text{Number of ways of drawing any two balls}}$

$$=> \frac{C_2^{11}}{C_2^{22}} = \frac{\frac{11 \times 10}{1 \times 2}}{\frac{22 \times 21}{1 \times 2}} = \frac{11 \times 10}{22 \times 21} = \frac{5}{21}$$

The correct answer is Option B.

68. In a basket, out of 12 balls, 7 are red and 5 are green.

Number of ways we can select 3 balls from 12 balls = $C_3^{12} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$

Number of ways of selecting 2 red balls and 1 green ball

$$= C_2^7 \times C_1^5 = \frac{7 \times 6}{1 \times 2} \times 5 = 21 \times 5 = 105$$

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So required probability = $\frac{C_2^7 \times C_1^5}{C_3^{12}} = \frac{105}{220} = \frac{21}{44}$

The correct answer is Option D.

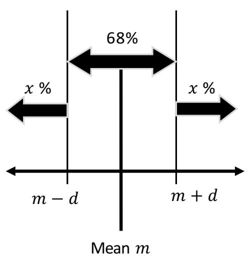
69. We know that the distribution is symmetric about the mean.

Thus, the percent of the distribution equidistant from the mean on either side of it is the same.

Let the percent of the distribution less than $(m + d)$ be $x\%$.

Thus, the percent of the distribution more than $(m - d)$ is also $x\%$.

The situation is shown in the diagram below.



Thus, we have

$x + 68 + x = 100$

$=> x = 16$

Thus, the percent of the distribution less than $(m + d) = x + 68 = 16 + 68 = 84\%$.

The correct answer is Option D.

70. The number of 6-member committees that can be formed out of the 21 members = C_6^{21}

$= \frac{21 \times 20 \times 19 \times 18 \times 17 \times 16}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

The number of 5-member committees that can be formed out of the 21 members = C_5^{21}

$= \frac{21 \times 20 \times 19 \times 18 \times 17}{5 \times 4 \times 3 \times 2 \times 1}$

Thus, the required ratio

$= \frac{\left(\frac{21 \times 20 \times 19 \times 18 \times 17 \times 16}{6 \times 5 \times 4 \times 3 \times 2 \times 1}\right)}{\left(\frac{21 \times 20 \times 19 \times 18 \times 17}{5 \times 4 \times 3 \times 2 \times 1}\right)}$

$= \frac{16}{6} = \frac{8}{3} = 8$ to 3

The correct answer is Option C.

71. There are two offices to which the four employees need to be assigned.

Thus, the number of options for each employee = 2 (since each employee can be assigned to any of the two offices)

Thus, the total number of ways of assigning the employees = $2 \times 2 \times 2 \times 2 = 2^4 = 16$.

Note: For n objects, each with r options, the total number of options = r^n

The correct answer is Option E.

72. We need to select 3 male officers from 5 male officers.

The number of ways of achieving it = $C_3^5 = \left(\frac{5 \times 4 \times 3}{3 \times 2 \times 1}\right) = 10$.

We also need to select 2 female officer from 3 female officers.

The number of ways of achieving it = $C_2^3 = C_1^3 = 3$.

Thus, the number of ways in which 3 male officers and 2 female office can be selected
= $10 \times 3 = 30$.

The correct answer is Option D.

73. We see that the earnings are \$1.20 per share, which is more than the expected earnings of \$0.90 per share.

Thus, the dividend distributed will be two-third of \$0.90 per share along with an additional payment.

The additional payment is \$0.05 per share for each additional \$0.15 per share earnings.

The additional earning per share = \$ (1.20 – 0.90) = \$0.30

Thus, the additional payment in dividend = \$ $\left(0.05 \times \left(\frac{0.30}{0.15}\right)\right)$ = \$0.10

Thus, the total dividend paid per share = \$ $\left(0.90 \times \frac{2}{3} + 0.10\right) = 0.60 + 0.10 =$ \$0.70; since the dividend distributed will be two-third of \$0.90 per share, thus it would be $0.90 \times \frac{2}{3} = 0.60$

Thus, total dividend paid to the person having 500 shares = \$ (0.70×500) = \$350.

The correct answer is Option C.

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74. Number of students on the committee G1 = 10.

As no member of G1 is in either of the other two groups, the above 10 students belong to only G1.

However, there may be an overlap with the students of G2 and G3.

Number of students in G2 = 10.
Number of students in G3 = 6.

We get the greatest number of students who would not be in any of the groups, if there is maximum overlap between the students of G2 and G3.

The maximum overlap between the students of G2 and G3 would be the minimum of the number of students in the two groups i.e. minimum of 6 and 10 = 6.

Thus, we have

$G2 \cap G3 = 6$

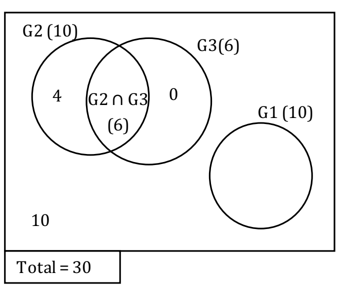
Thus, the number of students in G2 or G3 $= G2 + G3 - G2 \cap G3 = 10 + 6 - 6 = 10$

Thus, total number of students belonging to one of more groups

= G1 + G2 or G3 = 10 + 10 = 20.

Thus, maximum number of students who don't belong to any group = 30 – 20 = 10.

The above information can be represented in a Venn-diagram as shown below:



The correct answer is Option D.

75. Total number of toys = 1,000

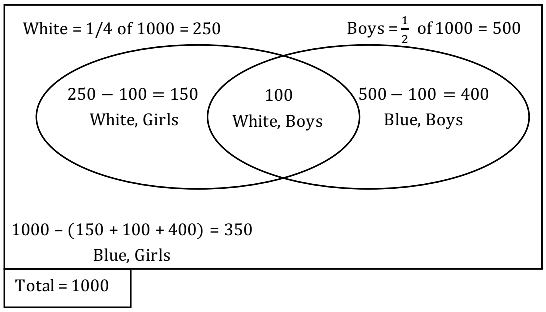
Number of white toys = 250

Number of blue toys = 750

Number of toys for boys = Number of toys for girls = $\frac{100}{2} = 500$

Number of white toys for boys = 100

Let us represent the above information using a Venn-diagram, as shown below:



Thus, from the above Venn-diagram, we have

Number of blue toys for girls = 350

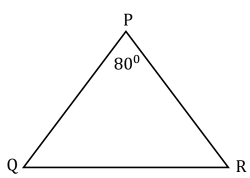
The correct answer is Option D.

76. Since the triangle PQR is isosceles, there can be three possible cases:

(1) $PQ = PR \neq QR$

$=> \angle PQR = \angle PRQ$

We know that: $\angle QPR = 80^\circ$



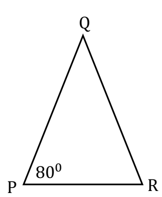
$=> \angle PQR + \angle PRQ = 180^\circ - 80^\circ = 100^\circ$

$=> \angle PRQ = \frac{100^\circ}{2} = 50^\circ$

(2) $QP = QR \neq PR$

$=> \angle QPR = \angle QRP$

We know that: $\angle QPR = 80^\circ$



$=> \angle QRP = 80^\circ$

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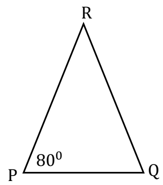
(3) RQ = RP ≠ QR



$=> \angle RPQ = \angle RQP = 80^\circ$ (given)

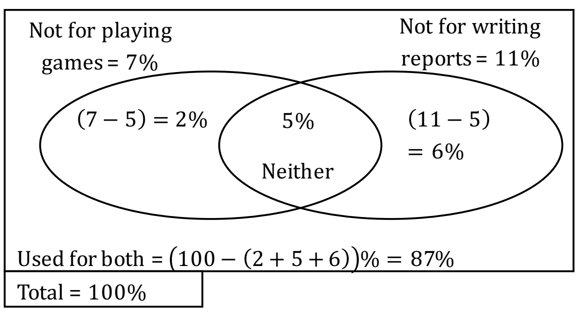
$=> \angle QRP = 180^\circ - (80^\circ + 80^\circ) = 20^\circ$

The correct answer is Option E.

77. We know that 95 percent teenagers have used a computer to play games or to write reports.

Thus, $(100 - 95) = 5\%$ teenagers have not used a computer for either of these purposes.

Let us represent the above information using a Venn-diagram, as shown below:



Thus, the percent of teenagers who have used a computer both to play games and to write reports = 87%.

The correct answer is Option D.

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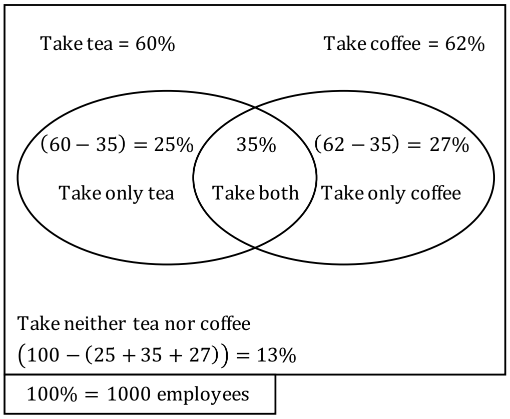
78. Let us represent the given information using a Venn-diagram, as shown below:



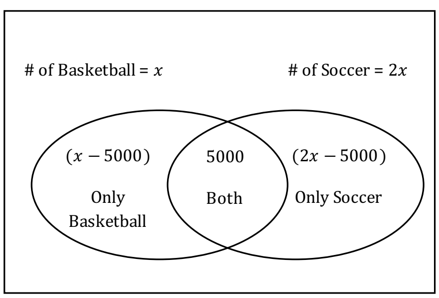
Thus, the number of employees who take neither tea nor coffee

= 13% of 1,000

= 130

The correct answer is Option B.

79. Let us represent the given information using a Venn-diagram, as shown below:



We know that the number of students who play both the sports is twice the number of students who play only basketball.

Thus, we have

$5,000 = 2\,(x - 5,000)$

=> $2x = 15,000$

=> $x = 7,500$

Thus, the number of students play only soccer = $2x - 5,000$

$= 10,000$

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The correct answer is Option C.

80. We know that the median is the middle-most value of any series/data set, but we do not know the value of x, so we cannot calculate exact value of Median; however we can surely find its range.

- Case 1:
  If x is smallest, the series would be x, 15, 20, 25 and median = average of 15 & 20 = 17.5–smallest median value.
- Case 2:
  If x is largest, the series would be 15, 20, 25, x and median = average of 20 & 25 = 22.5–largest median value.

Thus, the median would lie between 17.5 & 22.5, inclusive. Since only options I & II are in the range, Option C is correct.

The correct answer is Option C.

81. The question asks how many number of scores > (Mean + SD)?

$$\text{Mean} = \frac{46 + 54 + 58 + 67 + 77 + 79 + 97 + 98 + 99}{9} = 75$$

Mean + 1.SD = 75 + 19.51 = 94.51.

It is clear that three scores (97, 98, and 99) are greater.

The correct answer is Option C.

82. Let the sum of the 10 numbers other than x be s.

Thus, we have

$$x = 3 \times \left(\frac{s}{10}\right)$$

$$=> s = \frac{10x}{3}$$

Thus, the sum of all the 11 numbers in the list = $s + x = \frac{10x}{3} + x = \frac{13x}{3}$

Thus, the required fraction = $\frac{x}{\frac{13x}{3}} = \frac{3}{13}$.

The correct answer is Option B.

83. We have:

$w \le \frac{3 + 8 + w}{3} \le 3w$

=> $3w \le 11 + w \le 9w$

=> $2w \le 11 \le 8w$

=> $2w \le 11$ and $8w \ge 11$

=> $w \le \frac{11}{2} = 5\frac{1}{2}$

and

=> $w \ge \frac{11}{8} = 1\frac{3}{8}$

Since w is an integer, possible values of w are 2, 3, 4 and 5.

Thus, there are four possible values of w.

The correct answer is Option B.

84. Let the five integers be v, w, x, y and z such that $v > w > x > y > z$.

Thus, we need to find the least possible value of the largest among the five, i.e. v.

Let us assume that $v = w = x = y = z = 10$, because the average is 10.

Now, we reduce z by 2 and increase v by 2.

At the same time, we reduce y by 1 and increase w by 1.

Thus, we get the values:

$v = 12, w = 11, x = 10, y = 9, z = 8$

Thus, the least possible value of the greatest of the five numbers = 12.

The correct answer is Option B.

85. We have:

$\frac{p + q + 15}{3} = \frac{p + q + 15 + 35}{4}$

=> $4p + 4q + 60 = 3p + 3q + 150$

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=> $p + q = 90$

Thus, the average of p and $q = \frac{p+q}{2} = \frac{90}{2} = 45$.

The correct answer is Option B.

86. Let us recall the property of two-digit number:

"Difference between a particular two digit number and the number obtained by interchanging the digits of the same two digit number is always 9 times the difference between the digits."

Thus, the difference between actual amount and reversed amount = 54

= 9 × difference between the digits

=> Difference between the digits = $\frac{54}{9} = 6$

The difference between the digits of the number 6 is satisfied only by Option E.

The correct answer is Option E.

Alternate approach 1:

If we consider correct amount as $[xy] = 10x + y$, then interchanged amount becomes $[yx] = 10y + x$.

According to the given condition, difference between the new amount and the original amount is 54 cents.

=> $(10x + y) - (10y + x) = 54$

=> $10x - x - 10y + y = 54$

=> $9x - 9y = 54$

On dividing by 9, we have

$x - y = 6$.

Thus, the difference between the digits is 6, which is satisfied only by Option E.

Alternate approach 2:

Since the cash register contained 54 cents less than it should have as a result of this error, this implies that the tens digit of the correct amount must be greater than its units digit.

Only two options qualify. Let us analyze them:

(D) 75: $75 - 57 = 18 \neq 54$

(E) 93: $93 - 39 = 54$; correct answer

87. Let the number of pencils and erasers purchased be a and b, respectively.

Thus, total cost of the items = \$ $(0.7a + 0.5b)$

Thus, we have

$0.7a + 0.5b = 6.3$

$\Rightarrow 7a + 5b = 63$

It is clear that a and b are positive integers.

As a starting solution, we can see that 63 is divisible by 7 and hence we take:

$a = 9,\ b = 0$; however this is not possible as the number of erasers > 0.

The other solutions can be obtained by reducing the value of a by the coefficient of b i.e. 5, and increasing the value of b by the coefficient of a i.e. 7.

Thus, we have

$a = 9 - 5 = 4$
$b = 0 + 7 = 7$

If we apply the same approach again, then the value of a becomes $4 - 5 = -1$ (negative), which is not possible.

Hence, there is only one possible solution to the equation:

$a = 4,\ b = 7$

Thus, the total number of items purchased = $4 + 7 = 11$.

The correct answer is Option C.

88. We have:

$x + y + z = 2 \ldots$(i)
$x + 2y + 3z = 6 \ldots$(ii)

Multiplying (i) by 3 and subtracting (ii) from the result:

$3 \times (x + y + z = 2) - (x + 2y + 3z = 6)$

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=> $2x + y = 0$

=> $2x = -y$

=> $\frac{x}{y} = -\frac{1}{2}$

The correct answer is Option A.

89. Let the number of chairs sold last week be h.

Since the number of tables sold was 5 more than the chairs, the number of tables sold = $(h + 5)$.

Selling price of each table = \$150.

Selling price of each chair = \$85.

Thus, total sales revenue = $\$\left(85h + 150\left(h + 5\right)\right)$.

Thus, we have

$85h + 150\left(h + 5\right) = 1925$

=> $235h = 1,925 - 750$

=> $h = \frac{1,175}{235} = 5$

Thus, the number of chairs sold = 5.

Thus, the number of tables sold = 5 + 5 = 10.

Thus, the total number of tables and chairs sold = 5 + 10 = 15.

The correct answer is Option C.

Alternate approach:

We know that the number of tables sold is 5 more than the number of chairs sold.

The price of 5 tables = $\$\left(150 \times 5\right) = \$750$.

Removing this from the total, i.e. \$1,925, we are left with $\$\left(1,925 - 750\right) = \$1,175$.

This amount was obtained by selling equal numbers of tables and chairs.

Total price of one table and one chair = $\$\left(150 + 85\right) = \$235$.

Thus, number of items sold for $1,175

$= \frac{1,175}{235} \times 2 = 10$

Thus, total number of tables and chairs sold = 5 + 10 = 15.

90. We have:

$a = \sqrt{8ab - 16b^2}$

Squaring both the sides:

$a^2 = 8ab - 16b^2$

$=> a^2 - 8ab + 16b^2 = 0$

$=> (a - 4b)^2 = 0$

$=> a - 4b = 0$

$=> a = 4b$

The correct answer is Option D.

91. Given that,

$(x - 2)^2 = 9$

$=> x - 2 = \pm 3$

$=> x = 2 \pm 3$

$=> x = 5$ OR -1

Given that,

$(y - 3)^2 = 25$

$=> y - 3 = \pm 5$

$=> y = 3 \pm 5$

$=> y = 8$ OR -2

The minimum value of $\left(\frac{x}{y}\right)$ will be that value with the greatest magnitude of x, least magnitude of y and exactly one of x and y being negative in sign.

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Thus, we have

$$x = 5, y = -2 \Rightarrow \frac{x}{y} = -\frac{5}{2}$$

The maximum value of $\left(\frac{x}{y}\right)$ will be that value with the greatest magnitude of x, least magnitude of y and both x and y being simultaneously positive or negative in sign.

Thus, we have

$$x = 5, y = 8 \Rightarrow \frac{x}{y} = \frac{5}{8}$$

OR

$$x = -1, y = -2 \Rightarrow \frac{x}{y} = \frac{1}{2}$$

Among $\frac{5}{8}$ and $\frac{1}{2}$, the fraction $\frac{5}{8}$ is greater.

Thus, the required difference

$$= \frac{5}{8} - \left(-\frac{5}{2}\right)$$

$$= \frac{5}{8} + \frac{5}{2}$$

$$= \frac{25}{8}$$

The correct answer is Option E.

92. Given that,

$2x + 3y + xy = 12$

$\Rightarrow 2x + 3y + xy + 6 = 12 + 6 = 18$; (adding the product of the coefficients of x and y to both sides)

$\Rightarrow (2x + 6) + (xy + 3y) = 18$

$\Rightarrow 2(x + 3) + y(x + 3) = 18$

$\Rightarrow (x + 3)(y + 2) = 18$

Since x and y are positive integers, we must have:

$x + 3 > 3$, and

$y + 2 > 2$

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Possible ways of getting 18 are $1 \times 18, 2 \times 9$, and 3×6

Thus, the only possible solution is:

$x + 3 = 6 => x = 3$, and
$y + 2 = 3 => y = 1$

Thus, we have $x + y = 3 + 1 = 4$.

The correct answer is Option B.

93. We know that

$h = -2(t-5)^2 + 100 \ldots$(i)

First, we need to find the value of t so that the value of h is maximum.

In the expression for h, we have a term $-2(t-5)^2$

We know that: $(t-5)^2 \geq 0$ for all values of t (since it is a perfect square).

Thus, we have

$-2(t-5)^2 \leq 0$ for all values of t (multiplying with a negative reverses the inequality).

Thus, in order that h attains a maximum value, the term $-2(t-5)^2$ must be 0.

Thus, we have $-2(t-5)^2 = 0$

$=> t = 5$

Thus, h attains a maximum value at $t = 5$

Thus, two seconds after the maximum height is attained, i.e. at $t = 5 + 5 = 10$, we have the corresponding value of h (in feet) as:

$-2(10-5)^2 + 100$; substituting the value of $t = 10$ in eqn (i)

$= -2 \times 25 + 100$

$= 50$ feet

The correct answer is Option B.

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94. We know that:

$D(t) = -10(t-7)^2 + 100$, where $0 \le t \le 12$

We need to find the value of t so that the value of $D(t)$ is maximum.

In the expression for $D(t)$, we have a negative term $-10(t-7)^2$

We know that: $(t-7)^2 \ge 0$ for all values of t (since it is a perfect square).

Thus, we have

$-10(t-7)^2 \le 0$ for all values of t (multiplying with a negative reverses the inequality).

Thus, in order that $D(t)$ attains a maximum value, the term $-10(t-7)^2$ must be 0.

Thus, we have

$-10(t-7)^2 = 0$

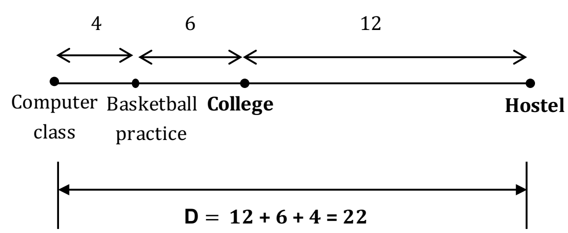
$\Rightarrow t = 7$

Thus, $D(t)$ attains a maximum value at $t = 7$ i.e. 7 hours past 12:00 a.m., i.e., 7:00 a.m.

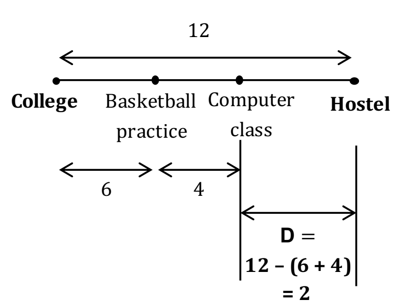
The correct answer is Option B.

95. The possible extreme scenarios are shown in the diagrams below:

(1) Maximum distance away from hostel:



(2) Minimum distance away from hostel:



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Thus, the maximum value of D is 22 and minimum value of D is 2

=> $2 \le D \le 22$

The correct answer is Option D.

96. As per given inequality: $|b| \le 12$, value of 'b' ranges from '−12' to '+12'. So, by putting these values in first equation $2a + b = 12$, we can form a table of consistent values of a & b.

a	***b***
0	12
1	10
2	8
3	6
4	4
5	2
6	0

a	***b***
Cont...	
7	−2
8	−4
9	−6
10	−8
11	−10
12	−12

So, a total of 13 ordered pairs are possible.

The correct answer is Option E.

Alternate approach:

We see that the value of b ranges from −12 to +12; this follows that b can have 25 number of integer values.

Now let us see how many integer values a can have.

$2a + b = 12 \Rightarrow a = \frac{12 - b}{2}$;

$\Rightarrow a = 6 - \frac{b}{2}$

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We see that for a to be an integer, $\frac{b}{2}$ must be an integer; this follows that b must be an even number.

Out of 25 possible values of b, 13 are even (including 0); so for a to be an integer, the set of arrangement can only have 13 ordered pairs.

97. Let the cost of an egg be $\$x$ and the cost of a sandwich be $\$y$.

Thus, we need to find the range within which $(4x + 3y)$ lies.

We have

$0.90 < 12x < 1.20$

$=> \frac{0.90}{3} < \frac{12x}{3} < \frac{1.20}{3}$

$=> 0.30 < 4x < 0.40 \ldots$(i)

Also, we have

$10 < 5y < 15$

$=> \frac{10}{5} < y < \frac{15}{5}$

$=> 2 < y < 3$

$=> 6 < 3y < 9 \ldots$(ii)

Adding (i) and (ii):

$6.30 < 4x + 3y < 9.40$

Thus, the correct answer is Option D.

98. Working with the options one at a time:

Comparing options A and B:

$(xy)^2 = x^2y^2 < x^2$ (since $0 < y < 1 => y^2$ is a fraction between 0 and 1)

Thus, Option B cannot have the greatest value.

Comparing options A and C:

$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2} = x^2 \times \left(\frac{1}{y^2}\right) > x^2$ (since $0 < y < 1 => 0 < y^2 < 1 => \frac{1}{y^2} > 1$)

Thus, Option A cannot have the greatest value.

Comparing options C and D:

$$\frac{x^2}{y} = \frac{x^2}{y^2} \times y = \left(\frac{x}{y}\right)^2 \times y < \left(\frac{x}{y}\right)^2 \text{ (since } 0 < y < 1 \Rightarrow y \text{ is a fraction between 0 and 1)}$$

Thus, Option D cannot have the greatest value.

Comparing options C and E:

$$x^2y = \frac{x^2}{y^2} \times y^3 = \left(\frac{x}{y}\right)^2 \times y^3 < \left(\frac{x}{y}\right)^2 \text{ (since } 0 < y < 1 \Rightarrow y^3 \text{ is a fraction between 0 and 1)}$$

Thus, Option E cannot have the greatest value.

The correct answer is Option C.

Alternate approach:

Since $x < 0$ and $0 < y < 1$ must be true for all the values x & y take, let us take convenient, smart values of x & y.

Say $x = -1$ & $y = \frac{1}{2}$

Let us calculate the values of each option.

(A) $x^2 = (-1)^2 = 1$

(B) $(xy)^2 = \left(-1 \times \frac{1}{2}\right)^2 = \frac{1}{4}$

(C) $\left(\frac{x}{y}\right)^2 = \left(\frac{-1}{\frac{1}{2}}\right)^2 = 4$: Maximum

(D) $\frac{x^2}{y} = \frac{(-1)^2}{\frac{1}{2}} = 2$

(E) $x^2y = (-1)^2 \times \frac{1}{2} = \frac{1}{2}$

99. Let the distance between the cities X and Y be d miles.

Range of speeds of Jack = 30 miles per hour to 45 miles per hour.

Time taken by Jack to cover the distance = 3 hours.

So, the range of distance between cities X and Y
= [30 × 3 miles] to [45 × 3 miles]

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= 90 miles to 135 miles

=> $90 < d < 135 \ldots$(i)

Range of speeds of Brian = 35 miles per hour to 55 miles per hour.

Time taken by Brian to cover the distance = 2 hours.

So, the range of distance between cities X and Y
= [35×2 miles] to [55×2 miles]

= 70 miles to 110 miles

=> $70 < d < 110 \ldots$(ii)

Thus, from (i) and (ii), we have

Range of distance between cities X and Y

= (Higher of the two minimum values) to (Lower of the two maximum values)

= (Higher of 90 and 70) to (Lower of 135 and 110)

= 90 miles to 110 miles

=> $90 < d < 110$

The only option which satisfies above inequality is 105 miles.

The correct answer is Option B.

100. Since DA = DC, triangle DAC is isosceles

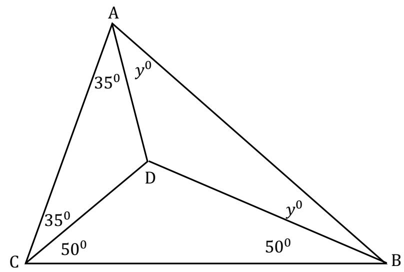
=> $\angle DAC = \angle DCA = 35°$

Since DB = DC, triangle DBC is isosceles

=> $\angle DBC = \angle DCB = 50°$

Since DA = DB, triangle DAB is isosceles

=> $\angle DBA = \angle DAB = y°$



Since sum of the internal angles of the triangle ABC is 180°, we have

$(35 + y) + (y + 50) + (50 + 35) = 180°$

$=> 2y = 10$

$=> y = 5°$

The correct answer is Option A.

Alternate approach:

Since DA = DB = DC, D is the center of the circumcircle of triangle ABC.

Thus, $\angle ADB = 2 \times \angle ACB = 2 \times (35 + 50)° = 170°$

Since triangle ADB is an isosceles triangle, we have

$$y = \frac{(180° - 170°)}{2} = 5°$$

101. Here we know that Brian's home and Andy's home are equidistant from John's home and distance between them is constant that is 8 miles. So if we join all these three places, we get an isosceles triangle.

So, here the property about lengths of sides of triangle, "Sum of any two sides is always greater than third side and positive difference between any two sides is always less than third side." is applicable.

• Statement I: 3 miles

Isosceles triangle is of sides: 3, 3, 8. Here $3 + 3 = 6 \ngtr 8$; so such a triangle does not exist. This is NOT the possible case.

• Statement II: 10 miles

Isosceles triangle is of sides: 10, 10, and 8. This triangle follows the above mentioned property. So this is a possible case.

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- Statement III: 12 miles

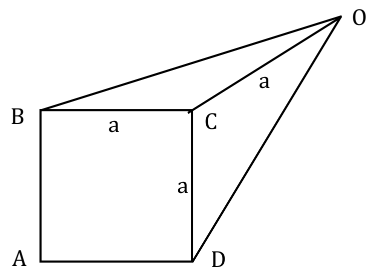
Isosceles triangle is of sides: 12, 12, and 8. This triangle follows the above mentioned property. So this is a possible case.

The correct answer is Option E.

Alternate approach:

The shortest required distance is when the John's home lies midway on the line joining Brian's home and Andy's home, i.e., $\frac{8}{2} = 4$ miles from either home. Thus, any value greater than or equal to 4 miles is possible.

102.



We observe that triangles BCO and DCO are isosceles.

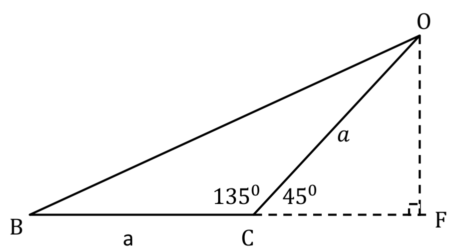
Also, triangle BCO ≅ triangle DCO

(Since CO is common, BC = CD and BO = DO)

Thus, ∠BCO = ∠DCO

$= \frac{360^\circ - 90^\circ}{2} = 135^\circ$

Considering triangle BCO alone, we have



In triangle COF:

$\angle OCF = 180^\circ - 135^\circ = 45^\circ$

$=> \angle COF = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

Thus, triangle COF is isosceles.

Let CF = OF = x

Thus, from Pythagoras' theorem in triangle COF:

$x^2 + x^2 = a^2$

$=> x = \frac{a}{\sqrt{2}}$

$=> \text{OF} = x = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2}$

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Thus, area of triangle BCO

$= \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times \text{BC} \times \text{OF}$

$= \frac{1}{2} \times a \times \frac{a\sqrt{2}}{2}$

$= \frac{a^2\sqrt{2}}{4}$

The correct answer is Option B.

Alternate approach:

From symmetry, we can say that OC, when extended would pass through A.

$\text{AC} = \sqrt{\text{AB}^2 + \text{BC}^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$

We know that if the height of two triangles is the same, the ratio of their area equals the ratio of their base.

Thus, we have

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle BOC}} = \frac{\text{Base of triangle ABC}}{\text{Base of triangle BOC}}$$

$$= \frac{\text{AC}}{\text{CO}}$$

$$= \frac{a\sqrt{2}}{a} = \frac{\sqrt{2}}{1}$$

$$\text{Area of triangle ABC} = \frac{\text{Area of square ABCD}}{2}$$

$$\frac{a \times a}{2} = \frac{a^2}{2}$$

Thus, we have

$$\frac{\left(\frac{a^2}{2}\right)}{\text{Area of triangle BOC}} = \frac{\sqrt{2}}{1}$$

$$=> \text{Area of triangle BOC} = \frac{a^2}{2\sqrt{2}} = \frac{a^2\sqrt{2}}{4}$$

103. Since z is the longest side in the right triangle, it must be the hypotenuse.

Thus, we have

$x^2 + y^2 = z^2 \ldots$(i)

Also, x and y are the perpendicular legs of the triangle.

Thus, area of the triangle:

$= \frac{1}{2} \times x \times y = 1$

$=> xy = 2 \ldots$(ii)

From (i), we have

$z^2 = x^2 + y^2$
$= (x + y)^2 - 2xy$
$= (x + y)^2 - 4$ (since $xy = 2$, from (ii))

Given $xy = 2$, the minimum value of $(x + y)$ occurs if $x = y$.

Let us verify:

Say, $x \neq y$:

$x = 2, y = 1 => x + y = 3$

However, if $x = y$:

$x = y = \sqrt{2} => x + y = 2\sqrt{2} \sim 2 \times 1.4 = 2.8 < 3$

However, the maximum value of $(x + y)$ is undefined and tends to infinity.

Say, we take $y = 100, x = 0.02 => x + y = 100.2$

Thus, the value of $(x + y)$ can be increased to any arbitrarily high value.

Thus, we can see that given $xy = 2$, $(x + y)$ has only a minimum value of $2\sqrt{2}$.

Hence, we have

$z^2 = (x + y)^2 - 4$

=> The minimum value of z^2 occurs when $(x + y)^2$ is minimum i.e. $(x + y)$ is minimum

=> Minimum value of $z^2 = \left(2\sqrt{2}\right)^2 - 4 = 4$

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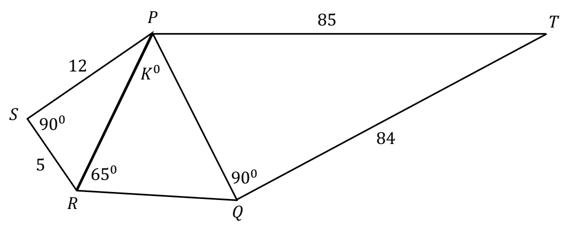
=> $z > 2$ (the equality does not hold since x and y cannot be equal since $x < y$)

The correct answer is Option A.

104. Let us bring out the figure.



In the right angled triangle PSR, as per Pythagoras theorem, we have

$PR^2 = PS^2 + RS^2$

=> $PR^2 = 12^2 + 5^2 = 169$

=> PR = 13

In the right angled triangle PQT, as per Pythagoras theorem, we have

$PQ^2 = PT^2 - QT^2$

=> $PQ^2 = 85^2 - 84^2 = 169$

=> PQ = 13

Thus, we have

PQ = PR

In a triangle, if two sides are equal, angles opposite to them are also equal.

=> $\angle PQR = \angle PRQ = 65^\circ$

=> $\angle QPR = 180^\circ - (\angle PQR + \angle PRQ)$

=> $\angle QPR = 180^\circ - (65^\circ + 65^\circ)$

=> $\angle QPR = 50^\circ$

=> $K^\circ = 50^\circ$

The correct answer is Option B.

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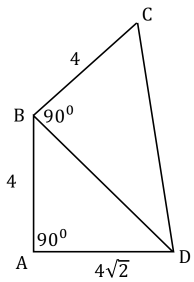
105. Let us bring out the figure.



In right angled triangle ABD, as per Pythagoras theorem:

$BD^2 = AB^2 + AD^2$

=> $BD^2 = 16 + 32 = 48$

=> $BD = 4\sqrt{3}$

In right angled triangle CBD, as per Pythagoras theorem:

$CD^2 = CB^2 + BD^2$

=> $CD^2 = 16 + 48 = 64$

=> CD =8

Thus, perimeter of triangle BCD

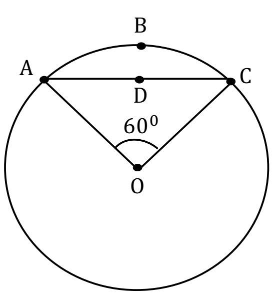
= BC + CD + BD

= $4 + 8 + 4\sqrt{3}$

= $12 + 4\sqrt{3}$

The correct answer is Option C.

106. Length of the arc ABC (which subtends 60° at the center) = $2\pi r \times \left(\frac{60}{360}\right)$, where r is the radius of the circle.



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Thus, we have

$$2\pi r \times \left(\frac{60}{360}\right) = 24\pi$$

$$=> r = 72$$

In triangle COA, we have AO = CO = radius of the circle

Hence, we have

$$\angle OCA = \angle OAC = \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

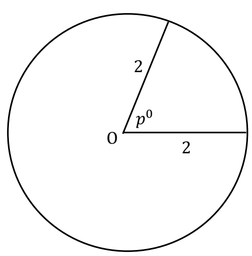
Thus, triangle COA is equilateral.

Thus, we have AC = CO = AO = radius of the circle = 72.

Thus, perimeter of the region ABCD = $(24\pi + 72) = 24(\pi + 3)$.

The correct answer is Option D.

107. Let us bring out the figure.



Area of a sector of a circle containing p° at the center is given by:

Area of the sector $= \pi \times (\text{radius})^2 \times \left(\frac{p}{360}\right)$

Thus, we have

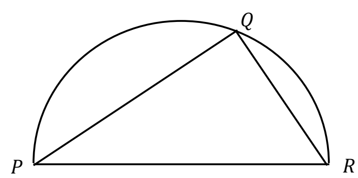
$$\pi \times (2)^2 \times \left(\frac{p}{360}\right) = \frac{\pi}{2}$$

$$=> p = \frac{360}{8}$$

$$= 45^\circ$$

The correct answer is Option C.

108. Let us bring out the figure.



Since the triangle PQR is inscribed in a **semicircle**, PR is the diameter of the semicircle.

Since the diameter subtends 90° at any point on the circumference, in triangle PQR, we have

$\angle PQR = 90^\circ$

Thus, from Pythagoras' theorem, we have

$PR^2 = PQ^2 + QR^2$

$=> PR^2 = 5^2 + 12^2 = 169$ (it is given that PQ = 5 and PR = 12)

$=> PR = 13$

=> Radius of the semicircle = $\frac{13}{2}$

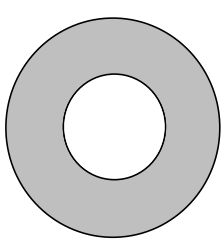
Thus, length of the arc PQR

$= \frac{\text{Circumference of the circle}}{2} = \frac{2\pi \times (\text{radius of the circle})}{2} = \pi \times (\text{radius of the circle})$

$= \frac{13\pi}{2}$

The correct answer is Option E.

109. The figure below demonstrate the two circles and the shaded region.



Let the radius of the outer circle be R and the radius of the inner circle be r.

Thus, area of the outer circle = πR^2

Area of the inner circle = πr^2

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Thus, area of the shaded region = $(\pi R^2 - \pi r^2)$

Since the area of the shaded region is 3 times the area of the smaller circle, we have

$\pi R^2 - \pi r^2 = 3\pi r^2$

=> $\pi R^2 = 4\pi r^2$

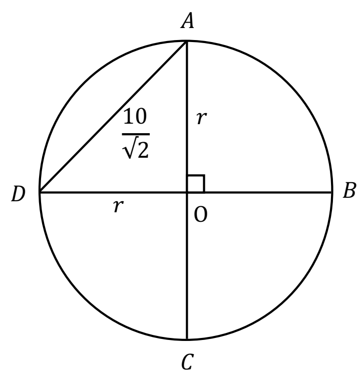
=> $R^2 = 4r^2$

=> $R = 2r$

=> $\frac{R}{r} = \frac{2}{1}$

The correct answer is Option C.

110.



In right angled triangle AOD, as per Pythagoras theorem, we have

=> $AD^2 = AO^2 + DO^2$

Since AD = $\frac{10}{\sqrt{2}}$ and AO = DO = radius = r, we have

=> $\left(\frac{10}{\sqrt{2}}\right)^2 = r^2 + r^2$

=> $2r^2 = 50$

=> $r = 5$

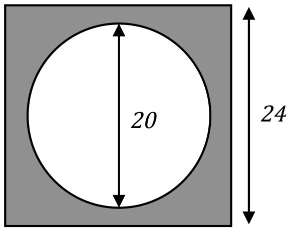
Thus, area of the circle

$= \pi r^2$

$= 25\pi$

The correct answer is Option C.

111. The diagram corresponding to the data provided is shown below:



We need to find what fraction the shaded area represents of the area of square tabletop.

Side of square table top is 24 inches.

Area of the square tabletop = $(\text{Side})^2$

=> $24^2 = 576$ square inches.

Diameter of circular cloth is 20 inches.

Area of the circular cloth = $\pi (r)^2$

=> $\pi \times \left(\frac{20}{2}\right)^2 =\approx 3.14 \times 100 = 314$ square inches.

Thus, area of the tabletop not covered by the cloth = $576 - 314 = 262$ square inches.

Thus, the required fraction = $\frac{\text{Area of the tabletop not covered by the cloth}}{\text{Area of square tabletop}}$

=> $\frac{262}{576} = 0.455$.

The only option satisfying is E, which is $\frac{9}{20} = 0.45$

The correct answer is Option E.

112. Let the length and breadth of the rectangular floor be x meters and y meters, where x and y are integers.

Since the perimeter of such floor is 16 meters, we have

$2(x + y) = 16$

=> $x + y = 8$

Thus, the possible cases are:

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(1) $(x, y) = (7, 1)$
=> Area of the floor = $1 \times 7 = 7$ square meters
=> Number of carpets required = $\left(\frac{7}{1 \times 1}\right) = 7$
=> Total cost of carpeting = \$ (7×6) = \$42

(2) $(x, y) = (6, 2)$
=> Area of the floor = $2 \times 6 = 12$ square meters
=> Number of carpets required = $\left(\frac{12}{1 \times 1}\right) = 12$
=> Total cost of carpeting = \$ (12×6) = \$72

(3) $(x, y) = (5, 3)$
=> Area of the floor = $3 \times 5 = 15$ square meters
=> Number of carpets required = $\left(\frac{15}{1 \times 1}\right) = 15$
=> Total cost of carpeting = \$ (15×6) = \$90

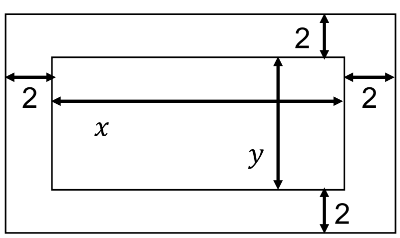
(4) $(x, y) = (4, 4)$
=> Area of the floor = $4 \times 4 = 16$ square meters
=> Number of carpets required = $\left(\frac{16}{1 \times 1}\right) = 16$
=> Total cost of carpeting = \$ (16×6) = \$96

Note: For a given perimeter, the area is the maximum if the length and breadth are equal.

The correct answer is Option D.

113. Let the length and width of the photograph be x centimeters and y centimeters respectively.

Thus, along with the border of 2 centimeters, the effective length becomes $(x + 4)$ centimeters and the effective width becomes $(y + 4)$ inches (since the border is along all sides) as shown in the figure below:



Thus, we have

$(x + 4)(y + 4) = a$

$=> xy + 4(x + y) = a - 16 \ldots$(i)

Again, along with the border of 4 centimeters, the effective length becomes $(x + 8)$ centimeters and the effective width becomes $(y + 8)$ centimeters.

Thus, we have

$(x + 8)(y + 8) = a + 100$

=> $xy + 8(x + y) = a + 100 - 64$
=> $xy + 8(x + y) = a + 36 \ldots$(ii)

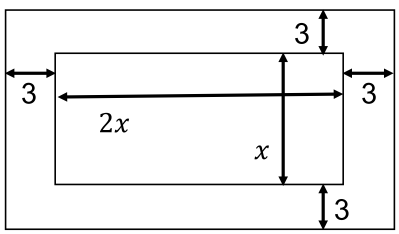
Subtracting (ii) from (i):

$xy - xy + 8(x + y) - 4(x + y) = (a + 36) - (a - 16)$
=> $4(x + y) = 36 + 16 = 52$
=> $2(x + y) = 26$

=> Perimeter of the photograph = $2(x + y) = 26$ centimeters.

The correct answer is Option D.

114. Let the width of the photograph without the border be x centimeters.



Thus, the length of the photograph without the border is $2x$ centimeters.

Thus, area of the above rectangle without the border = $2x \times x = 2x^2$ square centimeters.

Including the border, the length and width are $(2x + 3 + 3) = (2x + 6)$ and $(x + 3 + 3) = (x + 6)$ centimeters respectively.

Thus, area of the above rectangle including the border = $(2x + 6) \times (x + 6)$ square centimeters.

Thus, the area of the border

= (Area of the outer rectangle) – (Area of the inner rectangle)

$= (2x + 6) \times (x + 6) - 2x^2$

$= 2x^2 + 18x + 36 - 2x^2$

$= 18x + 36$

Thus, we have

$18x + 36 = 216$

=> $x = 10$

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Thus, the width of the photograph without the border = $x = 10$ centimeters.

The correct answer is Option A.

115. The volume of liquid is same in both the cylinders.

Volume of cylinder = $\pi \times r^2 \times h$

For first cylinder, we know that $d = 10 \;\; => \;\; r = 5$ inches and $h = 9$ inches.

Volume of liquid in the first cylinder = $v = \pi \times 5^2 \times 9 = 225\pi$ cubic inches.

Let the radius of the second cylinder be R inches and height be 4 inches.

Thus, volume of liquid in the second cylinder = $V = \pi \times R^2 \times 4 = 4R^2\pi$ cubic inches.

As per given data,

$v = V$

Thus, we have

$4R^2\pi = 225\pi$

$=> R^2 = \dfrac{225\ \pi}{4\ \pi}$

$=> R = \dfrac{15}{2} = 7.5$

Diameter of the second cylinder = $2 \times R = 2 \times 7.5 = 15$ inches

The correct answer is Option D.

116. Diameter of the smaller rim = 24 inches.

Circumference of the smaller rim = Distance covered by smaller ring in one rotation

$=> \;\; \pi \times 24 = 24\pi$ inch.

Number of rotations made per second by the smaller rim = r

D_1 = distance covered per minute by the smaller rim

= No. of revolutions per minute $\times$ Distance covered by smaller ring in one rotation

$=> 24\pi r$ inch.

Diameter of the larger rim = 36 inch.

Circumference of the larger rim = Distance covered by larger ring in one rotation

=> $\pi \times 36 = 36\pi$ inch.

Let the number of rotations made per minute by the larger rim be R.

D_2 = distance covered per minute by larger rim

= No. of rotations per minute $\times$ Distance covered by larger ring in one rotation

=> $36\pi R$ inch.

Hence, we have

$D_1 = D_2$

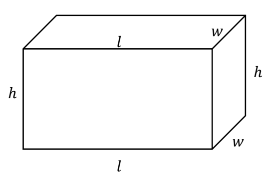
$24\pi r = 36\pi R$

=> $R = \frac{2r}{3}$

Thus, the number of rotations made per minute by the larger rim = $\frac{2r}{3}$

The correct answer is Option C.

117.



Let the dimensions of the length, width and height be l, w and h, respectively.

Since three faces shown have areas 12, 45, and 60, we have

Top face: $l \times w = 12 \ldots$(i)

Front face: $l \times h = 45 \ldots$(ii)

Right hand side face: $h \times w = 60 \ldots$(iii)

Multiplying the above three equations:

$l^2 \times w^2 \times h^2 = 12 \times 45 \times 60 = 3^4 \times 4^2 \times 5^2$

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$=> l \times w \times h = \sqrt{3^4 \times 4^2 \times 5^2} = 3^2 \times 4 \times 5$

Since the volume of the solid is given by $l \times w \times h$, we have

Volume of the solid = 180.

The correct answer is Option A.

118. The length of the line segment between 2 points $(x_1, y_1) = (-3, -6)$ and $(x_2, y_2) = (5, 0)$ is given by:

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - (-3))^2 + (0 - (-6))^2}$

$= \sqrt{8^2 + 6^2}$

$= 10$

Thus, the length of the diameter of the circle = 10.

Thus, the length of the radius of the circle = $\frac{10}{2} = 5$.

Thus, the area of the circle = $\pi \times 5^2 = 25\pi$.

The correct answer is Option C.

119. Equation of a line is: $y = mx + c$, where m is the slope and c is the Y-intercept.

Thus, we have the equation of the given line as:

$y = 3x + 4$

Let the required point on the above line be: $(a, 10)$.

Thus, we have

$10 = 3a + 4$

$=> a = 2$

Thus, the required X-coordinate of the point is 2.

The correct answer is Option A.

120. Equation of a line passing through a point (p, q) and slope m is given as:

$y - q = m\,(x - p)$

Thus, the equation of the line l that passes through the origin $(0,\ 0)$ and has slope 3 is:

$y - 0 = 3\,(x - 0)$

$=> y = 3x$

Since $(1, a)$ is a point on the line, we have

$a = 3 \times 1$

$=> a = 3 \ldots$(i)

Since $(b, 2)$ is a point on the line, we have

$2 = 3 \times b$

$=> b = \frac{2}{3} \ldots$(ii)

Thus, from (i) and (ii), we have

$$\frac{a}{b} = \frac{3}{\left(\frac{2}{3}\right)} = \frac{9}{2}$$

The correct answer is Option E.

121. The equation of a circle having center at (p, q) and radius r is:
$(x - p)^2 + (y - q)^2 = r^2$

Since the center of the circle is at $(3, 2)$, the equation of the circle is:
$(x - 3)^2 + (y - 2)^2 = r^2$

If a point (m, n) lies inside the circle $(x - p)^2 + (y - q)^2 = r^2$,
it must satisfy: $(m - p)^2 + (n - q)^2 < r^2$

Since $(-1,\ 2)$ lies inside the circle, it must satisfy:
$(-1 - 3)^2 + (2 - 2)^2 < r^2$

$=> r^2 > 16$

$=> r > 4 \ldots$(i)

If a point (m, n) lies outside the circle, it must satisfy:

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$(m - p)^2 + (n - q)^2 > r^2$

Since $(3, -4)$ lies outside the circle, it must satisfy:
$(3 - 3)^2 + (-4 - 2)^2 > r^2$

=> $r^2 < 36$

=> $-6 < r < 6$

Since r must be positive, we have
$0 < r < 6 \ldots$(ii)

Thus, from (i) and (ii), we have

$4 < r < 6$

From the options, only $r = 5$ satisfies the above inequality.

The correct answer is Option A.

Alternate approach:

Since $(-1, 2)$ lies inside, the distance between $(-1, 2)$ and the center $(3, 2)$ is less than the radius.

Similarly, since $(3, -4)$ lies outside, the distance between $(3, -4)$ and the center $(3, 2)$ is greater than the radius. Thus, we have

$\sqrt{[(-1 - 3)^2 + (2 - 2)^2]} < r < \sqrt{[(3 - 3)^2 + (-4 - 2)^2]}$

=> $\sqrt{16} < r < \sqrt{36}$

=> $4 < r < 6$ => $r = 5$ (only value among the options)

122. The three vertices are: $(a, b), (a, -b)$ and $(-a, -b)$

We know that $a < 0$ and $b > 0$

=> $-a > 0$ and $-b < 0$

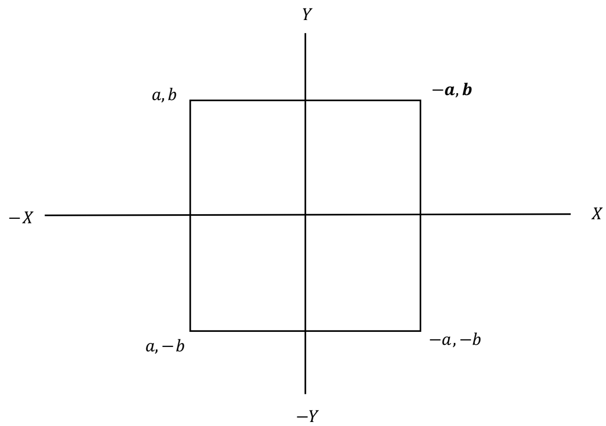
We can see that the distance of each of the vertices from the origin $(0, 0) = \sqrt{a^2 + b^2}$

Thus, the three vertices are equidistant from the origin.

Alternately, we can see that the midpoint of the diagonal joining (a, b) and $(-a, -b)$ is $(0, 0)$, i.e. the origin.

Thus, the centre of the square is at the origin.

Thus, the square would be positioned as shown in the diagram below:



Thus, the fourth vertex would be $(-a, b)$, which lies in the first quadrant.

Thus, the X-coordinate of the fourth vertex is positive and the Y-coordinate of the fourth vertex is also positive.

Thus, the only point which also lies in the same quadrant is $(6, 2)$.

The correct answer is Option E.

123. Distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between the points $(0, 0)$ and $(5, 5)$

$= \sqrt{(5 - 0)^2 + (5 - 0)^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

Distance between the points $(0, 0)$ and $(10, 0)$

$= \sqrt{(10 - 0)^2 + (0 - 0)^2}$

$= \sqrt{100}$

$= 10$

Note: The distance is obviously 10 since (10, 0) is a point on the X-axis at a distance of 10 to the right of the origin.

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Distance between the points $(10, 0)$ and $(5, 5)$

$= \sqrt{(10-5)^2 + (0-5)^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

Thus, the perimeter

$= 5\sqrt{2} + 10 + 5\sqrt{2}$

$= 10 + 10\sqrt{2}$

The correct answer is Option E.

124. Since the points are collinear, their slopes must be equal.

Slope of the line joining two points (x_1, y_1) and (x_2, y_2) is given by: $\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)$.

Slope of the line joining the points $(a, 0)$ and $(0, b)$

$= \dfrac{b-0}{0-a}$

$= -\dfrac{b}{a}$

Slope of the line joining the points $(a, 0)$ and $(1,\ 1)$

$= \dfrac{1-0}{1-a}$

$= \dfrac{1}{1-a}$

Thus, we have

$-\dfrac{b}{a} = \dfrac{1}{1-a}$

=> $-b \times (1-a) = a$

=> $-b + ab = a$

=> $b + a = ab$

=> $b = ab - a$

=> $b = a(b-1)$

$=> a = \frac{b}{b-1}$

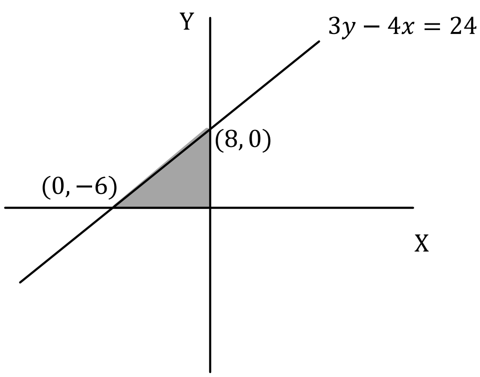
The correct answer is Option C.

125. The area of the triangle formed by the line with the two axes
$= \frac{1}{2} \times$ (Length of X intercept) $\times$ (Length of Y intercept)

We have:

$3y - 4x = 24 \ldots$(i)

The graph of the line and the area of the triangle formed by the line and the axes is shown in the diagram below:



To calculate the X-intercept:

Substituting $y = 0$ in (i):

$x = -\frac{24}{4} = -6$

Thus, the length of the X-intercept = 6.

To calculate the Y-intercept:

Substituting $x = 0$ in (i):

$y = \frac{24}{3} = 8$

Thus, the length of the Y-intercept = 8.

Thus, required area

$= \frac{1}{2} \times 6 \times 8 = 24$

The correct answer is Option C.

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4.2 Select One or Many Questions

126. There are two conditions that need to be satisfied:

(1) The denominator of the fraction is non-zero

(2) The denominator of the fraction under the root must be non-negative

The expression can be written as:

$$\sqrt{\frac{2}{(x-1)(x+2)}} = \frac{\sqrt{2}}{\sqrt{(x-1)(x+2)}}$$

Thus, from the conditions A and B above, we have:

$(x-1)(x+2) > 0$

$=> x-1 > 0$ AND $x+2 > 0$

OR

$x-1 < 0$ AND $x+2 < 0$

$=> x > 1$ AND $x > -2$

OR

$x < 1$ AND $x < -2$

$=> x > 1$

OR

$x < -2$

The correct answers are options C and F.

Alternate approach:

We have: $(x-1)(x+2) > 0$

Assuming this to be a quadratic EQUATION, we have:

$x = 1$ OR -2

Here, 1 is the greater of the two roots.

Since the inequality is of the greater than type, we can say that x must be greater than the largest root or lesser than the smallest root.

$=> x > 1$ OR $x < -2$

127. We know that, for a ratio $0 < \frac{x}{y} < 1$, if k is a positive number:

- $0 < \frac{x}{y} < \left(\frac{x+k}{y+k}\right) < 1$ ... (i)
- $0 < \left(\frac{x-k}{y-k}\right) < \frac{x}{y} < 1$ ... (ii)

Also, for a ratio $\frac{x}{y} > 1$, if k is a positive number:

- $1 < \left(\frac{x+k}{y+k}\right) < \frac{x}{y}$ ... (iii)
- $1 < \frac{x}{y} < \left(\frac{x-k}{y-k}\right)$ ... (iv)

In the above problem, we have:

The given ratio of marbles with Jack and Dave = $\frac{13}{9}$ $(> 1) =\sim 1.44$

Thus, after the inclusion of 10 marbles each, their marbles would increase by 10.

Hence, the final ratio must be smaller than $\frac{13}{9}$ (from relation (iii) above).

Working with the options, we have:

Option A:

$\frac{7}{4} = 1.75 \nless 1.44$ – Cannot be the ratio

Option B:

$\frac{7}{5} = 1.4 < 1.44$ – Can be the ratio

Option C:

$\frac{15}{11} = 1.36 < 1.44$ – Can be the ratio

Option D:

$\frac{31}{23} = 1.35 < 1.44$ – Can be the ratio

Option E:

$\frac{23}{8} = 2.875 \nless 1.44$ – Cannot be the ratio

Since the question asks for two answers and it is for sure that options A and E are the correct options, we need not check if other options B, C and D qualify to be qualified ratios.

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The correct answers are options A and E.

128. The population of the colony doubles every 2 hours.

The population of a colony of bacteria at 12:00 noon = 64×10^4.

Thus, the population at 8:00 pm, i.e. eight hours from now, i.e. 4 slots of 2 hours

$= 2\times$ Population at 6:00 pm

$= 2 \times 2\times$ Population at 4:00 pm

$= 2 \times 2 \times 2\times$ Population at 2:00 pm

$= 2 \times 2 \times 2 \times 2\times$ Population at 12:00 noon

$= 2^4 \times 64 \times 10^4$

$= 2^4 \times 2^6 \times 10^4$

$= 2^{10} \times 10^4$

Thus, square of the population at 8:00 pm

$= \left(2^{10} \times 10^4\right)^2$

$= 2^{20} \times 10^8$

Working with the options, we have:

Option A:

Population at 4 pm $= 2^2\times$ Population at 12:00 noon $= 2^2 \times 64 \times 10^4 = 2^8 \times 10^4$

Population at 6 pm $= 2^3\times$ Population at 12:00 noon $= 2^3 \times 64 \times 10^4 = 2^9 \times 10^4$

The product of the populations at 4 pm and 6 pm

$= (2^8 \times 10^4) \times \left(2^9 \times 10^4\right) = 2^{17} \times 10^8 \neq$ Square of the population at 8:00 pm – Does not satisfy

Option B:

Population at 10:00 pm $= 2^5\times$ Population at 12:00 noon $= 2^5 \times 64 \times 10^4 = 2^{11} \times 10^4$

The product of the populations at 6:00 pm and 10:00 pm

$= (2^9 \times 10^4) \times (2^{11} \times 10^4)$

$= 2^{20} \times 10^8$

= Square of the population at 8:00 pm – Satisfies

Option C:

Population at 12:00 midnight = 2^6 × Population at 12:00 noon = $2^6 \times 64 \times 10^4 = 2^{12} \times 10^4$

The product of the populations at 4:00 pm and 12:00 midnight

$= (2^8 \times 10^4) \times (2^{12} \times 10^4)$

$= 2^{20} \times 10^8$

= Square of the population at 8:00 pm – Satisfies

The correct answers are options B and C.

Alternate approach

Let the population at 12:00 noon be x.

Since the population doubles every 2 hours, successive values of the population every 2 hours form a geometric progression with first term $a = x$ and constant ratio $r = 2$

Thus, the values of the population at different times are:

- 2:00 pm: $2x$
- 4:00 pm: 2^2x
- 6:00 pm: 2^3x
- 8:00 pm: 2^4x
- 10:00 pm: 2^5x
- 12:00 midnight: 2^6x

Thus, we see that:

Square of the population at 8:00 pm = $(2^4x)^2 = 2^8x^2$

= Product of the populations at 6:00 pm and 10:00 pm = $2^3x \times 2^5x = 2^8x^2$

= Product of the populations at 4:00 pm and 12:00 midnight = $2^2x \times 2^6x = 2^8x^2$

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129. The ratio of the time durations = 2 : 3 : 5 : 6

If the first worker worked for 60 hours:

The modified ratio would be

= $\mathbf{2 \times 30}$ **:** $3 \times 30 : 5 \times 30 : 6 \times 30$

= 60 : 90 : 150 : 180

Thus, total time = 60 + 90 + 150 + 180 = 480 hours – Option E is possible

If the second person worked for 60 hours:

The modified ratio would be

= $2 \times 20 :$ $\mathbf{3 \times 20}$ $: 5 \times 20 : 6 \times 20$

= 40 : 60 : 100 : 120

Thus, total time = 40 + 60 + 100 + 120 = 320 hours – Option C is possible

If the third person worked for 60 hours:

The modified ratio would be

= $2 \times 12 : 3 \times 12 :$ $\mathbf{5 \times 12}$ $: 6 \times 12$

= 24 : 36 : 60 : 72

Thus, total time = 24 + 36 + 60 + 72 = 192 hours – Option B is possible

If the fourth person worked for 60 hours:

The modified ratio would be

= $2 \times 10 : 3 \times 10 : 5 \times 10 :$ $\mathbf{6 \times 10}$

= 20 : 30 : 50 : 60

Thus, total time = 20 + 30 + 50 + 60 = 160 hours – Option A is possible

The correct answers are options A, B, C and E.

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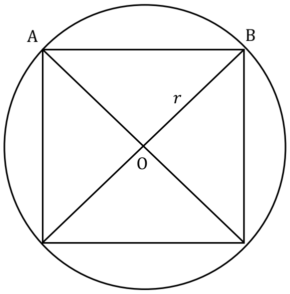
130. Let us bring out the figure.



Let O be the centre of the circle and the square.

Since the diagonals of a square are perpendicular, $\angle AOB = 90^\circ$

Length of an arc subtending angle θ° at the centre of the circle = $2\pi r \times \left(\frac{\theta}{360}\right)$

Thus, length of the minor arc AB

$= 2\pi r \times \left(\frac{90}{360}\right) = \frac{\pi r}{2}$

Thus, we have:

$4\pi < \frac{\pi r}{2} < 8\pi$

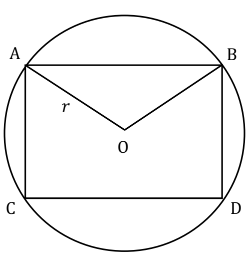
$=> 8 < r < 16$

$=> 16 < 2r =$ diameter < 32

Thus, the diameter of the circle lies between 16 and 32.

The correct answers are options B and C.

131. Refer to the following diagram.



In the diagram above, the rectangle is shown inscribed in the circle.

Since circumference of the circle is 4π, we have:

$2\pi r = 4\pi$

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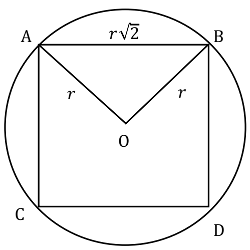
=> $r = 2$

The area of the rectangle would be minimum when the length tends to be equal to the diameter resulting in the width becoming almost 0.

Thus, the area would be a minimum, tending to zero.

The area of the rectangle would be the maximum when the length and width of the rectangle are equal.

Thus, the rectangle should actually be a square, as shown in the diagram below.



In triangle AOB, $\angle AOB = 90^\circ$

Thus, if AO = BO = r (radius of the circle), we have from Pythagoras' theorem:

$AB^2 = r^2 + r^2$

=> $AB = r\sqrt{2}$

Thus, area of the square = AB^2

$= \left(r\sqrt{2}\right)^2 = 2r^2$

$= 2 \times 2^2 = 8$

Thus, the area of the rectangle can be any value less than or equal to 8.

The correct answers are options A, B and C.

132. $\sqrt{810,000}$

$= \sqrt{9^2 \times 10^4}$

$= 9 \times 10^2$

$= 900$

Thus, there are 3 digits in x, each requiring 4 bits of memory.

Thus, total number of bits required

$= 3 \times 4 = 12$

Agian, $\sqrt{6,400}$

$= \sqrt{8^2 \times 10^2}$

$= 8 \times 10$

$= 80$

Thus, there could be 2 digits in x, each requiring 4 bits of memory.

Thus, total number of bits required

$= 2 \times 4 = 8$

Thus, the number of bits required would be either 8 or 12.

The correct answers are options B and D.

133. Total number of letters = $(n + 2)$

Since there are 2 A's (identical) and n B's (identical), total number of codes that can be created using all the letters

$= \frac{(n+2)!}{2!.n!}$

$= \frac{(n+2)(n+1) \times n!}{2!.n!}$

$= \frac{(n+2)(n+1)}{2}$

Thus, we have:

$\frac{(n+2)(n+1)}{2} < 66$

$=> n^2 + 3n + 2 < 132$

$=> n^2 + 3n - 130 < 0$

$=> (n - 10)(n + 13) < 0$

If this were a quadratic equality, the roots would have been -13 and 10.

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Since this is an inequality of the less than type, the value of n must lie between the two roots.

=> $-13 < n < 10$

Since n is the number of B's, it cannot be negative.

Thus, we have:

$0 < n < 10$

The correct answers are options A, B, C and D.

134. Total number of letters = 1 + 2 + 3 = 6

There are 2 Bs (identical) and 3 Cs (identical).

The different cases where 5 of the 6 letters are selected are shown in the table below:

Case #	Number of As selected	Number of Bs selected	Number of Cs selected	Selected letters	Number of sequences
A.	1	2	2	A, B, B, C, C	$\frac{5!}{2!2!} = \frac{120}{2 \times 2} = \mathbf{30}$
B.	1	1	3	A, B, C, C, C	$\frac{5!}{3!} = \frac{120}{6} = \mathbf{20}$
C.	0	2	3	B, B, C, C, C	$\frac{5!}{2!3!} = \frac{120}{2 \times 6} = \mathbf{10}$

Thus, the possible number of 5-letter sequences can be 10, 20 or 30.

The correct answers are options B, D and F.

135. Since the number leaves the remainder 1 when divided by 12 and by 8, the number would leave the same remainder 1 when divided by the LCM of 12 and 8, i.e. 24.

We need to find the numbers between 80 and 150, inclusive, that leave the remainder 1 that are in the format $(24n + 1)$; where n is positive integer.

The first such number is $(24 \times 4) + 1 = 97$

The other numbers are:

$24 \times 5 + 1 = 121$ and $24 \times 6 + 1 = 145$

Thus, the required numbers are: $97, \ 121, \ \text{and} \ 145$

The correct answers are options A, C and E.

136. We have 90 liters of 20% alcohol solution.

Thus, amount of alcohol = 20% of 90

$= \frac{20}{100} \times 90$

= 18 liters

Let x liters of pure alcohol be added.

Thus, the final amount of alcohol = $(x + 18)$ liters.

Total volume of the solution = $(x + 90)$ liters.

Since the final concentration of alcohol is at least 25%, we have:

$\frac{x + 18}{x + 90} \times 100\% \geq 25\%$

$=> \frac{x + 18}{x + 90} \geq \frac{25}{100} = \frac{1}{4}$

$=> 4x + 72 \geq x + 90$

$=> x \geq 6$

The correct answers are options D and E.

137. Probability that one marble is white and the other is red =

$$\frac{C_1^3 \times C_1^n}{C_2^{(3+n)}} = \frac{3 \times n}{\frac{(3 + n)(2 + n)}{1 \times 2}} = \frac{6n}{(3 + n)(2 + n)}$$

Probability that both the marbles are red =

$$\frac{C_2^n}{C_2^{(3+n)}} = \frac{\frac{n \times (n - 1)}{1 \times 2}}{\frac{(3 + n)(2 + n)}{1 \times 2}} = \frac{n \times (n - 1)}{(3 + n)(2 + n)}$$

It is given that $\frac{6n}{(3 + n)(2 + n)} > \frac{n \times (n - 1)}{(3 + n)(2 + n)}$

$6n > n \times (n - 1)$

$n < 7$

=> $3 < n < 7$; it is given that $n > 3$; thus, n can be 4, 5 or 6.

The correct answers are options C, D and E.

138. Let $x = 2.151515\ldots$ (i)

Since two digits 15 repeat indefinitely, let us multiply x with 100 so that the decimal part still has the same digits 15 repeating indefinitely.

Thus, we have:
$100x = 215.151515\ldots$ (ii)

Subtracting (i) from (ii), we obtain:
$100x - x = (215.151515\ldots) - (2.151515\ldots)$

=> $99x = 213$ (since the decimal parts of both are the same, they cancel out)

=> $x = \frac{213}{99} = \frac{71}{33}$

We see that 71 and 33 have no factors common.

Thus, if x is multiplied by 33, we get the integer 71.

Thus, if x is multiplied by any positive multiple of 33, then too, we will get an integer.

Thus, the required number is 33 or the multiples of 33.

From the options, we see that 33, 99 and 990 are multiples of 33.

The correct answers are options B, C and E.

139. We need to find integers n, between 10 and 20, such that n is not a factor of $(n - 1)!$.

$(n - 1)! = (n - 1) \times (n - 2) \times \cdots \times 2 \times 1$

We see that n, as a number, is not present in the above product.

However, it may be that the factors of n are present in the product above.

Say, for example, $n = 12$:
$(n - 1)! = (12 - 1)! = 11! = 11 \times 10 \times 9 \times 8 \times 7 \times \mathbf{6} \times 5 \times 4 \times 3 \times \mathbf{2} \times 1$

We see that, though 12 as a single number is not present in 11!, the factors of 12, i.e. 2 and 6 are present.

Thus, 12 is a factor of 11!

However, if we take $n = 11$:

$(n-1)! = (11-1)! = 10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

We see that 11 as a single number is not present in 10!, and since 11 is a prime number, it does not have any factors which can be present in 10!.

Thus, 11 is not a factor of 10!

Thus, we observe that n is not a factor of $(n-1)!$ only if n is a prime number.

(Note: The only exception to the above rule is when $n = 4$: Though 4 is not a prime number, 4 is not a factor of $(4-1)! = 3!$)

In our problem, the prime numbers between 10 and 20 $(10 < n < 20)$ are 11, 13, 17 and 19.

The correct answers are options A, C, F and G.

140. There are a total of n elements.

The number of ways in which 3 elements can be selected from n elements in the set

$$= C_3^n = \frac{n(n-1)(n-2)}{3 \times 2 \times 1}$$

$$= \frac{n(n-1)(n-2)}{6}$$

Let us now find the cases in which 3 elements are selected such that both 2 and 4 are also selected.

Since both 2 and 4 are already selected, we need to select only 1 more element from the remaining $(n-2)$ elements.

Thus, the number of ways

$$= C_1^{(n-2)} = (n-2)$$

Thus, the number of ways in which 3 elements can be selected so that both 2 and 4 are NOT simultaneously selected

= (Total # ways) – (# ways in which both 2 and 4 are selected)

$$= \frac{n(n-1)(n-2)}{6} - (n-2)$$

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$$= (n-2)\left[\frac{n(n-1)}{6} - 1\right]$$

$$= \frac{(n-2)(n^2-n-6)}{6}$$

$$= \frac{(n-2)(n+2)(n-3)}{6}$$

Thus, we have:

$$\frac{(n-2)(n+2)(n-3)}{6} < 35$$

Working with the options, we have:

Option A: $n = 4$:

$$\frac{(n-2)(n+2)(n-3)}{6} = \frac{2 \times 6 \times 1}{6} = 2 < 35 \text{ – Satisfies}$$

Option B: $n = 6$:

$$\frac{(n-2)(n+2)(n-3)}{6} = \frac{4 \times 8 \times 3}{6} = 16 < 35 \text{ – Satisfies}$$

Option C: $n = 7$:

$$\frac{(n-2)(n+2)(n-3)}{6} = \frac{5 \times 9 \times 4}{6} = 30 < 35 \text{ – Satisfies}$$

The correct answers are options A, B and C.

141. Number of floors Jane needs to cover = x.

Thus, time taken to walk down the steps for all x floors taking 30 seconds per floor = $30x$ seconds

Time taken to wait for the elevator and ride for x floors taking 2 seconds per floor = 7 minutes + $2x$ seconds

= $(420 + 2x)$ seconds

Thus, we have:

$30x > 420 + 2x$

$$=> x > \frac{420}{28} = 15$$

The correct answers are options C, D and E.

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142. Since the question asks us to find a percent value, we may assume any suitable value for tips since the initial value does not affect the final answer.

If the wages was 150 percent of the tips:

Wages was (100 + 50)% of the tips

$$= > \frac{\text{Wages}}{\text{Tips}} = \frac{100 + 50}{100} = 1 + \frac{1}{2} = \frac{3}{2}$$

Let the tips be \$2

Thus, the wages = \$3

Thus, income = \$5

Thus, required percent

$$= \frac{2}{5} \times 100$$

= 40%

If the wages was 166.67 percent of the tips:

Wages was (100 + 66.67)% of the tips

$$= > \frac{\text{Wages}}{\text{Tips}} = \frac{100 + 66.67}{100} = 1 + \frac{2}{3} = \frac{5}{3}$$

Let the tips be \$3

Thus, the wages = \$5

Thus, income = \$8

Thus, required percent

$$= \frac{3}{8} \times 100$$

= 37.5%

Thus, the required percent value lies between 37.5% and 40%, inclusive.

The correct answers are options C and D.

143. Minimum cost price of the bowl = \$60.

Let the selling price of the bowl be \$$x$.

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The mark-up amount = 25% of $x = \frac{x}{4}$

Thus, we have

Selling price = Mark-up + Cost price

=> Cost price = Selling price – Mark-up

$= >$ Cost price $= x - \frac{x}{4}$

Since the cost price is more than or equal to \$60, we have:

$x - \frac{x}{4} \geq 60$

$=> \frac{3x}{4} \geq 60$

$=> x \geq 80$

The correct answers are options C, D and E.

144. Cost price of the bowl = \$80.

Let the selling price of the bowl be $\$x$.

Let's calculate the selling price @20% mark-up. All the values of x more than and equal to the calculated value would be correct.

Thus, we have:

$x = 80 + 20\%$ of x

$=> x = 80 + \frac{x}{5}$

$=> \frac{4x}{5} = 80$

$=> x = \$100$

The minimum selling price = \$100. We see that only one option is \$100.

The correct answer is option A.

145. Since the question asks us to find a percent value, we may assume any suitable value of the number of stamps collected by Andy since the initial value does not affect the final answer.

Number of stamps collected by Andy = $\frac{2}{3}$ of that of Brian = $\frac{3}{4}$ of that of Suzy.

Let the number of stamps collected by Andy

= LCM of 2 and 3 (the numerators of the fractions) = 6

Thus, the number of stamps collected by Brian

$= \frac{3}{2} \times 6 = 9$

The number of stamps collected by Suzy

$= \frac{4}{3} \times 6 = 8$

Thus, total number of stamps = 6 + 9 + 8 = 23

The difference between the number of stamps collected by Andy and Brian as a percent of the total number of stamps

$= \frac{9-6}{23} \times 100 = \approx 13.00\%$

The difference between the number of stamps collected by Andy and Suzy as a percent of the total number of stamps

$= \frac{8-6}{23} \times 100 = \approx 8.70\%$

The difference between the number of stamps collected by Suzy and Brian as a percent of the total number of stamps

$= \frac{9-8}{23} \times 100 = \approx 4.35\%$

The correct answers are options B, E and F.

146. Probability of the event $= \frac{\text{Number of favorable cases}}{\text{Total number of cases}}$

Thus, the number of favorable cases = The number of ways in which both Kris and David can be selected

$= C_2^2 = 1$

Total number of cases = The number of ways in which any 2 can be selected from the $(n+2)$

$= C_2^{(n+2)} = \frac{(n+2) \times (n+2-1)}{2 \times 1} = \frac{(n+2)(n+1)}{2}$

Thus, required probability

$= \frac{1}{\left(\frac{(n+2)(n+1)}{2}\right)} = \frac{2}{(n+1)(n+2)} \ldots$ Option (B)

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$$= \frac{2}{n^2 + 3n + 2}$$

$$= \frac{2}{n(n+3) + 2} \ldots \text{Option (D)}$$

The correct answers are options B and D.

147. Since there are n sides in the polygon, the number of vertices is also n.

We need to find the number of diagonals that can be formed using the n vertices.

Number of ways in which any 2 vertices can be selected to form a line

$$= C_2^n = \frac{n(n-1)}{2}$$

Among the above number of lines, we have the n sides included; we must exclude them.

Thus, the number of diagonals = Total number of lines – Number of sides

$$= \frac{n(n-1)}{2} - n$$

$$= n\left(\frac{n-1}{2} - 1\right)$$

$$= \frac{n(n-3)}{2}$$

We know that the difference between the number of diagonals and the number of sides is 3.

Thus, we have the following 2 cases:

(A) Number of sides – Number of diagonals = 3

$$=> n - \frac{n(n-3)}{2} = 3$$

$$=> n\left(1 - \frac{n-3}{2}\right) = 3$$

$$=> \frac{n(5-n)}{2} = 3$$

$$=> 5n - n^2 = 6$$

$$=> n^2 - 5n + 6 = 0$$

$$=> (n-2)(n-3) = 0$$

$$=> n = 2 \text{ OR } 3$$

Since, to form a polygon, we need a minimum of 3 sides, the only value of $n = 3$

(B) Number of diagonals – Number of sides = 3

=> $\frac{n(n-3)}{2} - n = 3$

=> $n\left(\frac{n-3}{2} - 1\right) = 3$

=> $\frac{n(n-5)}{2} = 3$

=> $n^2 - 5n = 6$

=> $n^2 - 5n - 6 = 0$

=> $(n-6)(n+1) = 0$

=> $n = 6$ OR -1

Since the number of sides must be positive, the only value of $n = 6$

Thus, the possible values of n are 3 OR 6.

Alternately, we can simply use the values of n given in the options to calculate the number of diagonals and check whether the difference is 3.

For $n = 3$, the number of diagonals is 0 (a triangle has no diagonal) and hence, the difference is 3.

For $n = 6$, a hexagon, the number of diagonals = 3, and hence, the difference is also 3.

The correct answers are options A and D.

148. Let the total number of sold cakes = t.

Type of cake	Number of cakes sold	Revenue generated
Regular	$\frac{m}{100} \times t$	$\$\left(\frac{mt}{100} \times 1\right) = \$\left(\frac{mt}{100}\right)$
Premium	$t - \frac{m}{100} \times t = t\left(1 - \frac{m}{100}\right)$	$\$\left(1.25 \times t\left(1 - \frac{m}{100}\right)\right)$

Thus, total revenue

$= \$\left(\frac{mt}{100} + t\left(1.25 - \frac{1.25m}{100}\right)\right)$

$= \$\left(t\left(\frac{m}{100} + 1.25 - \frac{1.25m}{100}\right)\right)$

$= \$\left(t\left(1.25 - \frac{0.25m}{100}\right)\right)$

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Thus, we have:

Since we know that revenue from regular cakes = r% of total revenue, thus:

$$\frac{mt}{100} = r\% \text{ of } \left(t\left(1.25 - \frac{0.25m}{100}\right)\right)$$

$$=> \frac{mt}{100} = \frac{r}{100} \times t \times \left(1.25 - \frac{0.25m}{100}\right)$$

$$=> m = r\left(1.25 - \frac{0.25m}{100}\right) = r\left(\frac{5}{4} - \frac{m}{400}\right) = \frac{r\,(500 - m)}{400}$$

$$=> r = \frac{400m}{500 - m} \ldots \text{Option (D)}$$

$$= \frac{100 \times 4m}{100 \times \left(5 - \frac{m}{100}\right)}$$

$$= \frac{4m}{5 - \frac{m}{100}} \ldots \text{Option (A)}$$

The correct answers are options A and D.

Alternate approach:

Let us consider the total number of cakes sold is 100 and out of them 20% are regular cakes (m = 20%).

Thus, the number of regular cakes sold = 20% of 100 = 20

Revenue generated from sale of regular cakes = \$ (1×20) = \$20

Number of premium cakes sold = 100 − 20 = 80

Revenue generated from sale of premium cakes = \$ (1.25×80) = \$100

Thus total revenue = \$ $(20 + 100)$ = \$120

As r% of the total revenue comes from regular cakes, we have:

$$r = \frac{20}{120} \times 100 = 16.6\%$$

Now, let us substitute the value of m in the options and check which options give the result 16.6%

(A) $\frac{4m}{5 - \frac{m}{100}} = \frac{4 \times 20}{5 - \frac{20}{100}} = \frac{80}{5 - \frac{1}{5}} = \frac{400}{24} = 16.6\%$ – Correct

(B) $\frac{150m}{250 - m} = \frac{150 \times 20}{250 - 20} = \frac{300}{23} \neq 16.6\%$ – Incorrect

(C) $\frac{300m}{500-2m} = \frac{300 \times 20}{500-40} = \frac{600}{46} \neq 16.6\%$ – Incorrect

(D) $\frac{400m}{500-m} = \frac{400 \times 20}{500-20} = \frac{800}{48} = 16.6\%$ – Correct

(E) $\frac{500m}{625-m} = \frac{500 \times 20}{625-20} = \frac{10000}{605} \neq 16.6\%$ – Incorrect

(F) $\frac{20p}{25-\frac{m}{25}} = \frac{20 \times 20}{25-\frac{20}{25}} = \frac{400}{25-\frac{4}{5}} = \frac{2000}{121} \neq 16.6\%$ – Incorrect

149. Let the selling price per TV and earning per TV be \$$s$ and \$$e$, respectively.

Thus, the initial ratio of selling price per TV to earning per TV

$= \frac{s}{e}$

New selling price per TV

$= \$ \left((100+p)\% \text{ of } s\right)$

$= \$ \left(\frac{s\,(100+p)}{100}\right)$

New earning per TV

$= \$ \left((100+q)\% \text{ of } e\right)$

$= \$ \left(\frac{e\,(100+q)}{100}\right)$

Thus, the final ratio of selling price per TV to earning per TV

$$= \frac{\left(\frac{s\,(100+p)}{100}\right)}{\left(\frac{e\,(100+q)}{100}\right)}$$

$$= \frac{s\,(100+p)}{e(100+q)}$$

Thus, the percent change in the ratio

$$= \frac{\text{Final ratio} - \text{Initial ratio}}{\text{Initial ratio}} \times 100$$

$$= \left(\frac{\frac{s\,(100+p)}{e\,(100+q)} - \frac{s}{e}}{\frac{s}{e}}\right) \times 100\%$$

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$$= \left(\frac{(100 + p)}{(100 + q)} - 1\right) \times 100$$

$$= \frac{100\,(p - q)}{100 + q}\%\ldots\text{Option (D)}$$

$$= \frac{100\,(p - q)}{100\left(1 + \frac{q}{100}\right)}\%$$

$$= \frac{p - q}{\left(1 + \frac{q}{100}\right)}\%\ldots\text{Option (B)}$$

Or, continuing from option (D), we have:

$$\frac{100\,(p - q)}{100 + q}\%$$

$$= \frac{100q\left(\frac{p}{q} - 1\right)}{q\left(\frac{100}{q} + 1\right)}\%$$

$$= \frac{100\left(\frac{p}{q} - 1\right)}{\left(\frac{100}{q} + 1\right)}\%\ldots\text{Option (E)}$$

The correct answers are options B, D and E.

Alternate approach:

Let's consider selling price per TV initially as \$100.

Since there was an increase of p%, assuming $p = 20$, we have the new selling price equals to \$120.

Let's consider the initial earning per TV = \$50.

Since there was increase of q%, assuming $q = 10$, we have the new earning = \$55.

Initially, $\frac{\text{Selling price}}{\text{Earning}} = \frac{100}{50} = 2$

Finally, $\frac{\text{Selling price}}{\text{Earning}} = \frac{120}{55} = \approx 2.182$

Thus, the required percentage increase in the ratio = $\frac{2.182 - 2}{2} \times 100 = 9.1\%$

Now by putting $p = 20$ and $q = 10$ in the options, we have:

(A) $\frac{p}{q} = \frac{20}{10} = 2 \neq 9.1$ – Incorrect

(B) $\dfrac{p-q}{\left(1+\dfrac{q}{100}\right)}\% = \dfrac{20-10}{1+\dfrac{10}{100}} = \dfrac{10}{1.1} = \dfrac{100}{11} = \approx 9.1$ – Correct

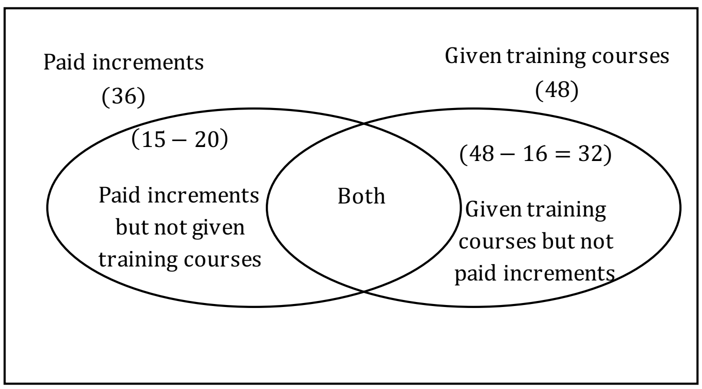
(C) $\dfrac{100\,(p-q)}{100+p}\% = \dfrac{100\,(20-10)}{100+20} = \dfrac{1,000}{120} \neq 9.1$ – Incorrect

(D) $\dfrac{100\,(p-q)}{100+q}\% = \dfrac{100\,(20-10)}{100+10} = \dfrac{1,000}{110} = \approx 9.1$ – Correct

(E) $\dfrac{100\left(\dfrac{p}{q}-1\right)}{\left(\dfrac{100}{q}+1\right)}\% = \dfrac{100\left(\dfrac{20}{10}-1\right)}{\dfrac{100}{10}+1} = \dfrac{100}{11} = \approx 9.1$ – Correct

(F) $\dfrac{100\,(p-q)}{100+p+q}\% = \dfrac{100\,(20-10)}{100+20+10} = \dfrac{1,000}{130} \neq 9.1$ – Incorrect

150. Let us represent the above information using a Venn-diagram, as shown below:



# of employees that were given pay increments and premium training courses

= 36 – (# of employees that were given pay increments but not premium training courses)

= 36 – 15 = 21 (Maximum)

OR

36 – 20 = 16 (Minimum)

Thus, the number of employees that were given premium training courses but did not pay increments

= 48 – (# of employees that were given pay increments and premium training courses)

= 48 – 16 = 32 (Maximum)

OR

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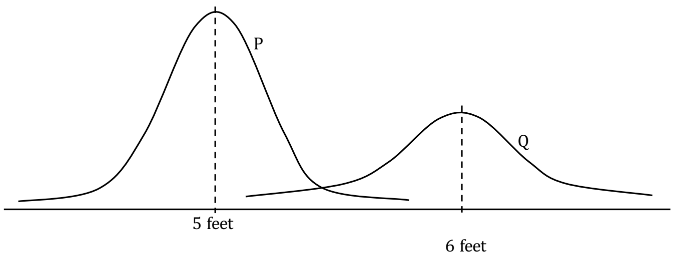
48 – 21 = 27 (Minimum)

The correct answers are options C, D and E.

151. Let us bring out the figure.



Working with the options:

Option A:

We can see that the curve for Q is more spread out than that of P.

Since standard deviation measures the spread of the data, we can say that Q has a larger standard deviation than P. – Correct

Option B:

The mean in a normal curve is at the point where the peak of the curve is attained.

Thus, mean for P is 5 feet while that for Q is 6 feet.

Thus, P does not have a larger mean than Q. – Incorrect

Option C:

In a normal distribution, the mean and median are the same point.

Thus, median for P is 5 feet while that for Q is 6 feet.

Thus, Q has a larger median than P. – Correct

Option D:

The percent of students within one standard deviation of the mean for any normal distribution is always the same, 68.23%. – Incorrect

Option E:

We can see that there in an overlap between the curves for the 7th and the 10th standards.

Thus, some students of the 7th grade may be taller than some students of the 10th grade. – Incorrect.

The correct answers are options A and C.

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152. Let the quantity of the sample be x.

Thus, amount of impurity present
= 10% of x

$$= \frac{10}{100} \times x$$

$$= \frac{x}{10}$$

After each purification process, the amount of impurity reduces by 60%.

Let the required number of times the purification has to be done = n.

Thus, the final amount of impurity

$$= \frac{x}{10} \times \left(1 - \frac{60}{100}\right)^n$$

Thus, we have:

$$\frac{x}{10} \times \left(1 - \frac{60}{100}\right)^n \leq 1\% \text{ of } x$$

$$=> \frac{x}{10} \times \left(1 - \frac{60}{100}\right)^n \leq \frac{x}{100}$$

$$=> \left(\frac{2}{5}\right)^n \leq \frac{1}{10}$$

$$=> \left(\frac{2 \times 2}{5 \times 2}\right)^n \leq \frac{1}{10}$$

$$=> \left(\frac{4}{10}\right)^n \leq \frac{1}{10}$$

$$=> 4^n \leq 10^{(n-1)}$$

As per the answer options, for $n = 2$, $4^2 = 16 \nleq 10^1$ – Does not satisfy

For $n = 3$, $4^3 \leq 10^2 => 64 \leq 100$ – Satisfies

Thus, we need to purify the water sample for a minimum of 3 times.

The correct answers are options B, C, D and E.

153. Since the product of the two integers should be negative, we need to select a positive integer from one set and a negative integer from the other set.

Since set M has all terms negative, we need to select the positive integer from set T.

Set T has $(n + 3)$ terms, of which, n are positive.

Number of ways of selecting a negative integer from set M

$= C_1^5 = 5$

Number of ways of selecting a positive integer from set T

$= C_1^n = n$

Thus, number of favorable cases = $5 \times n$.

Total number of cases

= (# of ways of selecting an integer from set M) $\times$ (# of ways of selecting an integer from set T)

$= C_1^5 \times C_1^{(n+3)} = 5 \times (n + 3)$

Thus, required probability

$= \frac{\text{Number of favorable cases}}{\text{Total number of cases}}$

$= \frac{5n}{5(n+3)}$

$= \frac{n}{(n+3)}$

Thus, we have:

$\frac{n}{n+3} > \frac{3}{5}$

$=> 5n > 3n + 9$

$=> n > \frac{9}{2} = 4.5$

Among the options, the eligible values for n are 5, 7, 8, and 9.

The correct answers are options B, C, D and E.

154. Machine Q produces 100 parts of product K in n minutes.

Since machine P produces parts twice as fast as machine Q does, time taken by machine P to produce 100 parts of product K

$= \frac{n}{2}$ minutes

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Since each part of product R takes $\frac{3}{2}$ times the time taken to produce each part of product K, the time taken by machine P to produce 100 parts of product R

$$= \frac{n}{2} \times \frac{3}{2} = \frac{3n}{4} \text{ minutes}$$

Thus, the number of parts of product R produced by machine P in $\frac{3n}{4}$ minutes = 100.

Thus, the number of parts produced in t minutes

$$= \frac{100}{\left(\frac{3n}{4}\right)} \times t$$

$$= 100 \times \frac{4}{3n} \times t$$

$$= \frac{400t}{3n} \ldots \text{Option (A)}$$

$$= \frac{t}{\left(\frac{3n}{400}\right)}$$

$$= \frac{t}{0.0075n} \ldots \text{Option (E)}$$

The correct answers are options A and E.

155. There are three equal terms (lowest) and five equal terms (greatest).

Let the lowest term be a.

=> The greatest term = $3a$.

Thus, the 8 terms in the set X are:

$a, a, a, 3a, 3a, 3a, 3a, 3a$

Thus, the median is the average of the $\left(\frac{8}{2}\right)^{\text{th}}$ and $\left(\frac{9+1}{2}\right)^{\text{th}}$ terms

= average of 4^{th} and 5^{th} term

$$= \frac{3a + 3a}{2} = 3a$$

Thus, to determine the median, we need to determine the value of a.

Working with the options:

Option A:

For any four terms, the average will be the lowest when the terms selected are:

a, a, a, $3a$

Since the average is 12, we have:

$\frac{a + a + a + 3a}{4} = 12$

=> $a = 8$

=> Median = $3a = 24$ – Sufficient

Option B:

The range of all the terms in the set

= Greatest term – Smallest term

= $3a - a$

= $2a$

= Twice the value of the least term

Thus, this statement does NOT offer any new information using which we can solve for a. – Insufficient

Option C:

The statement implies that $3a - a = 16$

=> $2a = 16$ => $a = 8$

=> Median = $3a = 3 \times 8 = 24$ – Sufficient

The correct answers are options A and C.

156. Total charge for Mike = \$ $(18 + xd)$

Total charge for Tom = \$ $(25 + yd)$

Thus, we have:

$18 + xd = 25 + yd$

=> $(x - y)d = 7$

=> $d = \frac{7}{x - y}$

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Thus, the charge Mike (equal to that of Tom) had to pay

$= \$(18 + xd)$

$= \$\left(18 + x \times \left(\frac{7}{x-y}\right)\right) \ldots$ Option (C)

$= \$\left(18 + \frac{7x}{x-y}\right)$

$= \$\left(\frac{25x - 18y}{x-y}\right) \ldots$ Option (E)

The correct answers are options C and E.

157. Let the price of the smallest size of bottle = $\$x$.

Successive sizes of the bottles are priced at $\$d$ more than the immediately preceding size.

Thus, the prices of the bottles form an arithmetic progression whose first term $a = x$ and the constant difference = d.

The n^{th} term in arithmetic progression is given by $(a + (n-1)\,d)$

Thus, the price of the bottle of the largest size, i.e. the 6^{th} bottle

$= \$\,(x + (6-1)\,d)$

$= \$(x + 5d)$

Thus, the mean price of the 6 bottles

$= \frac{\text{Price of the first bottle} + \text{Price of the sixth bottle}}{2}$

$= \$\left(\frac{x + (x + 5d)}{2}\right)$

$= \$\frac{(2x + 5d)}{2}$

Thus, the total price of all 6 bottles

= Average price of a bottle × Number of bottles

$= \$\left(\frac{(2x + 5d)}{2} \times 6\right)$

$= \$(6x + 15d)$

Thus, we have:

$6x + 15d = 75$

$=> 2x + 5d = 25 \ldots$ (i)

Since price of each bottle is an integer (obviously positive), x must be a positive integer.

Also, since price of each bottle is \$$d$ MORE than the next one below it in size, d is a positive integer.

In (i), since 25 is a multiple of 5 and the coefficient of d is 5, x must also be a multiple of 5.

Thus, possible positive integer solutions of (i) are:

$x = 5,\ d = 3$

OR

$x = 10,\ d = 1$

Thus, possible values of x (price of the smallest bottle) are 5 or 10.

The correct answers are options E and G.

158. Distance covered by Katie in 15 minutes traveling at 50 miles per hour

$= 50 \times \frac{15}{60}$

= 12.5 miles

Thus, when Jack crossed the milestone, the distance between Katie and Jack = 12.5 miles.

Since both Katie and Jack are traveling in the same direction, their relative speed

= Speed of Jack – Speed of Katie

= 60 – 50 = 10 miles per hour.

The distance between them will be 10 miles in two cases:

(A) Jack is 10 miles behind Katie

Thus, Jack gained 12.5 – 10 = 2.5 miles over Katie

Thus, time taken

$= \frac{\text{Distance}}{\text{Relative speed}}$

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$= \frac{2.5}{10}$

= 0.25 hours

= 0.25 × 60 minutes

= 15 minutes

(B) Jack is 10 miles ahead of Katie

Thus, Jack gained 10 – (– 12.5) = 22.5 miles over Katie

Thus, time taken

$= \frac{\text{Distance}}{\text{Relative speed}}$

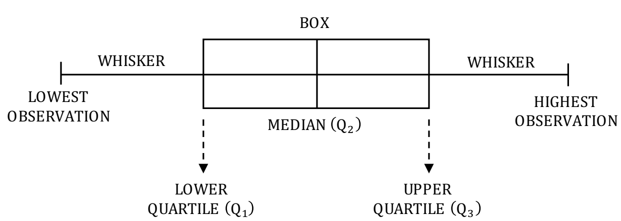
$= \frac{22.5}{10}$ hours

$= \frac{22.5}{10} \times 60$ minutes

= 135 minutes

The correct answers are options B and D.

159. The information that can be obtained from a box-and-whisker plot is shown in the figure below:



A box-and-whisker plot is a convenient means of graphically representing data using quartiles.

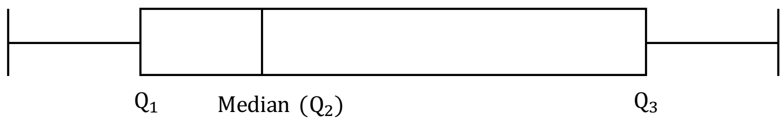
We have three major points: the first middle point (the median, or Q_2), and the middle points of the two halves, the lower quartile (Q_1) and the upper quartile (Q_3), which divide the entire data set into quarters, called 'quartiles'.

Q_1 is the middle number for the first half of the list, Q_2 is the middle number for the whole list, and Q_3 is the middle number for the second half of the list.

The box in the middle, i.e. from Q_1 to Q_3 represents the middle 50 percent of the data, also known as the Inter-Quartile-Range ($Q_3 - Q_1$).

Thus, the box is used to identify each of the two middle quartile groups of data, and the 'whiskers' extend outward from the boxes to the lowest and highest values observed.

Thus, we have:



Working with the options:

Option A:

Median of the whole set = Q_2

Median of the lower half of the data = Q_1

Median of the upper half of the data = Q_3

We can observe that, in the given box-and-whisker plot, Q_2 is closer to Q_1 than to Q_3.

Thus, the median of the whole set is closer to the median of the lower half of the data than it is to the median of the upper half of the data. – Correct

Option B:

Standard deviation is ‘0’ only for data which has the same value, i.e. no spread.

The box-and-whisker plot above clearly shows the spread of the data.

Hence, the standard deviation must be greater than zero. – Correct

Option C:

In a box-and-whisker plot, no information can be derived about the mean.

Thus, the mean need not be mid-way of the box.

Thus, we cannot determine whether the mean of the whole set is greater than the median. – Incorrect

The correct answers are options A and B.

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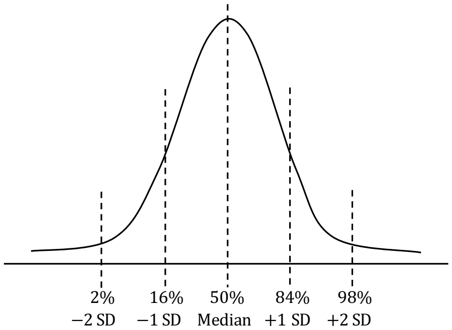
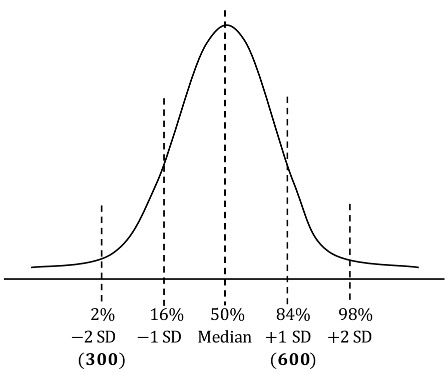
160. A normally distributed data is shown below:



We know that the 2^{nd} percentile is 300 and the 84^{th} percentile is 600.

Note that the 2^{nd} percentile corresponds to (-2)SD, while 84^{the} percentile corresponds to $(+1)$SD.

Thus, we have the following diagram:



Thus, the difference between 300 and 600 is equal to 3 SD

Thus, we have:

$3 \times SD = 600 - 300$

$=> 3 \times SD = 300$

$=>$ SD $= \frac{300}{3} = 100$

Thus, the mean = $300 + 2 \times$ SD

$= 300 + 2 \times 100$

$= 500$

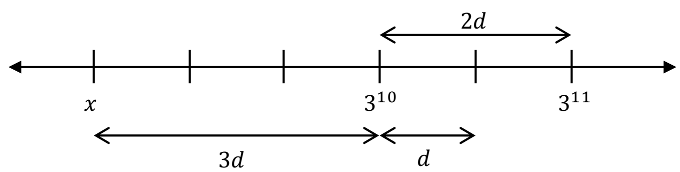
Thus, we have:

The value, (-1)SD from the mean = 500 – 100 = 400;

Thus, the scores within one SD from the mean would be between 400 and 600, inclusive.

The correct answers are options C, D and E.

161. Let us bring out the number line.



Let the length of the line between the two consecutive tick marks be d.

Thus, the length of the line segment between the points 3^{10} and 3^{11} is $2d$.

Thus, we have:

$2d = 3^{11} - 3^{10}$

$= 3 \times 3^{10} - 3^{10}$

$\Rightarrow 2d = 3^{10}(3-1) = 2 \times 3^{10}$

$\Rightarrow d = 3^{10}$

$\Rightarrow 3d = 3 \times 3^{10} = 3^{11}$

Since the length of the line segment between the points x and 3^{10} is $3d$, we have:

$x = 3^{10} - 3d = 3^{10} - 3^{11}$

$= 3^{10} - 3 \times 3^{10} = 3^{10}(1-3)$

$= -2 \times 3^{10}$

Working with the options:

(A) $x = -3^{11}$ – Incorrect

(B) $x = -2\left(3^{10}\right)$ – Correct

(C) $x^3 = \left(-2 \times 3^{10}\right)^3 = -8 \times 3^{30} < x = -2 \times 3^{10}$ – Correct

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(D) $x^5 = \left(-2 \times 3^{10}\right)^5 = -32 \times 3^{50}$

$x^3 = \left(-2 \times 3^{10}\right)^3 = -8 \times 3^{30}$

Thus, $x^5 \ngtr x^3$ – Incorrect

The correct answers are options B and C.

162. $a_{n+1} = 1 + \frac{1}{a_n}$

$a_1 = 1$

If $n = 1:\ a_2 = 1 + \frac{1}{a_1} = 1 + 1 = 2$

If $n = 2:\ a_3 = 1 + \frac{1}{a_2} = 1 + \frac{1}{2} = \frac{3}{2}$

If $n = 3:\ a_4 = 1 + \frac{1}{a_3} = 1 + \frac{1}{\left(\frac{3}{2}\right)} = 1 + \frac{2}{3} = \frac{5}{3}$

If $n = 4:\ a_5 = 1 + \frac{1}{a_4} = 1 + \frac{1}{\left(\frac{5}{3}\right)} = 1 + \frac{3}{5} = \frac{8}{5}$

Working with the options:

(A) $\left(a_4 = \frac{5}{3}\right) > \left(a_3 = \frac{3}{2}\right)$ – Correct

(B) $\left(a_5 = \frac{8}{5}\right) \ngtr \left(a_4 = \frac{5}{3}\right)$ – Incorrect

(C) $a_5 = \frac{8}{5}$ – Correct

The correct answers are options A and C.

163. Let the time taken by the man while traveling by air and while traveling by train be t hours and T hours, respectively.

Since the total time is 8 hours, we have:

$T + t = 8 \ldots$(i)

The man would have saved $\frac{4}{5}$ of the time he was in train had he travelled all the way by air.

=> Time saved = $\frac{4T}{5}$

Again, we know that he would reach 4 hours early had he travelled entirely by air.

Thus, the time saved is 4 hours

=> $\frac{4T}{5} = 4$

=> $T = 5$

=> $t = 8 - 5 = 3$ ...using equation (i)

Had the man travelled entirely by air, he would have taken 8 – 4 = 4 hours.

Thus, speed of the man by air = $\frac{720}{4}$ = 180 miles per hour.

Thus, the distance, in miles, originally travelled by the man by air = $180 \times t$

$= 180 \times 3 = 540$

Working with the options:

(A) Incorrect

(B) Correct

(C) Distance travelled by train = 720 – 540 = 180 miles.

Thus, distance travelled by air = $540 = 3 \times 180$ – Correct

(D) $\frac{\text{Distance travelled by air}}{\text{Total distance}} = \frac{540}{720} = \frac{3}{4} \neq \frac{1}{3}$ – Incorrect

The correct answers are options B and C.

Alternate approach:

Total time taken by the man traveling partly by air and partly by train = 8 hours.

Had he travelled all the distance by air, he would have saved 4 hours.

Thus, time required to travel 720 miles by air = $8 - 4 = 4$ hours.

Thus speed through air = $\frac{\text{Distance}}{\text{time}} = \frac{720}{4}$ = 180 miles per hour.

Let's consider, for t hours man travelled by train.

Thus, $\frac{4}{5}t = 4 => t = 5$

Thus time for which distance is covered by air = $8 - 5 = 3$

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Thus, distance travelled by air in 3 hours = $3 \times 180 = 540$ miles . . . Option B

Thus, distance travelled by train in 5 hours = $720 - 540 = 180$ miles.

Thus we can say that distance travelled by air is thrice of train . . . Option C

164. $x^2y^2 - xy = 6$

$=> x^2y^2 - xy - 6 = 0$

$=> x^2y^2 - 3xy + 2xy - 6 = 0$

$=> xy\,(xy - 3) + 2\,(xy - 3) = 0$

$=> (xy + 2)\,(xy - 3) = 0$

$=> xy = -2$ OR 3

$=> y = -\frac{2}{x}$ OR $\frac{3}{x}$

The correct answers are options B and C.

Alternate approach:

We can assume a convenient value of x and cross-check with the options for the value of y.

Say, $x = 1$, then

Option A: $y = \frac{6}{1} = 6$

Since $xy = 1 \times 6 = 6 \neq 0$, it satisfies the first condition ($xy \neq 0$), but $x^2y^2 - xy = 1^2 \times 6^2 - 1 \times 6 = 30 \neq 6$, we discard this option.

Option B: $y = \frac{-2}{1} = -2$

Since $xy = 1 \times -2 = -2 \neq 0$, it satisfies the first condition, and $x^2y^2 - xy = 1^2 \times (-2)^2 - 1 \times -2 = 4 + 2 = 6$, we accept this option.

Option C: $y = \frac{3}{1} = 3$

Since $xy = 1 \times 3 = 3 \neq 0$, it satisfies the first condition, and $x^2y^2 - xy = 1^2 \times 3^2 - 1 \times 3 = 9 - 3 = 6$, we accept this option.

165. Since we need to find the average speed, we may assume any suitable value of the distance.

Since we need to take $\frac{1}{3}$ of the distance, and also divide a part of the distance by 12, 20 and 40 (since speeds are given as 20, 12 and 40 miles per hour), it is best to assume the distance to be a multiple of 120 (LCM of 20, 12 and 40).

To be on the safer side, let us assume the distance to be 360 miles, a multiple of LCM = 120.

Distance travelled at 20 miles per hour = $\frac{1}{3} \times 360 = 120$ miles.

Thus, time taken = $\frac{\text{Distance}}{\text{Speed}} = \frac{120}{20} = 6$ hours . . . (i)

Distance left to be covered = 360 – 120 = 240 miles.

Distance travelled at 12 miles per hour = $\frac{1}{2} \times$ distance left to be covered = $\frac{1}{2} \times 240 = 120$ miles.

Thus, time taken = $\frac{\text{Distance}}{\text{Speed}} = \frac{120}{12} = 10$ hours . . . (ii)

Distance left to be covered = 240 – 120 = 120 miles.

This distance is travelled at 40 miles per hour.

Thus, time taken = $\frac{\text{Distance}}{\text{Speed}} = \frac{120}{40} = 3$ hours . . . (iii)

From (i) and (iii):

The time taken to travel the part of the distance at 40 miles per hour was $\frac{1}{2}$ of the time taken to cover the part of the distance at 20 miles per hour . . . Option A

From (i), (ii) and (iii), we have:

Total time taken = 6 + 10 + 3 = 19 hours ≠ 18 hours – Option B is incorrect.

Thus, average speed

$= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{360}{19}$

= 18.9 miles per hour (we can calculate the value easily using a calculator)
≠ 20 hours – Option C is incorrect.

The correct answer is option A.

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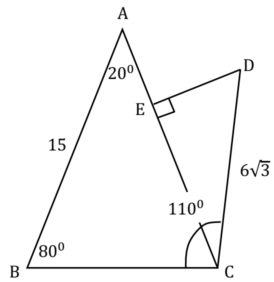
166. Let us bring out the figure.



In triangle ABC, we have:

$\angle ACB = 180^\circ - (\angle ABC + \angle CAB)$

$= 180^\circ - (80^\circ + 20^\circ) = 80^\circ$

Thus, triangle ABC is isosceles with AB = AC = 15

$\angle DCE = 110^\circ - \angle ACB$

$= 110^\circ - 80^\circ = 30^\circ$

=> $\angle CDE = 180^\circ - (\angle DCE + \angle DEC)$

$= 180^\circ - (30^\circ + 90^\circ) = 60^\circ$

Thus, triangle DEC is a 60-90-30 triangle.

DC is the hypotenuse of the triangle DEC.

In a 60-90-30 triangle, we know that the side opposite to the 60° is $\frac{\sqrt{3}}{2}$ times the side opposite to the 90°.

Thus, we have:

$CE = \frac{\sqrt{3}}{2} \times CD$

=> $CE = \frac{\sqrt{3}}{2} \times 6\sqrt{3} = 9$

=> $AE = AC - CE$

=> $AE = 15 - 9 = 6$

Working with the options:

Option A:

$3\sqrt{2} = 3 \times 1.41 = 4.23$

$4\sqrt{3} = 4 \times 1.73 = 6.92$

Thus, $3\sqrt{2} < (AE = 6) < 4\sqrt{3}$ – Correct

Option B:

AE = 6 and CE = 9 => $AE \not> CE$ – Incorrect

Option C:

In triangle CDE, DE is the side opposite to 30°

=> $DE = \frac{1}{2} \times CD = \frac{1}{2} \times 6\sqrt{3} = 3\sqrt{3} \neq AE$ – Incorrect

Option D:

$AE^2 + CD^2 = 6^2 + (6\sqrt{3})^2 = 36 + 108 = 144 = 12^2$ – Correct

The correct answers are options A and D.

167. The center of the circle is at O $(0,\ 0)$.

Since A lies on the circle, OA = radius of the circle

From the figure: OA = radius = 10

Since P $(8,\ k)$ lies on the circumference of the circle, OP = radius of the circle

=> $\sqrt{(8-0)^2 + (k-0)^2} = 10$

=> $\sqrt{64 + k^2} = 10$

=> $64 + k^2 = 100$ (squaring both sides)

=> $k^2 = 36$

=> $k = \pm 6$

(A) If the co-ordinates of P are $(8,\ -6)$:

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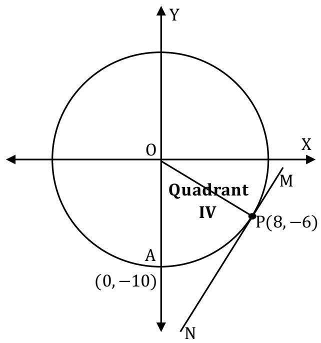

Thus, slope of the line OP

$$= \frac{\text{Difference between y} - \text{cordinates}}{\text{difference between x} - \text{cordinates}}$$

$$= \frac{0 - (-6)}{0 - 8} = -\frac{3}{4}$$

Whenever two lines l_1 (slope m_1) and l_2 (slope m_2) are perpendicular, product of the slopes is equal to -1, i.e. $m_1 \times m_2 = -1$.

Let's recall the property that the line joining the center of the circle and the point of contact with a tangent is always perpendicular to the tangent.

Thus, the tangent (MN) at P is perpendicular to the radius (OP):

$$=> \text{Slope of MN} = -\frac{1}{\text{Slope of OP}}$$

$$= -\frac{1}{\left(-\frac{3}{4}\right)} = \frac{4}{3}$$

(B) If the co-ordinates of P are $(8,\ 6)$:

In the same way as above, we have:

Slope of the line OP

$$= \frac{0 - 6}{0 - 8} = \frac{3}{4}$$

Since the tangent (M′N′) at P is perpendicular to the radius (OP), we have:

$$\text{Slope of M}'\text{N}' = -\frac{1}{\text{Slope of OP}}$$

$$= -\frac{1}{\left(\frac{3}{4}\right)} = -\frac{4}{3}$$

The correct answers are options B and E.

168. Given: $y = \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\left(2 + \cfrac{1}{\ldots\infty}\right)}}}$

$$y = \cfrac{1}{2 + \left(\cfrac{1}{2 + \cfrac{1}{\left(2 + \cfrac{1}{\ldots\infty}\right)}}\right)}$$

$y = \dfrac{1}{2+y}$; since the expression goes to infinity.

$2y + y^2 = 1$

=> $y^2 = 1 - 2y \ldots$ Option (B)

=> $y^2 + 2y = 1$

=> $y^2 + 2y + 1 = 1 + 1$ (adding 1 to both sides to form perfect square)

=> $(y+1)^2 = 2$

=> $y + 1 = \pm\sqrt{2}$

=> $y = \pm\sqrt{2} - 1$

Since y consists of all positive terms, $y > 0$

=> $y = \sqrt{2} - 1 \ldots$ Option (C)

The correct answers are options B and C.

169. We are given that the probability of Andy winning the competition is $\frac{1}{a}$, thus, the probability of Bob winning the competition is $\frac{1}{a} \times 2 = \frac{2}{a}$, and the probability of Chad winning the competition is $\frac{1}{a} \times 3 = \frac{3}{a}$.

Similarly, the probability of Andy NOT winning the competition is $1 - \frac{1}{a} = \frac{a-1}{a}$, thus, the probability of Bob winning the competition is $1 - \frac{2}{a} = \frac{a-2}{a}$, and the probability of Chad winning the competition is $1 - \frac{3}{a} = \frac{a-3}{a}$.

Scenario 1: Andy wins and others lose:

Required probability = $\frac{1}{a} \times \frac{a-2}{a} \times \frac{a-3}{a} = \frac{(a-2)(a-3)}{a^3}$ – Option A is correct.

Scenario 2: Bob wins and others lose:

Required probability = $\frac{a-1}{a} \times \frac{2}{a} \times \frac{a-3}{a} = \frac{2(a-1)(a-3)}{a^3}$ – Option B is correct.

Scenario 3: Chad wins and others lose:

Required probability = $\frac{a-1}{a} \times \frac{a-2}{a} \times \frac{3}{a} = \frac{3(a-1)(a-2)}{a^3}$ – Option C is correct.

The correct answers are options A, B and C.

170. $f(x) = \frac{1}{x}$

$g(x) = \frac{x}{x^2 + 1}$

Working with the options:

Option A:

$g(2) = \frac{2}{2^2 + 1} = \frac{2}{5}$

$=> f(g(2)) = f\left(\frac{2}{5}\right)$

$= \frac{1}{\left(\frac{2}{5}\right)}$

$= \frac{5}{2}$ – Correct

Option B:

$g(f(x)) = g\left(\frac{1}{x}\right)$

$= \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^2 + 1}$

$= \frac{\left(\frac{1}{x}\right)}{\left(\frac{1+x^2}{x^2}\right)}$

$= \frac{1}{x} \times \frac{x^2}{1 + x^2}$

$= \frac{x}{1 + x^2} \ldots$ (i)

$\neq x$ – Incorrect

Option C:

From (i) above, we have:

$g(f(x)) = g(x)$ – Correct

The correct answers are options A and C.

171. $P(r) = \frac{8r}{1-r}$

$=> P(r^2) = \frac{8r^2}{1-r^2}$

Also, $P\left(\frac{1}{2}\right) = \frac{8 \times \left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)} = 8$

Thus, we have:

$P(r^2) = \frac{1}{2} \times (8)$

$=> \frac{8r^2}{1-r^2} = 4$

$=> 8r^2 = 4 - 4r^2$

$=> 12r^2 = 4$

$=> r^2 = \frac{1}{3}$

$=> r = \pm\frac{1}{\sqrt{3}}$

The correct answers are options A and E.

172. We have:

$f(x) = ax^2 + b$

Thus, $3f(x) + 2f(-x)$

$= 3(ax^2 + b) + 2\left(a(-x)^2 + b\right)$

$= 3(ax^2 + b) + 2(ax^2 + b)$

$= 5(ax^2 + b)$

Since $3f(x) + 2f(-x) = 5x^2 - 10$ for all values of x, we have:

$5(ax^2 + b) = 5x^2 - 10$

$=> 5(ax^2 + b) = 5(x^2 - 2)$

$=> ax^2 + b = x^2 - 2$

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Comparing coefficients on LHS and RHS:

$a = 1$ & $b = -2$

Thus, $f(x) = x^2 - 2$

Alternately, we see that:

$f(-x) = f(x)$

Thus, $3f(x) + 2f(-x) = 5x^2 - 10$

=> $5f(x) = 5x^2 - 10$

=> $f(x) = x^2 - 2$

Since $f(x) = 7$, we have:

$x^2 - 2 = 7$

=> $x^2 = 9$

=> $x = \pm 3$

The correct answers are options B and D.

173. $\sqrt{x} = 25$

=> $x = 25^2 = (5^2)^2$

=> $x = 5^4$

Thus, we have:

$N = x^3 - x^2$

$= (5^4)^3 - (5^4)^2$

$= 5^{12} - 5^8$

$= 5^8 (5^4 - 1)$

$= 5^8 (625 - 1)$

$= 624 (5^8)$

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$= 6 \times 104 \times 5^8$

$= (2 \times 3) \times (2^3 \times 13) \times 5^8$

$= 2^4 \times 3 \times 5^8 \times 13$

Working with the options:

(A) 5^{12} is not a factor of N. – Incorrect

(B) 5^{10} is not a factor of N. – Incorrect

(C) $12 \times 5^8 = 2^2 \times 3 \times 5^8$

We can observe that the above is a factor of N. – Correct

(D) 31 is not a factor of N. – Incorrect

(E) $39 \times 5^7 = 3 \times 13 \times 5^7$

We can see that the above is a factor of N. – Correct

The correct answers are options C and E.

174. $f(x) = x^3 - kx^2 + 2x$

$=> f(-x) = (-x)^3 - k(-x)^2 + 2(-x)$

$= -x^3 - kx^2 - 2x$

Since $f(-x) = -f(x)$, we have:

$-x^3 - kx^2 - 2x = -(x^3 - kx^2 + 2x)$

$=> 2kx^2 = 0$

$=> k = 0$

Thus, we have:

$f(x) = x^3 + 2x$

Working with the options, we have:

(A) $f(-3) = (-3)^3 + 2(-3) \neq 3$ – Incorrect

(B) $f(1) = 1^3 + (2 \times 1) = 3$ – Correct

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(C) $f(k-1) = f(0-1) = f(-1) = (-1)^3 + 2(-1) \neq 3$ – Incorrect

(D) $f(1+k) = f(1+0) = f(1) = 1^3 + (2 \times 1) = 3$ – Correct

The correct answers are options B and D.

175. We have:

$f(x) = ax^2 + bx + c$

Thus, $f(2) = a \times 2^2 + b \times 2 + c = 8$

$=> 4a + 2b + c = 8$

Let's analyze the above linear equation keeling in mind that a, b and c are positive integers.

Since $4a$ & $2b$, and 8 are even integers, c must also be even integer. So, the minimum value of c must be 2. With $c = 2$, we have $a = b = 1$. There is no other possibility.

Let's find out the sum of any two among a, b and c.

1. $a + b = 1 + 1 = 2$ – Option A is correct.

2. $a + c = 1 + 2 = 3$ – Option B is correct.

3. $c + b = 2 + 1 = 3$ – We already saw that Option B is correct.

The correct answers are options A and B.

176. $f(x-1) = 2x^2 - 3x + 3$

We need to express the RHS in terms of $(x - 1)$.

$f(x-1) = 2((x-1)+1)^2 - 3((x-1)+1) + 3$; rewriting x as $x - 1 + 1$.

Thus, $f(x) = 2(x+1)^2 - 3(x+1) + 3$

$=> f(x) = 2(x^2 + 2x + 1) - 3x - 3 + 3$

$=> f(x) = 2x^2 + 4x + 2 - 3x$

$=> f(x) = 2x^2 + x + 2$

$=> f(-x) = 2(-x)^2 + (-x) + 2$; replacing x with $-x$

$=> f(-x) = 2x^2 - x + 2$

Since $f(x) = 2f(-x) - 1$, we have:

$2x^2 + x + 2 = 2(2x^2 - x + 2) - 1$

$=> 2x^2 + x + 2 = (4x^2 - 2x + 4) - 1$

$=> 2x^2 - 3x + 1 = 0$

$=> (2x - 1)(x - 1) = 0$

$=> x = \frac{1}{2}$ OR 1

The correct answers are options A and B.

177. $K = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$= \frac{8 + 4 + 2 + 1}{16}$

$= \frac{15}{16}$

$= 0.9375 < 1$

Let's see all the options.

(A) 0 and $\frac{7}{9}$ => 0 and 0.78: We see that $K = 0.9375$ is out of the range.

(B) $\frac{1}{10}$ and $\frac{12}{17}$ => 0.10 and 0.706: We see that $K = 0.9375$ is out of the range.

(C) $\frac{1}{4}$ and $\frac{17}{18}$ => 0.25 and 0.944: We see that $K = 0.9375$ falls in this range.

(D) $\frac{3}{10}$ and $\frac{23}{21}$ => 0.30 and 1.09: We see that $K = 0.9375$ falls in this range.

(E) $\frac{11}{12}$ and $\frac{23}{21}$ => 0.917 and 1.09: We see that $K = 0.9375$ falls in this range.

The correct answers are options C, D and E.

178. Let the smallest term be a.

=> The greatest term = $5a$.

Thus, the four terms in the set X are $a,\ 5a,\ 5a,\ 5a$

Thus, the median is the average of the $\left(\frac{4}{2}\right)^{\text{th}}$ and $\left(\frac{4}{2} + 1\right)^{\text{th}}$ terms

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= average of 2nd and 3rd terms

$= \frac{5a + 5a}{2} = 5a$

So, if we get the value of a, we get the answer.

Let's take each option one by one.

A. Arithmetic mean (Average) of the set

The average of all the terms in the set

$= \overline{x} = \frac{a + 5a + 5a + 5a}{4} = 4a$

If the value of $4a$ is known, we get the value of a. – Correct

B. Range of the set

Range of the set = $5a - a = 4a$.

As with Option A, if the value of $4a$ is known, we get the value of a. – Correct

C. Value of any one of the terms

If we know the value of $5a$ or a, we get the answer. – Correct

The correct answers are options A, B and C.

179. We have:

$h(x) = 2^{px+1}$

$g(x) = 2^{px} + 1$

Since $g(k) = 2h(k) - 2$, we have:

$2^{pk} + 1 = 2\left(2^{pk+1}\right) - 2$

$=> 2^{pk} + 1 = 2\left(2^{pk} \times 2^1\right) - 2$

$=> 2^{pk} + 1 = 4\left(2^{pk}\right) - 2$

$=> 4\left(2^{pk}\right) - 2^{pk} = 1 + 2$

$=> 3\left(2^{pk}\right) = 3$

=> $2^{pk} = 1 = 2^0$

=> $pk = 0$ (Option C is correct)

=> $p = 0$ OR $k = 0$ OR $p = k = 0$

Working with the remaining two statements:

Option A:

We cannot determine whether $p = 0$. Hence, we cannot determine the value of 2^p. – Incorrect

Option B:

Since $pk = 0$ => $k = 0$, given that $p \neq 0$. – Correct

The correct answers are options B and C.

180. We have:

$f(x) = ax^2 + bx \ldots$(i)

$f(x+1) = f(x) + x + 1 \ldots$(ii)

Substituting $x = 0$ in (i):

$f(0) = 0 \ldots$(iii)

Substituting $x = 1$ in (i):

$f(1) = a + b \ldots$(iv)

Substituting $x = 0$ in (ii):

$f(0+1) = f(0) + 1$

=> $f(1) = f(0) + 1$

=> $a + b = 0 + 1 \ldots$using (iii) and (iv)

=> $a + b = 1 \ldots$(Option A)

Substituting $x = -1$ in (i):

$f(-1) = a(-1)^2 + b(-1)$

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$\Rightarrow f(-1) = a - b \ldots$(v)

Substituting $x = -1$ in (ii):

$f(-1+1) = f(-1) + (-1) + 1$

$\Rightarrow f(0) = f(-1)$

$\Rightarrow 0 = a - b \ldots$using (iii) and (v)

$\Rightarrow a = b$

Since $a + b = 1$, we have:

$a = b = \frac{1}{2} \ldots$(Options C and E)

The correct answers are options A, C and E.

181. We know that for any two numbers a and b:

$||a| - |b|| \leq |a - b|$

(i) $||a| - |b|| = |a - b|$, possible only if a and b are of the same sign (both positive, or both negative)

(ii) $||a| - |b|| < |a - b|$, if a and b are of opposite signs (one positive, the other negative)

If we substitute: $a = x - 1$ and $b = x - 5$

$\Rightarrow |x - 1| - |x - 5| \leq |(x - 1) - (x - 5)|$

$\Rightarrow |x - 1| - |x - 5| \leq 4$

This satisfies the given inequality (without the equality sign).

Since the inequality is of the strictly 'less than' type, we have, from (ii) above:

(A) $x - 1 < 0$ AND $x - 5 > 0$

$\Rightarrow x < 1$ AND $x > 5$ – Not possible

OR

(B) $x - 1 > 0$ AND $x - 5 < 0$

$\Rightarrow x > 1$ AND $x < 5$

=> $1 < x < 5$

=> Possible integer values of x are: 2, 3, and 4

Working with the options:

(A) Possible integer values of x are: 2, 3, 4, 5, 6, etc. – Does not satisfy
(B) Possible integer values of x are: 2 and 3 – Satisfies
(C) Possible integer values of x are: 3, 2 and 1 – Does not satisfy
(D) Possible integer values of x are: 3 and 4 – Satisfies

The correct answers are options B and D.

182. $n = \frac{18!}{15!}$

$= \frac{18 \times 17 \times 16 \times 15!}{15!}$

$= 18 \times 17 \times 16$

$= (2 \times 3^2) \times 17 \times 2^4$

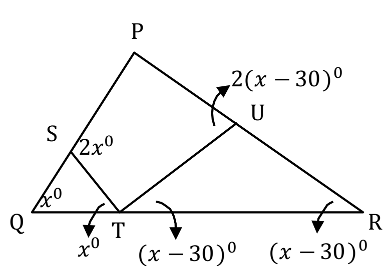
$= 2^5 \times 3^2 \times 17$

Working with the options:

(A) $51 = 3 \times 17$ – Satisfies
(B) $136 = 2^3 \times 17$ – Satisfies
(C) $153 = 3^2 \times 17$ – Satisfies
(D) $216 = 2^3 \times 3^3$ – Does not satisfy since 3^3 is not a factor of n

The correct answers are options A, B and C.

183. Let us bring out the figure.



Let $\angle SQT = x°$

=> $\angle STQ = x°$, since SQ = ST (given)

$\angle PRQ = (x - 30)°$, since $\angle PQR = \angle PRQ + 30°$ (given)

=> $\angle UTR = (x - 30)°$, since TU = UR (given)

For triangle SQT:

$\angle PST = \angle SQT + \angle STQ$, since exterior angle equals the sum of the interior opposite angles

=> $\angle PST = 2x°$

Again, for triangle UTR:

$\angle PUT = \angle URT + \angle UTR$, since exterior angle equals the sum of the interior opposite angles

=> $\angle PUT = 2(x - 30)°$

In quadrilateral PUTS, sum of the interior angles = 360°

Since $\angle SPU + \angle STU = 220°$, we have:

$\angle PST + \angle PUT = 360° - 220° = 140°$

=> $2x + 2(x - 30) = 140$

=> $4x = 200$

=> $x = 50$

Thus, from the diagram, we can see that the angles equal to 50° are $\angle SQT$ and $\angle STQ$.

The correct answers are options A and D.

184. We have: $3^{6x} = 8100$

$$\left(3^{(x-n)}\right)^3 = 3^{3(x-n)} = 3^{3x-3n} = \frac{3^{3x}}{3^{3n}}$$

$$= \frac{\left(3^{(6x)}\right)^{\frac{1}{2}}}{3^{3n}} = \frac{\sqrt{3^{6x}}}{3^{3n}} = \frac{\sqrt{8100}}{3^{3n}} = \frac{90}{3^{3n}}$$

Thus, we have:

$$\frac{10}{3} \leq \frac{90}{3^{3n}} \leq 10$$

Dividing by 10 throughout:

=> $\frac{1}{3} \le \frac{9}{3^{3n}} \le 1$

=> $\frac{1}{3} \le \frac{3^2}{3^{3n}} \le 1$

=> $3^{-1} \le 3^{2-3n} \le 3^0$

Since bases are same, we can compare the exponents.

Thus, $-1 \le 2 - 3n \le 0$

Multiplying throughout by (-1):

=> $1 \ge 3n - 2 \ge 0$

Adding 2 throughout:

=> $3 \ge 3n \ge 2$

=> $1 \ge n \ge \frac{2}{3}$

The correct answers are options A, B, C and D.

185. $P = 4^3 \times 25^2$

$= 2^6 \times 5^4 = (2 \times 5)^4 \times 2^2$

$= 4 \times 10^4$

Thus, we have:

$10^x + n = 4 \times 10^4$

=> $n = 4 \times 10^4 - 10^x$

Possible values of x and n are:

(1) $x = 1$ => $n = 4 \times 10^4 - 10 = 40000 - 10 = 39990$: Option B is correct
(2) $x = 2$ => $n = 4 \times 10^4 - 10^2 = 40000 - 100 = 39900$: Option C is correct
(3) $x = 3$ => $n = 4 \times 10^4 - 10^3 = 40000 - 1000 = 39000$: Option E is correct
(4) $x = 4$ => $n = 4 \times 10^4 - 10^4 = 40000 - 10000 = 30000$: Option F is correct
(5) $x = 5$ or above => n becomes negative and hence, not possible

The correct answers are options B, C, E and F.

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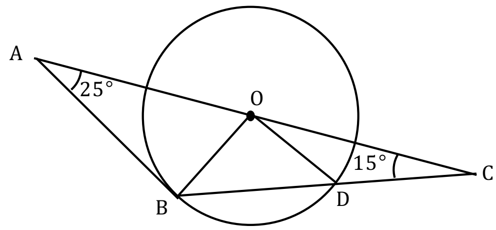
186. Let us bring out the figure.



Since AB is tangent to the circle at B, $\angle ABO = 90^\circ$

Thus, for triangle ABO:

$\angle BOC = \angle BAO + \angle ABO$, since exterior angle equals the sum of the interior opposite angles

=> $\angle BOC = 25^\circ + 90^\circ$, since $\angle BAO = 25^\circ$ (given)

=> $\angle BOC = 115^\circ$ ... Option A is incorrect

In triangle BOC:

$\angle OBC = 180^\circ - (\angle BOC + \angle OCB) = 180^\circ - (115^\circ + 15^\circ)$

=> $\angle OBC = 50^\circ$ ... Option B is correct

In triangle BOD, BO = DO = radius of the circle

Thus, we have:

$\angle ODB = \angle OBD = 50^\circ$ ... Option C is correct

=> $\angle BOD = 180^\circ - (\angle ODB + \angle OBD)$

=> $\angle BOD = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$ ... Option D is incorrect

For triangle ODC:

$\angle ODB = \angle COD + \angle DCO$, since exterior angle equals the sum of the interior opposite angles

=> $\angle COD = \angle ODB - \angle DCO = 50^\circ - 15^\circ$, since $\angle DCO = 15^\circ$ (given)

=> $\angle COD = 35^\circ$ ... Option E is correct

The correct answers are options B, C and E.

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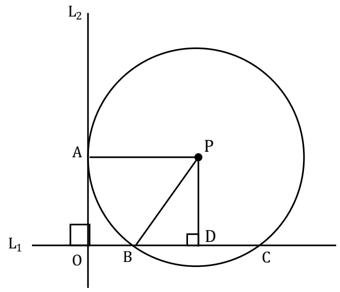
187. Let us bring out the figure.



Since PD is perpendicular to the chord BC, we have:

$BD = DC = \frac{BC}{2} = \frac{16}{2} = 8$

Also: PD = OA = 6 – Option C is correct

Thus, in right angled triangle PBD, we have:

$PB^2 = PD^2 + BD^2 = 6^2 + 8^2 = 100$

=> PB = radius of the circle = 10 – Option A is correct

OB = OD − BD = AP − BD = Radius of the circle − BD

=> OB = 10 − 8 = 2 – Option B is correct

In right angled triangle OPD:

$OP^2 = OD^2 + PD^2 = AP^2 + PD^2 = (\text{Radius of the circle})^2 + PD^2$

$=> OP^2 = 10^2 + 6^2 = 136$

$=> OP = \sqrt{136} = 2\sqrt{34}$ – Option D is correct

The correct answers are options A, B, C and D.

188. In any triangle, sum of any two sides is greater than the third side.

Thus, we have the following cases:

(A) $x + 5 > 15 => x > 10 \ldots$ (i)

(B) $x + 15 > 5 => x > -10$, which is obvious since x,being the length of the side of a triangle, must be positive.

(C) $5 + 15 > x => x < 20 \ldots$ (ii)

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We also have:

$5 < x < 15 \ldots$(iii)

Thus, from (i), (ii) and (iii), we have:

$10 < x < 15$

Thus, the possible integer values of x are: 11, 12, 13 and 14.

The correct answers are options E, F, G and H.

Alternate approach:

For any two sides of a triangle, the third side always lies in between the difference between the other two sides and the sum of those two sides. Thus:

Difference between two sides < Third side < Sum of the two sides

=> $15 - 5 < x < 15 + 5$

=> $10 < x < 20 \ldots.$(i)

However, it's given that: $5 < x < 15 \ldots.$(ii)

From (i) and (ii),

$10 < x < 15$

Thus, the possible integer values of x are: 11, 12, 13 and 14.

189. Working with the options:

Option A:

$h + h^2 > 0$

$$h + h^2 = \left(h^2 + 2 \times h \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right) - \left(\frac{1}{2}\right)^2$$

$$= \left(h + \frac{1}{2}\right)^2 - \frac{1}{4}$$

Maximum value (less than 0) : $@h = -1$ OR 0

$$\left(h + \frac{1}{2}\right)^2 - \frac{1}{4} => \left(-1 + \frac{1}{2}\right)^2 - \frac{1}{4} = 0 \text{ OR } \left(0 + \frac{1}{2}\right)^2 - \frac{1}{4} = 0$$

Minimum value greater than $\left(-\frac{1}{4}\right)$: $@h = -\frac{1}{2}$:

$\left(h + \frac{1}{2}\right)^2 - \frac{1}{4} => \left(-\frac{1}{2} + \frac{1}{2}\right)^2 - \frac{1}{4} = -\frac{1}{4}$

Thus, we have:

$-\frac{1}{4} < h + h^2 < 0$

$=> h + h^2 \not> 0$ – Incorrect

Option B:

$\frac{1}{h} < h$

Taking reciprocal throughout:

$-\infty < \frac{1}{h} < -1$

Thus, from the above two inequalities, we have:

$\frac{1}{h} < h$ – Correct

Option C:

$\frac{1}{h^2} > \frac{1}{h}$

Since h is negative, $\frac{1}{h}$ must also be negative.

However, $\left(\frac{1}{h^2}\right)$ must be positive since it is the square of a number.

Thus, we have:

$\frac{1}{h^2} > \frac{1}{h}$ – Correct

The correct answers are options B and C.

Alternate approach:

Let us pick any value of h and evaluate each of the answer options.

Since $-1 < h < 0$, let us assume that $h = -\frac{1}{2}$:

- Option A: $h + h^2 = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4} \not> 0$ – Incorrect

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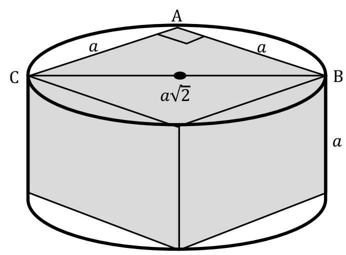
- Option B: $\frac{1}{h} = -2 < h = -\frac{1}{2}$ – Correct
- Option C: $\frac{1}{h^2} = (-2)^2 = 4 > \frac{1}{h} = -2$ – Correct

190. Let us consider the situation where the largest possible cube (shaded in grey) fits perfectly in a particular cylinder as shown in the diagram below; however, the question does not state that we must fit in a largest possible cube into the cylinder.



Let the edge of the cube be a.

Since the cube has to have a perfect fit (minimum non-utilization of cylinder space), we must have:

Height of the cylinder = Edge of the cube = a

In right angled triangle CAB:

$CB^2 = CA^2 + AB^2 = a^2 + a^2 = 2a^2$

=> CB = $a\sqrt{2}$

Thus, the diameter of the cylinder = $a\sqrt{2}$

=> Radius of the cylinder = $\frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Thus, volume of the cylinder

$= \pi \times \text{radius}^2 \times \text{height}$

$= \pi \times \left(\frac{a}{\sqrt{2}}\right)^2 \times a$

$= \frac{\pi a^3}{2}$

$= \frac{3}{2}a^3$

Volume of the cube = a^3

Thus, volume of the cylinder not occupied by the cube

$$= \frac{3}{2}a^3 - a^3$$

$$= \frac{a^3}{2}$$

Thus, the required percent

$$= \frac{\text{Volume of the cylinder not occupied by the cube}}{\text{Volume of the cylinder}} \times 100$$

$$= \frac{\left(\frac{a^3}{2}\right)}{\left(\frac{3}{2}a^3\right)} \times 100\%$$

$$= 33.3\%$$

The above situation depicts the case where minimum volume of the cylinder as a percent of the total volume of the cylinder is unutilized, i.e. not covered by the cube.

Thus, in any other scenario, the required percent value would be either greater than or equal to 33.3%

The only possible values from the answer options are 36% and 42%.

The correct answers are options D and E.

191. We have:

$0 < p < 1 \ldots \text{(i)}$

$1 < q < 2 \ldots \text{(ii)}$

Working with the options:

Option A:

Taking reciprocal throughout in (ii):

$\frac{1}{2} < \frac{1}{q} < 1 \ldots \text{(iii)}$

Multiplying (i) and (iii):

$0 < \frac{p}{q} < 1$ – Satisfies

Option B:

Multiplying (i) and (ii):

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$0 < pq < 2$ – Does not necessarily satisfy

If $0 < pq < 1$, the answer is yes; however, if $1 < pq < 2$, the answer is no.

Option C:

Multiplying (-1) throughout in (i):

$-1 < -p < 0 \ldots$(iv)

Adding (ii) and (iv):

$0 < q - p < 2$ – Does not necessarily satisfy

If $0 < q - p < 1$, the answer is yes; however, if $1 < q - p < 2$, the answer is no.

The correct answer is option A.

Alternate approach:

We have:

$0 < p < 1 \ldots$(i)

$1 < q < 2 \ldots$(ii)

Working with the options:

Option A:

Since p is smaller than q, any number chosen for p must be smaller than any number selected for q.

Hence, we have $\frac{p}{q} < 1$ – Satisfies

Option B:

Let us work with some numbers:

(1) $p = 0.1,\ q = 1.9 \Rightarrow pq = 0.19 < 1$
(2) $p = 0.8,\ q = 1.5 \Rightarrow pq = 1.2 \not< 1$

Thus, pq may be greater than 1 or less than 1 – Does not satisfy

Option C:

Let us work with some numbers:

(1) $p = 0.1,\ q = 1.9 \Rightarrow q - p = 1.8 \nless 1$

(2) $p = 0.8,\ q = 1.5 \Rightarrow q - p = 0.7 < 1$

Thus, $(q - p)$ may be greater than 1 or less than 1 – Does not satisfy

192. $N = (9,999 \times 74) - \left(10^5 - 26\right)$

$= (10,000 - 1) \times 74 - 10^5 + 26$

$= 74 \times 10^4 - 74 - 10 \times 10^4 + 26$

$= 10^4\,(74 - 10) - 74 + 26$

$= 10^4 \times 64 - 48$

$= 640,000 - 48$

$= 639,952$

Thus, the sum of digits of $N = d = 6 + 3 + 9 + 9 + 5 + 2 = 34$

Thus, the factors of d are: 1, 2, 17 and 34.

The correct answers are options A, E and G.

193. Since there are n sides in the polygon, the number of vertices is also n.

We need to find the number of diagonals that can be formed using the n vertices.

Number of ways in which any 2 vertices can be selected to form a line segment
$= C_2^n = \frac{n\,(n-1)}{2}$

The line segments can be either a side or a diagonal.

Among the above lines, we have the n sides included.

Thus, the number of diagonals = Total number of line segments – Number of sides

$$= \frac{n\,(n-1)}{2} - n$$

$$= n\left(\frac{n-1}{2} - 1\right)$$

$$= \frac{n\,(n-3)}{2}$$

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Thus, we have:

Number of diagonals = $\frac{n(n-3)}{2}$, which is a prime number

The numerator is a product of 2 terms.

Since the number of diagonals is a prime number, one of the terms in the numerator must cancel out with the 2 in the denominator (since a prime number does not have any two different positive factors other than 1 and itself).

The above is possible under the following situations:

(A) $n - 3 = 2$ (Note: $n = 2$ is not possible since then, $(n - 3)$ becomes negative)

$=> n = 5$ (A pentagon, which has 5 sides, has 5 diagonals)

(B) One of the terms in the numerator is 1 and the other is a prime multiple of 2

$=> n - 3 = 1$ (Note: $n = 1$ is not possible since then, $(n - 3)$ becomes negative)

$=> n = 4$ (A quadrilateral, which has 4 sides, has 2 diagonals)

Thus, the possible values of n are 4 or 5.

The correct answers are options B and C.

Alternate approach:

Working with each option, we have:

(A) $n = 3$ => Number of diagonals = $\frac{n(n-3)}{2} = 0 \neq$ prime – Does not satisfy

(B) $n = 4$ => Number of diagonals = $\frac{n(n-3)}{2} = \frac{4 \times 1}{2} = 2 =$ prime – Satisfies

(C) $n = 5$ => Number of diagonals = $\frac{n(n-3)}{2} = \frac{5 \times 2}{2} = 5 =$ prime – Satisfies

(D) $n = 6$ => Number of diagonals = $\frac{n(n-3)}{2} = \frac{6 \times 3}{2} = 9 \neq$ prime – Does not satisfy

(E) $n = 7$ => Number of diagonals = $\frac{n(n-3)}{2} = \frac{7 \times 4}{2} = 14 \neq$ prime – Does not satisfy

194. Let the 3 numbers be a, b and c.

Since the pair-wise GCD of the above numbers is 7, we have:

$a = 7x, b = 7y$ and $c = 7z$, where x, y and z have no common factors, .i.e., every pair of numbers formed from x, y and z is a 'co-prime' pair.

Since the product of the 3 numbers is 8,232, we have:

$8,232 = (7x)(7y)(7z)$

$=> xyz = \frac{8,232}{7^3} = \frac{1,176}{7^2} = \frac{168}{7} = 24$

Thus, we have the following restrictions:

(i) $xyz = 24$
(ii) x, y and z have no common factors other than 1
(iii) $x \neq y \neq z$ (since the numbers are distinct)

$=> x = 1, y = 3, z = 8$

Note: In any other combination, x, y and z would have common factors present, for example: (1, 6, 4), (1, 2, 12), (2, 3, 4), etc.

Thus, the numbers are:

$a = 7x = 7$

$b = 7y = 21$

$c = 7z = 56$

The correct answers are options A, C and G.

195. We have: $4x < x < x^3 < x^2$

Thus, we have:

$4x < x$

$=> 4x - x < 0 => 3x < 0$

$=> x < 0 \ldots$(i)

Also, we have:

$x^3 < x^2$

Dividing both sides by x^2:

$=> x < 1 \ldots$(ii)

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Also, we have:

$x < x^3$

(1) If $x > 0$: Canceling x, we have: $1 < x^2 \Rightarrow x > 1$ or $x < -1 \Rightarrow x > 1$

(2) If $x < 0$: Canceling x, we have: $x^2 < 1 \Rightarrow -1 < x < 1 \Rightarrow -1 < x < 0$

Thus, from (1) and (2): $x > 1$ OR $-1 < x < 0 \ldots$(iii)

Combining (i), (ii) and (iii):

$-1 < x < 0$

Thus, the possible values of x among the options are $-\frac{2}{3}$, $-\frac{1}{\sqrt{3}}$, and $-\frac{1}{2}$.

The correct answers are options D, E and F.

Alternate approach:

We have: $4x < x < x^3 < x^2$

We work with each option and verify the given inequality.

(A) $x = -\sqrt{3} = \approx -1.73$:
$x = -1.73 \nless x^3 = -5.18$ – Does not satisfy

(B) $x = -\frac{3}{2} = \approx -1.5$:
$x = -1.5 \nless x^3 = -3.37$ – Does not satisfy

(C) $x = -1$:
$\Rightarrow x = -1 \nless x^3 = -1$ – Does not satisfy

(D) $x = -\frac{2}{3} = \approx -0.67$:
$4x = -2.68 < x = -0.67 < x^3 = -0.3 < x^2 = 0.45$ – Satisfies

(E) $x = -\frac{1}{\sqrt{3}} = \approx -0.58$:
$4x = -2.32 < x = -0.58 < x^3 = -0.19 < x^2 = 0.33$ – Satisfies

(F) $x = -\frac{1}{2} = -0.5$:
$4x = -2 < x = -0.5 < x^3 = -0.125 < x^2 = 0.25$ – Satisfies

196. Let the two numbers be a and b.

Since the GCD of the above numbers is 15, we have:

$a = 15x$ and $b = 15y$, where x and y have no common factors, i.e. they are co-prime to one another.

Thus, the LCM of the numbers = $15xy$

Thus, we have:

$15xy = 180$

$=> xy = 12$

Since x and y have no common factors, the possible values are:

(A) $x = 1,\ y = 12 => a = 15 \times 1 = 15,\ b = 15 \times 12 = 180$

(B) $x = 3,\ y = 4 => a = 15 \times 3 = 45,\ b = 15 \times 4 = 60$

Thus, there are two pairs of such numbers: (15, 180) and (45, 60)

The correct answers are options B and F.

Alternate approach:

Let the two numbers be a and b.

Since the GCD of the above numbers is 15, we have:

$a = 15x$ and $b = 15y$, where x and y have no common factors.

Let's recall the property about any two numbers a and b:

$a \times b = \text{LCM}(a, b) \times \text{GCD}(a, b)$

$=> 15x \times 15y = 180 \times 15$

$=> xy = 12$

Since x and y have no common factors, the possible values are:

(A) $x = 1,\ y = 12 => a = 15 \times 1 = 15,\ b = 15 \times 12 = 180$
(B) $x = 3,\ y = 4 => a = 15 \times 3 = 45,\ b = 15 \times 4 = 60$

Thus, there are 2 pairs of such numbers: (15, 180) and (45, 60)

197. Let the consecutive integers be: $x,\ x + 1, x + 2$, and $x + 3$.

The possible values of $ab + cd$ are shown below (while assigning values, one should note that interchanging the values of a and b or c and d doesn't make a difference):

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a	b	c	d	$ab + cd$
x	$x + 1$	$x + 2$	$x + 3$	$x(x + 1) + (x + 2)(x + 3) = (2x^2 + 6x + 6) = p$
x	$x + 2$	$x + 1$	$x + 3$	$x(x + 2) + (x + 1)(x + 3) = (2x^2 + 6x + 3) = q$
x	$x + 3$	$x + 1$	$x + 2$	$x(x + 3) + (x + 1)(x + 2) = (2x^2 + 6x + 2) = r$

Thus, the possible positive differences are:

(A) $p - q = (2x^2 + 6x + 6) - (2x^2 + 6x + 3) = 3$
(B) $q - r = (2x^2 + 6x + 3) - (2x^2 + 6x + 2) = 1$
(C) $p - r = (2x^2 + 6x + 6) - (2x^2 + 6x + 2) = 4$

The correct answers are options B, D and E.

Alternate approach:

Since we have numerical options provided, it does not matter which set of consecutive integers are selected.

Thus, we take the consecutive integers as 1, 2, 3 and 4.

Possible values of $(ab + cd)$ are:

a	b	c	d	$ab + cd$
1	2	3	4	$14 = p$
1	3	2	4	$11 = q$
1	4	2	3	$10 = r$

Thus, the possible positive differences are:

(A) $p - q = 14 - 11 = 3$
(B) $q - r = 11 - 10 = 1$
(C) $p - r = 14 - 10 = 4$

198. We have: $(p - q)$ is even

=> p and q are both even

OR

p and q are both odd

Also, we have: $\frac{p}{q}$ is even

=> p is even and q is an odd factor of p, i.e. p is even and q is odd

OR

p is even and q is an even factor of p, i.e. p and q are both even

Thus, combining the above results, we have:

p and q are both even, i.e. p is even and q is an even factor of p

In such an event, since p is divisible by q and the result is also even, p must have a higher exponent of 2 than q would have.

For example, if $p = 2^3$, then $q = 2^2$ or 2, etc.

Thus, the minimum exponent of 2 in p is 2.

=> $p = 2^2k = 4k$, where k is a positive integer

Working with the options:

(A) $\frac{p}{2} = \frac{4k}{2} = 2k$ = even – Does not satisfy

(B) If $q = 2$ => $\frac{q}{2} = 1$ = odd – Satisfies

(C) If $p = 4,\ q = 2$ => $\frac{a+b}{2} = 3$ = odd – Satisfies

(D) If $q = 4$ => $\frac{q+2}{2} = 3$ = odd – Satisfies

(E) $\frac{p+2}{2} = \frac{4k+2}{2} = 2k+1$ = odd – Satisfies

The correct answers are options B, C, D and E.

199. On tossing the coin once, probability of getting heads = $P(H) = \frac{1}{2}$

Thus, on tossing the coin once, probability of getting tails = $P(T) = 1 - P(H) = 1 - \frac{1}{2} = \frac{1}{2}$

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Thus, we need to find, on tossing the coin 5 times, the probability that it will land heads up on 3 flips AND tails up on 2 flips.

The 3 flips where the coin lands heads up could be any 3 of the 5 flips.

Thus, the number of ways of getting 3 heads in 5 flips

$= C_3^5 = C_{(5-3)}^5 = C_2^5 = \frac{5 \times 4}{2 \times 1} = 10$

For any one of the above cases, the corresponding probability

$= P(H) \times P(H) \times P(H) \times P(T) \times P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{32}$

Thus, the required probability

$= \frac{1}{32} + \frac{1}{32} + \ldots (10 \text{ times})$

$= 10 \times \frac{1}{32}$

$= \frac{5}{16}$

Since the required probability is given as $\frac{a}{b}$, where a and b are positive integers less than 50, we have:

- $\frac{a}{b} = \frac{5}{16} \Rightarrow a + b = 5 + 16 = 21$
- $\frac{a}{b} = \frac{5 \times 2}{16 \times 2} = \frac{10}{32} \Rightarrow a + b = 10 + 32 = 42$
- $\frac{a}{b} = \frac{5 \times 3}{16 \times 3} = \frac{15}{48} \Rightarrow a + b = 15 + 48 = 63$
- $\frac{a}{b} = \frac{5 \times 4}{16 \times 4} = \frac{20}{64}$ => Since $b > 50$, we cannot accept this or any subsequent value

The correct answers are options C, E and G.

200. Among the 6 letters, 4 are consonants (B, C, D and F) and 2 are vowels (A and E).

(A) Number of ways in which 4-letter codes can be generated from the 6 letters containing only 1 vowel:

We can select 3 consonants from the 4 consonants in $C_3^4 = C_{(4-3)}^4 = C_1^4 = 4$ ways

We can select 1 vowel from the 2 vowels in $C_1^2 = 2$ ways

Thus, the number of selections = $4 \times 2 = 8$

The 4 letters selected above can be arranged in $4! = 24$ ways.

Thus, the number of 4-letter codes formed = $8 \times 24 = 192$.

(B) Number of ways in which 4-letter codes can be generated from the 6 letters containing both vowels:

We can select 2 consonants from the 4 consonants in $C_2^4 = \frac{4 \times 3}{2 \times 1} = 6$ ways

We can select both vowels from the 2 vowels in $C_2^2 = 1$ way

Thus, the number of selections = $6 \times 1 = 6$

The 4 letters selected above can be arranged in $4! = 24$ ways.

Thus, the number of 4-letter codes formed = $6 \times 24 = 144$.

The correct answers are options A and C.

201. There are 3 different colors used and blue features in the 2^{nd}, 3^{rd}, 4^{th} and 6^{th} positions in each set of 6 beads.

Thus, the positions for blue are: $6k + 2,\ 6k + 3,\ 6k + 4$ (where k is a non-negative integer) and $6k$ (where k is a positive integer).

Working with the options:

(A) 38 => $38 = 6 \times 6 + 2 \equiv 6k + 2$ – Satisfies
(B) 82 => $82 = 6 \times 13 + 4 \equiv 6k + 4$ – Satisfies
(C) 102 => $102 = 6 \times 17 \equiv 6k$ – Satisfies
(D) 119 => $119 = 6 \times 19 + 5 \equiv 6k + 5$ – Does not satisfy
(E) 125 => $125 = 6 \times 20 + 5 \equiv 6k + 5$ – Does not satisfy

The correct answers are options A, B and C.

202. Let the first and the second term be x and y, respectively.

Third term = $x + y$

Fourth term = $y + (x + y) = x + 2y$

Fifth term = $(x + y) + (x + 2y) = 2x + 3y$

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Thus, we have:

$2x + 3y = 18$

$=> y = \frac{18 - 2x}{3}$

Since y is a positive integer, and 18 is divisible by 3, $2x$ must also be divisible by 3.

Thus, x must be a multiple of 3.

Thus, we have the following values of x and y:

(A) $x = 3 : y = \frac{18 - 6}{3} = 4$

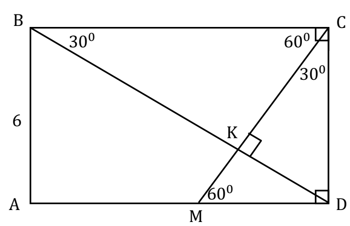
(B) $x = 6 : y = \frac{18 - 12}{3} = 2$

(C) $x = 9 : y = \frac{18 - 18}{3} = 0$ – cannot be accepted since y should be a positive integer

Thus, the possible values of the second term, y are 2 or 4

The correct answers are options B and D.

203. Let us bring out the figure.



In right angled triangle BCK:

$\angle BCK = 180^\circ - (\angle CBK + \angle CKB)$

$= 180^\circ - (30^\circ + 90^\circ) = 60^\circ$

$\angle MCD = \angle BCD - \angle BCK$

$= 90^\circ - 60^\circ = 30^\circ$

$\angle CMD = \angle BCM = 60^\circ$ (they form a pair of alternate angles since BC is parallel to AD)

Thus, triangle CDM is a 30-90-60 triangle.

=> The side opposite 60° is $\sqrt{3}$ times the side opposite 30°.

$=> CD = \sqrt{3} \times DM$

$= \ > AB = \sqrt{3} \times DM$ (since ABCD is a rectangle, AB = CD)

$=> 6 = \sqrt{3} \times DM$

$= \ > DM = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ ... Option A is correct and Option B is incorrect.

In triangle DKM:

$\angle MDK = \angle CBD = \ 30^\circ$ (they form a pair of alternate angles since BC is parallel to AD)

Thus, triangle DKM is a 30-90-60 triangle

=> The side opposite 30° is $\frac{1}{2}$ of the side opposite 90°

$= > MK = \frac{1}{2} \times DM$

$=> MK = \frac{1}{2} \times 2\sqrt{3} = \sqrt{3}$ ... Option C is correct and option D is incorrect.

The correct answers are options A and C.

204. Let us factorize 126:

$126 = 2 \times 3^2 \times 7$

Since we only need the sum of digits, which digit occupies a particular position in the number is not important.

For example, 1,367 and 3,176 are different numbers, each having the product of digits equals 126 and theie sum equals 17.

Thus, we simply need the number of ways in which 126 can be obtained as a product of 4 DISTINCT digits from 1 to 9.

The possible ways in which distinct digits that can be used are shown below:

(A) 1, 3, 6, 7 => Sum of digits = 17
(B) 1, 2, 7, 9 => Sum of digits = 19

The correct answers are options B and D.

205. We have:

$f(x) = x^2 + 3$

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$g(x) = 3f(x) = 3(x^2 + 3)$

Since $g(x) = 84$, we have:

$3(x^2 + 3) = 84$

=> $x^2 = 25$

=> $x = \pm 5$

Thus, we have:

$f(x-1) = (x-1)^2 + 3$

If $x = 5$: $f(x-1) = (5-1)^2 + 3 = 19$

If $x = -5$: $f(x-1) = (-5-1)^2 + 3 = 39$

The correct answers are options B and D.

206. The question asks about the range (from the options given) where the probability of the number being present is the greatest.

Wider the range, greater will be the probability that the number will lie in that range.

For example:

The probability that the number will lie between 0 and $\frac{2}{3}$ = 1, since the number is chosen lies in that range.

The probability that the number will lie between 0 and $\frac{1}{3}$ will be less than 1, since the range from 0 to $\frac{1}{3}$ is a part of the range 0 to $\frac{2}{3}$, implying that there is a chance of the selected number being in the range from $\frac{1}{3}$ to $\frac{2}{3}$.

Working with the options:

(A) 0 and $\frac{7}{20}$: The gap = $\frac{7}{20} - 0 = \frac{7}{20}$

(B) $\frac{1}{5}$ and $\frac{1}{4}$: The gap = $\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$

(C) $\frac{3}{20}$ and $\frac{1}{5}$: The gap = $\frac{1}{5} - \frac{3}{20} = \frac{1}{20}$

(D) $\frac{1}{4}$ and $\frac{3}{10}$: The gap = $\frac{3}{10} - \frac{1}{4} = \frac{1}{20}$

(E) $\frac{3}{10}$ and $\frac{9}{20}$: The gap = $\frac{9}{20} - \frac{3}{10} = \frac{3}{20}$

(F) $\frac{3}{10}$ and $\frac{13}{20}$: The gap = $\frac{13}{20} - \frac{3}{10} = \frac{7}{20}$

Thus, we observe that options A and F, each represents the largest gap, hence would be our required range.

The correct answers are options A and E.

207. Let the required quantity be k.

Thus, according to the problem, we have:

$$k + \frac{x+1}{x} = k\left(\frac{x}{x-1}\right)$$

$$=> k\left(\frac{x}{x-1}\right) - k = \frac{x+1}{x}$$

$$=> k\left(\frac{x}{x-1} - 1\right) = \frac{x+1}{x}$$

$$=> k\left(\frac{1}{x-1}\right) = \frac{x+1}{x}$$

$$=> k = (x-1)\left(\frac{x+1}{x}\right) = \frac{(x-1)(x+1)}{x} \ldots \text{Option B is correct}$$

$$=> k = \frac{x^2-1}{x} = x - \frac{1}{x}$$

Thus, no other options match with the expression for k.

The correct answer is option B.

Alternate approach:

Let us take a value of x, say $x = 2$.

Thus, we need to find the quantity, say k, that can be added to $\left(\frac{x+1}{x} = \frac{3}{2}\right)$ or multiplied with $\left(\frac{x}{x-1} = 2\right)$ to obtain the same result.

Thus, we have:

$$\frac{3}{2} + k = 2k$$

$$=> k = \frac{3}{2}$$

We substitute the value of x in the options to check which option(s) give us $k = \frac{3}{2}$.

(A) $k = 1 + \frac{1}{x-1} = 1 + \frac{1}{2-1} \neq \frac{3}{2}$ – Does not satisfy

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(B) $k = \frac{(x-1)\,(x+1)}{x} = \frac{(2-1)\,(2+1)}{2} = \frac{3}{2}$ – Satisfies

(C) $k = \frac{x}{x-1} = \frac{2}{2-1} \neq \frac{3}{2}$ – Does not satisfy

(D) $k = -\,(x+1) = -(2+1) \neq \frac{3}{2}$ – Does not satisfy

(E) $k = x^2 - \frac{1}{x^2} = 2^2 - \frac{1}{2^2} \neq \frac{3}{2}$ – Does not satisfy

208. Since the sum of the squares of x and y is 20, we have:

$x^2 + y^2 = 20 \ldots$(i)

Since the sum of the reciprocals of x and y is 2, we have:

$\frac{1}{x} + \frac{1}{y} = 2$

$=> \frac{x+y}{xy} = 2$

$=> x + y = 2xy$

Squaring both sides:

$(x+y)^2 = (2xy)^2$

$=> x^2 + y^2 + 2xy = 4x^2y^2$

Substituting $x^2 + y^2 = 20$ from (i):

$20 + 2xy = 4x^2y^2$

$=> 2x^2y^2 - xy - 10 = 0$

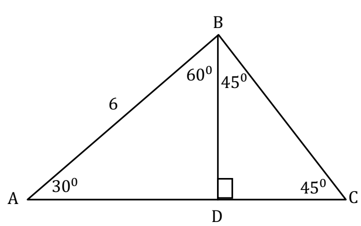
$=> 2x^2y^2 - 5xy + 4xy - 10 = 0$

$=> (2xy - 5)(xy + 2) = 0$

$=> xy = \frac{5}{2}$ OR -2

The correct answers are options A and D.

209. Let us bring out the figure.



In right angled triangle ABD:

$\angle ABD = 180^\circ - (\angle BAD + \angle BDA)$

$= 180^\circ - (30^\circ + 90^\circ)$

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$= 60^\circ$

Thus, triangle ADB is a 30-90-60 triangle

=> The length of the side opposite $30^\circ = \frac{1}{2}$ times the length of the side opposite 90°

$=> BD = \frac{1}{2} \times AB$

$=> BD = \frac{1}{2} \times 6 = 3$ ... Option A is correct and option B is incorrect.

In triangle BDC:

$\angle DBC = \angle ABC - \angle ABD$

$= 105^\circ - 60^\circ$

$= 45^\circ$

$\angle DCB = 180^\circ - (\angle DBC + \angle BDC)$

$= 180^\circ - (45^\circ + 90^\circ)$

$= 45^\circ$

Thus, triangle BDC is isosceles

$= > CD = BD = 3$ ... Option C is correct and option D is incorrect.

$=> BC^2 = CD^2 + BD^2 = 3^2 + 3^2 = 18$

$=> BC = 3\sqrt{2}$ ... Option F is correct and option E is incorrect.

The correct answers are options A, C and F.

210. Each child must get at least one gift.

Thus, if we give one gift to each child, two gifts will be left over.

These last two gifts can be distributed in the following ways:

(A) Given to any one child:

Thus, $(n - 1)$ children get one gift each and one child gets three gifts.

We can select the child who gets two gifts in $C_1^n = n$ ways

The gifts can be assigned to the children in only one way since all the gifts are identical, and hence there is no need to select the gifts.

Thus, the number of ways = $n \times 1 = n$

(B) Given to two children, one to each:

Thus, $(n-2)$ children get one gift each and two children get two gifts each.

We can select the two children who get two gifts in $C_2^n = \frac{n(n-1)}{2}$ ways

The gifts can be assigned to the children in only one way since all the gifts are identical, and hence there is no need to select the gifts.

Thus, the number of ways = $\frac{n(n-1)}{2} \times 1 = \frac{n(n-1)}{2}$

Thus, the total number of ways

$= n + \frac{n(n-1)}{2} = \frac{2n + n(n-1)}{2}$

$= \frac{n(n+1)}{2}$

Thus, we have:

$\frac{n(n+1)}{2} = 15$

=> $n(n+1) = 30$

=> $n^2 + n - 30 = 0$

=> $(n-5)(n+6) = 0$

=> $n = 5$ OR -6

Since n must be positive, we have:

$n = 5$

Alternately, we can simply say that, since $n(n+1) = 30$, and n is a positive integer, $5(5+1) = 5 \times 6$ is the only possibility.

=> $n = 5$

The correct answer is option C.

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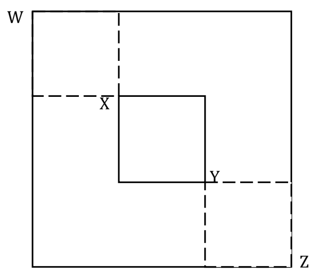
Alternate approach:

We know that the number of ways is simply $(C_1^n + C_2^n)$

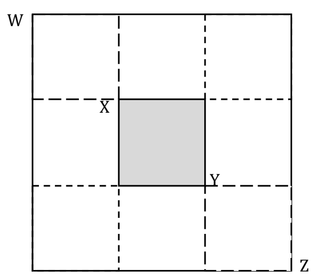
Working with the options, we find the value of n which satisfies: $(C_1^n + C_2^n) = 15$

For $n = 5$: $(C_1^n + C_2^n) = \left(C_1^5 + C_2^5\right) = 5 + 10 = 15$

211. The above question can be simply solved by imagining a small square having corners W and X, and another small square having corners Y and Z, as shown below:



Placing two more identical squares at the other two corners of the larger square, we have:



Thus, it is clear from the above diagram that the larger square consists of 9 smaller squares of equal area.

Thus, the area of the smaller square is $\frac{1}{9}$ of the area of the larger square. – Option A is correct and Option B is incorrect.

Also, it is clear that one side of the larger square is thrice that of the smaller square.

Thus, the perimeter of the smaller square is $\frac{1}{3}$ of the perimeter of the larger square. – Option D is correct and Option C is incorrect.

The correct answers are options A and D.

Alternate approach:

Let WX = XY = YZ = a

Thus, length of the diagonal of the smaller square = XY = a

Thus, length of the side of the smaller square = $\frac{a}{\sqrt{2}}$

Thus, Area of the smaller square = $\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^2}{2}$

Length of the diagonal of the larger square = WZ = $3a$

Thus, length of the side of the larger square = $\frac{3a}{\sqrt{2}}$

Thus, Area of the larger square =$\left(\frac{3a}{\sqrt{2}}\right)^2 = \frac{9a^2}{2}$

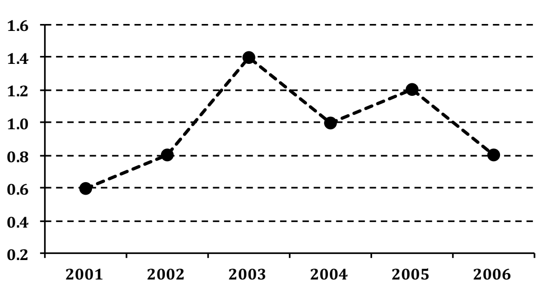
Thus, we have:

$$\frac{\text{Area of the smaller square}}{\text{Area of the larger square}} = \frac{\left(\frac{a^2}{2}\right)}{\left(\frac{9a^2}{2}\right)} = \frac{1}{9}$$

Also, we have:

$$\frac{\text{Perimeter of the smaller square}}{\text{Perimeter of the larger square}} = \frac{4 \times \left(\frac{a}{\sqrt{2}}\right)}{4 \times \left(\frac{3a}{\sqrt{2}}\right)} = \frac{1}{3}$$

212. Let us bring out the figure.



Exports in 2001 = \$60 million

$$\frac{\text{Imports in 2001}}{\text{Exports in 2001}} = 0.6$$

=> Imports in 2001 = 0.6 $\times$ Exports in 2001

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=> Imports in 2001 = 0.6×60 = \$36 million

We know that exports increased by \$2 million every year.

Thus, we have:

Year 2005:

Exports in 2005 = \$ $(60 + 4 \times 2)$ million = \$68 million

Given that $\frac{\text{Imports in 2005}}{\text{Exports in 2005}} = 1.2$

=> Imports in 2005 = $1.2 \times$ Exports in 2005

=> Imports in 2005 = 1.2×68 = \$81.6 million

Thus, percent increase in imports from 2001 to 2005

$$= \frac{\text{Imports in 2005} - \text{Imports in 2001}}{\text{Imports in 2001}} \times 100$$

$$= \frac{81.6 - 36}{36} \times 100\%$$

= 126.7% – Option F is correct

Year 2006:

Exports in 2006 = \$ $(60 + 5 \times 2)$ million = \$70 million

Given that $\frac{\text{Imports in 2006}}{\text{Exports in 2006}} = 0.8$

=> Imports in 2006 = $0.8 \times$ Exports in 2006

=> Imports in 2006 = 0.8×70 = \$56 million

Thus, percent increase in imports from 2001 to 2006

$$= \frac{\text{Imports in 2006} - \text{Imports in 2001}}{\text{Imports in 2001}} \times 100$$

$$= \frac{56 - 36}{36} \times 100\%$$

= 55.5% – Option B is correct

The correct answers are options B and F.

213. Since the problem asks for a percent value, we can assume any suitable value since the initial value does not affect the final answer.

Thus, let the total number of people be 100.

Number of people who said that they would vote for X = M% of 100 = M.

Number of people who did not say that they would vote for X = $(100 - M)$.

Since among those who said they would vote for X, N% actually voted for X, number of such people who actually voted for X

= N% of M

$= \frac{NM}{100}$

Since among those who did not say they would vote for X, P% actually voted for X, number of such people who actually voted for X

= P% of $(100 - M)$

$= \frac{P(100 - M)}{100}$

Thus, total number of people who voted for X = $\left(\frac{NM}{100} + \frac{P\,(100 - M)}{100}\right)$

Thus, the percent of people who voted for X

$$= \frac{\left(\frac{NM}{100} + \frac{P\,(100 - M)}{100}\right)}{100} \times 100\%$$

$$= \left(\frac{NM}{100} + \frac{P\,(100 - M)}{100}\right)\%$$

$$= \left(\frac{NM + P\,(100 - M)}{100}\right)\% \text{ – Option A is correct}$$

$$= \left(\frac{NM - PM + 100P}{100}\right)\%$$

$$= \left(\frac{M\,(N - P)}{100} + P\right)\% \text{ – Option D is correct}$$

The correct answers are options A and D.

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214. Let us bring out the table.

	Bank X	
Attribute	**2011**	**2012**
Loan Sanctions	650	750
Total Revenue	1,600	1,800

Since Loan Sanctions increased by at least 20% from 2012 to 2013, the minimum value of Loan Sanctions in 2013

$= 750 + 20\%$ of 750

$= 750\left(1 + \frac{20}{100}\right)$

$= 750 \times \frac{6}{5}$

$= \$900$ million

Thus, we have:

Loan Sanctions in 2013 $\geq \$900$ million ... (i)

Since Total Revenue increased by at most 50% from 2012 to 2013, the maximum value of Total Revenue in 2013

$= 1,800 + 50\%$ of $1,800$

$= 1,800\left(1 + \frac{50}{100}\right)$

$= 1,800 \times \frac{3}{2}$

$= \$2,700$ million

Thus, we have:

Total Revenue in 2013 $\leq \$2,700$ million ... (i)

Thus, from (i) and (ii), we obtain the MINIMUM value of the required ratio of Loan Sanctions and Total Revenue

$= \frac{900}{2,700}$

$= \frac{1}{3}$

Thus, we have:

$\frac{\text{Loan Sanctions}}{\text{Total Revenue}} \geq \frac{1}{3}$

The correct answers are options B, C and D.

215. We need to select $(3 + m)$ employees from 10 available employees, of which 4 are women and 6 are men, such that there are m men and 3 women in any group.

Number of ways of selecting m men out of 6 = C_m^6

Number of ways of selecting 3 women out of 4 = $C_3^4 = C_{(4-3)}^4 = C_1^4 = 4$

Thus, number of such ways = $C_m^6 \times 4$

Thus, we have:

$C_m^6 \times 4 = 60$

$=> C_m^6 = 15$

Working with the options, we have:

(A) $m = 1 => C_m^6 = C_1^6 \neq 15$ – Does not satisfy

(B) $m = 2 => C_m^6 = C_2^6 = \frac{6 \times 5}{2 \times 1} = 15$ – Satisfies

(C) $m = 3 => C_m^6 = C_3^6 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \neq 15$ – Does not satisfy

(D) $m = 4 => C_m^6 = C_4^6 = C_{(6-4)}^6 = C_2^6 = 15$ – Satisfies

The correct answers are options B and D.

216. Since the problem asks about a percent value, we can assume any suitable value since the initial value does not affect the final answer.

Let us assume that the quantity of coal mined daily as 1,000 tons.

(Note: We use 1,000 instead of 5,500 to make the calculations easier)

Thus, quantity of pure metal present in the coal

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= 0.5% of 1,000

$= \frac{0.5}{100} \times 1,000$

= 5 tons

Metal lost during mining in a day

= x% of 5

$= \frac{x}{100} \times 5$

$= \frac{x}{20}$ tons

Thus, quantity of metal successfully mined daily = $\left(5 - \frac{x}{20}\right)$ tons.

It is given that the number of days, n, required for the pure metal to be extracted at the mine is equal to the daily amount of coal mined.

Thus, quantity of metal successfully extracted in n days = $n \times \left(5 - \frac{x}{20}\right)$ tons, which is equal to the quantity of coal extracted in a day.

$= \; > n \times \left(5 - \frac{x}{20}\right) = 1,000$

$=> n = \frac{1,000}{\left(5 - \frac{x}{20}\right)}$

$=> n = \left(\frac{20,000}{100 - x}\right)$

Working with the options, we have:

(A) $x = 10; \; n = 200$:

Substituting $x = 10$ in (i), we have:

$n = \frac{20,000}{100 - 10} = \frac{20,000}{90} \neq 200$ – Does not satisfy

(B) $x = 20; \; n = 250$:

Substituting $x = 20$ in (i), we have:

$n = \frac{20,000}{100 - 20} = \frac{20,000}{80} = 250$ – Satisfies

(C) $x = 40; \; n = 333\frac{1}{3}$:

Substituting $x = 40$ in (i), we have:

$n = \frac{20,000}{100 - 40} = \frac{20,000}{60} = 333\frac{1}{3}$ – Satisfies

(D) $x = 50;\ n = 400$:

Substituting $x = 50$ in (i), we have:

$n = \frac{20,000}{100 - 50} = \frac{20,000}{50} = 400$ – Satisfies

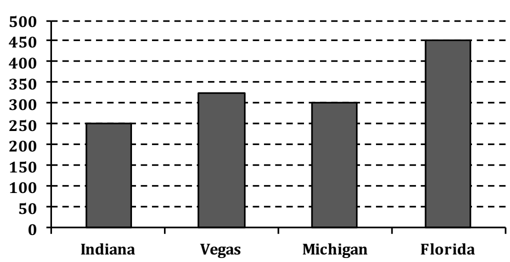
The correct answers are options B, C and D.

217. The different sums of money Charles spends are shown in the table below:

Packet size	**Number of packets required for 2,000 Balloons**	**Price per packet**	**Total price**
1 balloon	$2,000/1 = 2,000$	$1	$\$(2,000 \times 1) = \$2,000$
10 balloons	$2,000/10 = 200$	$9	$\$(200 \times 9) = \$1,800$
100 balloons	$2,000/100 = 20$	$75	$\$(20 \times 75) = \$1,500$
1,000 balloons	$2,000/1,000 = 2$	$600	$\$(2 \times 600) = \$1,200$

The correct answers are options B, C, E and F.

218. Let us bring out the bar-chart.



Number of customers in Indiana and Michigan together

$= (250 + 300)$ thousand

= 550 thousand

Thus, 550 thousand represents 20% of the total number of customers of the company

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=> 20% of total customers of the company = 550 thousand

=> Total customers of the company

$= 550 \times \frac{100}{20}$

= 2,750 thousand

Indiana:

Number of customers in Indiana = 250 thousand

Thus, the required percent

$= \frac{\text{Customers in Indiana}}{\text{Total customers}} \times 100$

$= \frac{250}{2,750} \times 100\%$

= 9.1% ... Option G is correct

Vegas:

Number of customers in Vegas = 325 thousand

Thus, the required percent

$= \frac{\text{Customers in Vegas}}{\text{Total customers}} \times 100$

$= \frac{325}{2,750} \times 100\%$

= 11.8% ... Option D is correct

Michigan:

Number of customers in Michigan = 300 thousand

Thus, the required percent

$= \frac{\text{Customers in Michigan}}{\text{Total customers}} \times 100$

$= \frac{300}{2750} \times 100\%$

= 10.9% ... Option E is correct

Florida:

Number of customers in Florida = 450 thousand

Thus, the required percent

$$= \frac{\text{Customers in Florida}}{\text{Total customers}} \times 100$$

$$= \frac{450}{2750} \times 100\%$$

= 16.4% ... Option A is correct

The correct answers are options A, D, E and G.

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219. Working with the options:

(A) Company X in 2010 – Required ratio = $\frac{80}{250} \times 100 = 32\%$

(B) Company X in 2011 – Required ratio = $\frac{100}{450} \times 100 = 22.2\%$

(C) Company Y in 2010 – Required ratio = $\frac{128}{400} \times 100 = 32\%$

(D) Company Y in 2011 – Required ratio = $\frac{215}{650} \times 100 = 33.1\%$

The correct answer is option D.

220. Working with the options:

- Option A:
  Required difference = 450 – 150 = \$300 million $\ngtr$ \$650 million – Does not satisfy

- Option B:
  Since the percent increase in Net Profits from 2010 to 2011 was the same as that from 2011 to 2012, we have:

  $$\frac{\text{Net Profit in 2011}}{\text{Net Profit in 2010}} = \frac{\text{Net Profit in 2012}}{\text{Net Profit in 2011}}$$

  $$=> \frac{150}{125} = \frac{\text{Net Profit in 2012}}{150}$$

  $$=> \text{Net Profit in 2012} = \frac{150 \times 150}{125} = \$180 \text{ million}$$

  Since the Total Income in 2012 is double the Total Income in 2011, we have:

  Total income in 2012 = 2×450 = \$900 million

  Required difference = 900 – 180 = \$720 million > \$650 million – Satisfies

- Option C:
  Required difference = 650 – 250 = \$400 million $\ngtr$ \$650 million – Does not satisfy

- Option D:
  Since the percent increase in Net Profits from 2010 to 2011 was the same as that from 2011 to 2012, we have:

  $$\frac{\text{Net Profit in 2011}}{\text{Net Profit in 2010}} = \frac{\text{Net Profit in 2012}}{\text{Net Profit in 2011}}$$

  $$=> \frac{250}{170} = \frac{\text{Net Profit in 2012}}{250}$$

  $$=> \text{Net Profit in 2012} = \frac{250 \times 250}{170} = 367.6 \approx \$368 \text{ million}$$

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Since the Total Income in 2012 is double the Total Income in 2011, we have:

Total income in 2012 = 2×650 = \$1,300 million

Required difference = 1,300 – 368 = \$932 million > \$650 million – Satisfies

The correct answers are options B and D.

221. Let the total number of books be n.

We know that, if the books in the bookshelf are divided into 3 stacks each having an equal number of books, 2 books are left over.

Thus, we have:

$n = 3k + 2$, where k is the number of books in each stack

We also know that, if the books in the bookshelf are divided into a number of stacks of 7 books each, 2 books are left over.

Thus, we have:

$n = 7l + 2$, where l is the number of stacks

Thus, we have:

n is two more than a multiple of the LCM of 3 and 7

=> $n = 21k + 2$

Since $n < 50$, the possible values of n are:

- For $k = 1$: $n = 21 \times 1 + 2 = 23$
- For $k = 2$: $n = 21 \times 2 + 2 = 44$

The correct answers are options B and D.

222. Let the number of hours worked in a week be w.

We know that:

- If $w \leq t$: Raymond's earnings = \$ xw.
- If $w > t$: Raymond's earnings = \$ $(xt + 2(w - t))$.

We have:

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$w = (t - 3)$; i.e. $w < t$

Thus, Raymond's earnings = \$ $(x(t - 3))$

Thus, we have:

$x(t - 3) = 14$

Possible values of x and t are:

- $x = 14$; $(t - 3) = 1 \Rightarrow t = 4$: Not possible since $t > 4$
- $x = 7$; $(t - 3) = 2 \Rightarrow t = 5$: Possible
- $x = 2$; $(t - 3) = 7 \Rightarrow t = 10$: Possible
- $x = 1$; $(t - 3) = 14 \Rightarrow t = 17$: Possible

The correct answers are options A, D and F.

223. We have:

$k^{\#} = k^2 + 1$

Since $17 \le k^{\#} \le 37$, we have:

$17 \le k^2 + 1 \le 37$

$\Rightarrow 16 \le k^2 \le 36$

$\Rightarrow 4 \le k \le 6$ OR $-6 \le k \le -4$

Thus, the possible positive values of k are: 4, 5, and 6.

The correct answer is option D.

224. We know that two sides of the triangle are $2x$ and $(5x + 1)$.

Let the third side be s.

Since sum of two sides is greater than the third side, we have:

- $2x + (5x + 1) > s \Rightarrow s < 7x + 1 \ldots$(i)
- $s + 2x > 5x + 1 \Rightarrow s > 3x + 1 \ldots$(ii)
- $s + (5x + 1) > 2x \Rightarrow s > -(3x + 1)$ – This implies that s is greater than a negative quantity (since $x > 0$), which is obvious

Thus, from (i) and (ii), we have:

$3x + 1 < s < 7x + 1 \ldots$(iii)

We must not simplify above inequality by cancelling '1' as $3x + 1$ and $7x + 1$ are whole values of sides.

Working with the options, we observe that:

- Option B: $(3x)$ is definitely less than $(3x + 1)$ – Not possible
- Option C: $(7x + 3)$ is definitely greater than $(7x + 1)$ – Not possible

Each of the other options can be valid depending on the value of x. Let us verify:

- Option A: For $x = 3$:

Relation (iii): $3 \times 3 + 1 < s < 7 \times 3 + 1 \Rightarrow 10 < s < 22$

$s = 2x + 5 = 11$ – Satisfies

Alternatively, we can check the inequalities:

- $3x + 1 < s \Rightarrow 3x + 1 < 2x + 5 \Rightarrow x < 4$
- $s < 7x + 1 \Rightarrow 2x + 5 < 7x + 1 \Rightarrow x > \dfrac{4}{5}$

Thus, any value of x between $\frac{4}{5}$ and 4 would satisfy relation (iii).

- Option D: For $x = \dfrac{1}{2}$:

Relation (iii): $3 \times \dfrac{1}{2} + 1 < s < 7 \times \dfrac{1}{2} + 1 \Rightarrow 2.5 < s < 4.5$

$s = 8x = 4$ – Satisfies

Alternatively, we can check the inequalities:

- $3x + 1 < s \Rightarrow 3x + 1 < 8x \Rightarrow x > \dfrac{1}{5}$
- $s < 7x + 1 \Rightarrow 8x < 7x + 1 \Rightarrow x < 1$

Thus, any value of x between $\frac{1}{5}$ and 1 would satisfy relation (iii).

The correct answers are options A and D.

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225. $\left(-\frac{1}{5}\right)^{|n|}$

$= (-1)^{|n|}\left(\frac{1}{5}\right)^{|n|}$

Since $|n|$ is always non-negative, the above expression will attain larger values for larger 'even' values of $|n|$, thereby resulting into a positive value of the above expression.

Thus, we have: $n = -2$ OR 2.

The correct answers are options B and C.

Alternate approach:

Working with the options, we have:

(A) $n = -3 \Rightarrow (-1)^{|n|}\left(\frac{1}{5}\right)^{|n|} = -\frac{1}{125}$

(B) $n = -2 \Rightarrow (-1)^{|n|}\left(\frac{1}{5}\right)^{|n|} = \frac{1}{25}$

(C) $n = 2 \Rightarrow (-1)^{|n|}\left(\frac{1}{5}\right)^{|n|} = \frac{1}{25}$

(D) $n = 3 \Rightarrow (-1)^{|n|}\left(\frac{1}{5}\right)^{|n|} = -\frac{1}{125}$

226. $|x^2 - 10| < 6$

$\Rightarrow -6 < x^2 - 10 < 6$

Adding 10 throughout:

$\Rightarrow 4 < x^2 < 16$

Taking square root throughout:

$\Rightarrow -4 < x < -2$

OR

$\Rightarrow 2 < x < 4$

Thus, possible integer values of x are -3 and 3

The correct answers are options B and E.

Alternatively, you can plug-in values from the options and check which values qualify for the absolute inequality $|x^2 - 10| < 6$. Since for x^2 in the inequality, it does not matter whether x

is positive or negative, you should check only options A, B and C as Option A would yield the same result as option D, Option B as Option E and Option C as Option F.

227. We have:

$u = x^2 + 2|x| + 1$

We know that:

$x^2 \geq 0$ and $|x| \geq 0$ for all values of x

Thus, u is the sum of two non-negative terms and a positive integer.

Thus, the value of u is always positive – Option A is correct

The minimum value of u occurs when $x^2 = |x| = 0 \Rightarrow x = 0$

The corresponding minimum value of u is 1 – Option B is correct

$u = x^2 + 2|x| + 1$

$\Rightarrow u = |x|^2 + 2|x| + 1$

$\Rightarrow u = (|x| + 1)^2$

Thus, the value of u is always a perfect square – Option C is correct

The correct answers are options A, B and C.

228. We know that AAB is a three-digit number where $B = 2A$.

Thus, the possible values of AAB are:

- For $A = 1 : AAB = 112$
- For $A = 2 : AAB = 224 = 112 \times 2$
- For $A = 3 : AAB = 336 = 112 \times 3$
- For $A = 4 : AAB = 448 = 112 \times 4$

A cannot be 5, since if it so, B would be 10, not a single digit.

We can see that all the above numbers have 112 common.

Since $112 = 16 \times 7 = 2^4 \times 7$, the prime factors of 112 are 2 and 7.

Thus, the prime numbers that are common factors of all such numbers are 2 and 7.

The correct answers are A and D.

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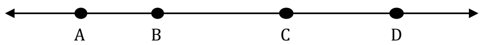
229. Let us bring out the figure.



We know that:

$AB = \frac{1}{2}CD$

$=> CD = 2AB \ldots (i)$

$BD = \frac{3}{2}AC \ldots (ii)$

$BC = 24 \ldots (iii)$

From (ii):

$=> BC + CD = \frac{3}{2}(AB + BC)$

Using (i) and (iii):

$=> 24 + 2AB = \frac{3}{2}(AB + 24)$

$=> 48 + 4AB = 3AB + 72$

$=> AB = 24$

$=> CD = 2 \times AB = 2 \times 24 = 48$

Also, $AC = AB + BC = 24 + 24 = 48$

Thus, we have:

$AC = CD = 48$

The correct answers are B and D.

230. We know that the average number of correct answers was 50 and the median number of correct answers was 40.

Since there were 9 candidates, the median value would be that for the $\left(\frac{9+1}{2}\right)^{th} = 5^{th}$ student (after arranging in ascending or descending order).

Thus, the 5^{th} student had 40 correct answers.

Thus, at least one student had exactly 40 correct answers – Option C is correct

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Let the average of the number of correct answers of the first 4 students be f, and the average of the number of correct answers of the last 4 students be l.

Since the average number of correct answers is 50, we have:

$$\frac{4f + 40 + 4l}{9} = 50$$

$$=> 4f + 4l = 410$$

$$=> f + l = 102.5 \ldots \text{(i)}$$

Since the number of correct answers of the first four students must be less than or equal to 40, we have:

$$f \le 40$$

Thus, from (i):

$$l \ge 102.5 - 40$$

$$=> l \ge 62.5$$

Thus, the average number of correct answers of the last 4 students is at least 62.5.

Thus, at least one student must have more than 60 correct answers. – Option A is correct

From the results $f \le 40$ and $l \ge 62.5$, if we assume that the nine scores are: 40, 40, 40, 40, 40, 62.5, 62.5, 62.5, and 62.5, we can invalidate option B & D.

The correct answers are options A and C.

231. If Suzy returns at the end of the day 2, she would have stayed for 2 days.

Since she would return home at the end of the first day on which it rained, it must not have rained on day 1 and rained on day 2.

Let the probability of rain on each day = $P(R) = x$

=> Probability that there is no rain on a particular day

$$= P\left(\overline{R}\right) = 1 - P(R)$$

$$= 1 - x$$

Thus, probability that she returns at the end of day 2

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= Probability that there is no rain on day 1 AND there is rain on day 2

$= P\left(\overline{R}\right) \times P(R)$

$= (1 - x)x$

Since the above probability is 0.24, we have:

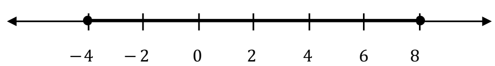
$(1 - x)x = 0.24$

Since $0.24 = 0.6 \times 0.4 = (1 - 0.4) \times 0.4 = 0.6 \times (1 - 0.6)$, we have:

$x = 0.4$ OR 0.6

The correct answers are options B and D.

232. Let us bring out the figure.



The highlighted region is $-4 \le x \le 8$

Working with the options:

(A) $|x| \le 2 \Rightarrow -2 \le x \le 2$ – Does not satisfy

(B) $|x| \le 5 \Rightarrow -5 \le x \le 5$ – Does not satisfy

(C) $|x| \le 9 \Rightarrow -9 \le x \le 9$ – Does not satisfy

(D) $|x - 2| \le 4 \Rightarrow -4 \le x - 2 \le 4 \Rightarrow -2 \le x \le 6$ – Does not satisfy

(E) $|x - 2| \le 6 \Rightarrow -6 \le x - 2 \le 6 \Rightarrow -4 \le x \le 8$ – Satisfies

(F) $(x - 2)^2 - 6 \le 30$
$\Rightarrow (x - 2)^2 \le 36$
$\Rightarrow -6 \le x - 2 \le 6$
$\Rightarrow -4 \le x \le 8$ – Satisfies

The correct answers are options E and F.

Alternate approach:

You can plug-in the extreme values from the highlighted region of the graph. The values are: −4 & 8. The correct option must result only in 'equality' and not in 'less than or equal to' since these are the extreme values.

Let's us plug-in $x = -4$ & 8 in each option and check which option(s) are correct.

(A) $|x| \le 2$:

- $@x = -4,\ |-4| = 4 \ne 2 \Rightarrow$ inequality does not hold true

- $@x = 8,\ |8| = 8 \neq 2$ => inequality does not hold true, not the correct answer.

(B) $|x| \leq 5$:

- $@x = -4,\ |-5| = 4 \neq 5$ => inequality does not hold true not the correct answer.
- $@x = 8,\ |8| = 8 \neq 5$ => inequality does not hold true, not the correct answer.

(C) $|x| \leq 9$

- $@x = -4,\ |-4| = 4 \neq 9$ => inequality does not hold true, not the correct answer.
- $@x = 8,\ |8| = 8 \neq 9$ => inequality does not hold true, not the correct answer.

(D) $|x - 2| \leq 4$

- $@x = -4,\ |-4 - 2| = 6 \neq 4$ => inequality does not hold true, not the correct answer.
- $@x = 8,\ |8 - 2| = 6 \neq 4$ => inequality does not hold true

(E) $|x - 2| \leq 6$

- $@x = -4,\ |-4 - 2| = 6 = 6$ => inequality holds true
- $@x = 8,\ |8 - 2| = 6 = 6$ => inequality holds true, correct answer

(F) $(x - 2)^2 - 6 \leq 30 = (x - 2)^2 \leq 36$

- $@x = -4,\ (-4 - 2)^2 = 36 = 36$ => inequality holds true
- $@x = 8,\ (8 - 2)^2 = 36 = 36$ => inequality holds true, correct answer

233. We know that 10 gram of the food supplement contains 9% of the minimum daily requirement of protein and 11% of the minimum daily requirement of vitamin C.

Let x grams of the food supplement are consumed so that at least the minimum daily requirements of protein and vitamin C are provided.

Thus, x grams of the health food contain $(9x/10)$% of the minimum daily requirement of protein and $(11x/10)$% of the minimum daily requirement of vitamin C.

Thus, we have:

$9x/10 \geq 100$ AND $11x/10 \geq 100$ (since we may exceed the minimum daily requirement)

$$=> x \geq \frac{1,000}{9} \text{ AND } x \geq \frac{1,000}{11}$$

Thus, the values of x satisfying the above inequalities are:

$$x \geq \frac{1,000}{9} = 111.11$$

The correct answers are option C and D.

Alternate approach:

You would notice that between the two protein and vitamin C, the food supplies comparatively lesser amount of protein (9% of protein vs 11% of Vit. C), so if one has ample supplies of protein, he can be assured of the ample supplies of vitamin C.

Thus, the limiting case is protein.

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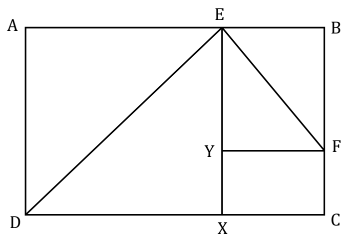
Since 10 gm of the food provides only 9% of the minimum requirement protein, minimum intake of food = $\frac{1,000}{9}$ = 111.11 gm

234.



Let us draw the line EX perpendicular to DC and FY perpendicular to EX, as shown above.

Since area of triangle ADE is 20, area of triangle DEX is also 20.

=> Area of AEXD = 40

Again, since area of triangle BEF is 8, area of triangle EFY is also 8.

=> Area of EBFY = 16

Since FC $<$ BF, we have:

FC $\times$ FY $<$ BF $\times$ FY

=> Area of CFYX $<$ Area of EBFY

=> Area of CFYX $<$ 16

Thus, area of rectangle ABCD

= Area of AEXD + Area of EBFY + Area of CFYX

= 40 + 16 + Area of CFYX

= 56 + Area of CFYX

Since Area of CFYX $<$ 16, we have:

The area of ABCD $<$ 56 + 16 = 72

Thus, the area of ABCD is less than 72, but greater than 56.

The correct answers are options C and D.

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235. Time taken by the first pipe to fill half the entire tank = 3 hours.

Thus, time taken by the first pipe to fill the entire tank

$= 2 \times 3$

= 6 hours

We can have the following scenarios:

- The time taken by both pipes to fill the tank together will be the minimum when the second pipe has the maximum efficiency.

Thus, we need to assume that it fills the full tank (less than half full) in slightly more than 6 hours.

Thus, we have two pipes, the first one can fill the tank in exactly 6 hours and the other in slightly more than 6 hours.

Thus, time taken by both pipes will be half the time taken by each alone, i.e. $\frac{6}{2}$ = slightly more than 3 hours.

- The time taken by both pipes to fill the tank together will be the maximum when the second pipe has the minimum efficiency.

Thus, we need to assume that it fills 'slightly less than a half' full tank in 6 hours, or, as an extreme case, that it fills half the full tank in 6 hours.

Thus, the second pipe fills the entire tank in $2 \times 6 = 12$ hours. (Since the tank is slightly less than half full, the actual time would be slightly less than 12)

Thus, we have two pipes, one of which can fill the tank in 6 hours and the other in slightly less than 12 hours.

Thus, in 1 hour, fraction of the tank filled by both = $\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$ (The actual fraction is slightly greater than $\frac{1}{4}$ since the fraction filled by the second pipe would be slightly greater than $\frac{1}{12}$)

Thus, time taken by both pipes to fill the tank = $\frac{1}{\left(\frac{1}{4}\right)}$ = 4 hours

(The actual time would be slightly less than 4 hours)

Thus, the time taken would be in between 3 hours to 4 hours, exclusive.

The correct answer is option C.

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236. Let us analyze the situations of John and Bob:

John:

John's score = 75

Mean score of all students = 70

Difference between John's score and the mean score = 75 – 70 = 5

Standard deviation of all scores = 2

Thus, number of 'Standard deviations' that John's score is more than the mean score

$= \frac{5}{2} = 2.5$

Bob:

Bob's score = 72

Mean score of all students = 65

Difference between Bob's score and the mean score = 72 – 65 = 7

Standard deviation of all scores = 3

Thus, number of 'Standard deviations' that Bob's score is more than the mean score

$= \frac{7}{3} = 2.33 < 2,5$

Thus, John's score (2.5*Standard deviation) is farther away from the mean than is Bob's score (2.33*Standard deviation). – Option A is correct

Considering Option B:

Since John's score is higher than the mean score, there may or may not be a student who has scored higher than what John has scored. – Option B is incorrect

Considering Option C:

Since Bob's score is higher than the mean score, there must be a student who has scored less than the mean score, and hence, has scored less than Bob's score. – Option C is correct

The correct answers are options A and C.

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237. Let the five positive distinct integers be k, m, r, s and t such that $k < m < r < s < t$; where $t = 40$.

Since the average of the five positive integers is 16, we have:

$$\frac{k + m + r + s + t}{5} = 16$$

$=> k + m + r + s + t = 80$

Since $t = 40$, we have:

$k + m + r + s = 80 - 40 = 40 \ldots$(i)

Also, we have:

$k < m < r < s < t$

$=> k < m < r < s < 40$

The median is the middle term when the numbers are arranged in order.

Thus, the median is r.

We need to determine the maximum and minimum possible values of the median.

Thus, we have the following scenarios:

- Since we need to maximize r, and at the same time keep all the numbers positive integers, we have:

  $k = 1, m = 2$

  Thus, from (i), we have:

  $r + s = 40 - (1 + 2) = 37$

  Since $r < s$, we have:

  $r = 18, s = 19$

  Thus, the maximum value of the median is 18.

- Since we need to minimize r, and at the same time keep all the numbers positive integers, we have:

  $k = 1, m = 2$ $(=> r > 2)$

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Thus, from (i), we have:

$r + s = 40 - (1 + 2) = 37$

Since $r < s$, we have:

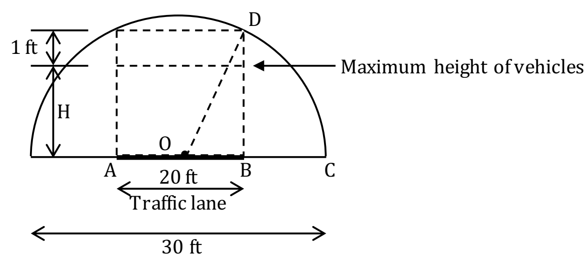
$r = 3,\ s = 34$

Thus, the minimum value of the median is 3.

Thus, the median can lie between 3 and 18, inclusive.

The correct answers are B, C, D and E.

238.



Let O be the center of the tunnel (and hence, of the traffic lane AB).

We need to find the maximum height of the vehicles allowed in the tunnel so that there is a clearance of 1 feet from the top of the tunnel.

Let the height of vehicles allowed be H feet

(Note: At the center of the tunnel, i.e. above O, the height is larger than at the sides of the traffic lane, i.e. above A or B. Thus, in order to find the maximum height of vehicles allowed, we must take the positions A or B)

In right angled triangle OBD:

$$OB = \frac{AB}{2} = \frac{20}{2} = 10$$

$$OD = OC = \frac{30}{2} = 15$$

Thus, from Pythagoras' theorem:

$$BD^2 = OD^2 - OB^2$$

$$= 15^2 - 10^2$$

$= 125$

$=> BD = 5\sqrt{3}$

Thus, we have:

$H = BD - 1 = 5\sqrt{3} - 1$

Thus, the maximum allowed height would be $(5\sqrt{3} - 1)$ feet.

The correct answers are options C, D, E, and F.

239. $f(x) = -\frac{1}{x}$

$=> f(p) = -\frac{1}{p} = -\frac{1}{2}$

$=> p = 2$

Also, we have:

$f(pq^2) = -\frac{1}{pq^2} = -\frac{1}{18}$

$=> pq^2 = 18$

$=> q^2 = \frac{18}{p} = \frac{18}{2} = 9$

$=> q = \pm 3$

The correct answers are options A and D.

240. $f(x) = \sqrt{x} - 10$

$=> f(q) = \sqrt{q} - 10$

Since $p = f(q)$, we have:

$p = \sqrt{q} - 10$

$=> \sqrt{q} = p + 10$

Squaring both sides, we have:

$=> q = (p + 10)^2$ – Option B is correct

$= (p^2 + 20p + 100)$

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$= (p^2 - 20p + 100) + 40p$

$\Rightarrow q = (p - 10)^2 + 40p$ – Option D is correct

The correct answers are options B and D.

Alternate approach:

We know that:

$\sqrt{q} = p + 10$

Let us take a set of values of p and q which satisfy the above equation; say $p = 0$; thus, $q = 100$.

Substituting the above values in each option, we have:

(A) $\sqrt{p + 10} = \sqrt{10} \neq q$ – Does not satisfy

(B) $(p + 10)^2 = 100 = q$ – Satisfies

(C) $\sqrt{p^2 + 10} = \sqrt{10} \neq q$ – Does not satisfy

(D) $(p - 10)^2 + 40p = 100 = q$ – Satisfies

(E) $p^2 + 10 = 10 \neq q$ – Does not satisfy

(F) $(p - \sqrt{10})(p + \sqrt{10}) + 20 = 10 \neq q$ – Does not satisfy

241. Slope of the line L passing through $(0, 2)$ and $(-3, 0)$

$= \frac{2 - 0}{0 - (-3)}$

$= \frac{2}{3}$

Thus, the slope of the line parallel to the above line L $= \frac{2}{3}$

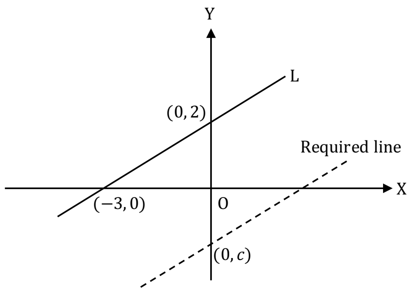
Thus, the equation of the line parallel to the line L is

$y = \frac{2}{3}x + c$

$\Rightarrow 3y = 2x + 3c$

$\Rightarrow 3y - 2x = 3c$, where c is the intercept on the Y-axis

Since the line should intersect the positive direction of the X-axis, it must intersect the negative direction of the Y-axis as shown in the figure below:



Thus, the value of c should be negative.

Thus, the equation of the parallel line is:

$3y - 2x =$ (a negative quantity)

The correct answers are options B and E.

242. Number of defects in the first 6 cars = 9, 7, 10, 4, 6 and n, respectively.

Thus, the total number of defects = $(9 + 7 + 10 + 4 + 6 + n) = 36 + n$

Thus, the mean number of defects

$$= \frac{36 + n}{6}$$

We know that the mean number of defects and the median number of defects are the same.

Working with the options:

(A) $n = 3$:

$$\text{Mean} = \frac{36 + 3}{6} = 6.5$$

The number of defects, when arranged in order, is: 3, 4, 6, 7, 9 and 10

The median is the average of the two middle terms.

Thus, we have:

$$\text{Median} = \frac{\left\{\left(\frac{6}{2}\right)^{\text{th}}\text{term} + \left(\frac{6}{2} + 1\right)^{\text{th}}\text{term}\right\}}{2} = \frac{\left\{3^{\text{rd}}\text{ term} + 4^{\text{th}}\text{ term}\right\}}{2} = \frac{6 + 7}{2} = 6.5$$

Thus, Mean = Median – Satisfies

(B) $n = 7$:

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Mean = $\frac{36 + 7}{6} = 7.16$

The number of defects, when arranged in order, is: 4, 6, 7, 7, 9 and 10

The median is the average of the two middle terms.

Thus, we have:

$$\text{Median} = \frac{\left\{\left(\frac{6}{2}\right)^{\text{th}} \text{term} + \left(\frac{6}{2} + 1\right)^{\text{th}} \text{term}\right\}}{2} = \frac{\left\{3^{\text{rd}} \text{ term} + 4^{\text{th}} \text{ term}\right\}}{2} = \frac{7 + 7}{2} = 7$$

Thus, Mean $\neq$ Median – Does not satisfy

(C) $n = 12$:

Mean = $\frac{36 + 12}{6} = 8$

The number of defects, when arranged in order, is: 4, 6, 7, 9, 10 and 12

The median is the average of the two middle terms.

Thus, we have:

$$\text{Median} = \frac{\left\{\left(\frac{6}{2}\right)^{\text{th}} \text{term} + \left(\frac{6}{2} + 1\right)^{\text{th}} \text{term}\right\}}{2} = \frac{\left\{3^{\text{rd}} \text{ term} + 4^{\text{th}} \text{ term}\right\}}{2} = \frac{7 + 9}{2} = 8$$

Thus, Mean = Median – Satisfies

The correct answers are options A and C.

243. Working with the options:

(A) The unit digit of p or of q or of both p and q may be '0'.
In that case, the above statement is not correct.

For example: $p = 870,\ q = 950 \Rightarrow p + q = 1,820$, a four-digit number.

Here, the unit digit of $(p + q)$ is NOT greater than the unit digit of either p or q (in fact, it is equal).

Thus, Option A is not correct.

(B) Since the digits in the unit place of p and q are not known, the digit in the tens place of $(p + q)$ could be 2 (if there is no carry from the unit digits) or 3 (if there is a carry of '1' from the unit digits of p and q).

Note: The maximum possible carry from the unit digits when two numbers are added is ‘1’.

For example:

- $p = 870,\ q = 950 => p + q = 1,820$, the tens digit is 2.
- $p = 875,\ q = 956 => p + q = 1,831$, the tens digit is 3.

Thus, Option B is correct.

(C) Since p and q are three-digit numbers, $p < q$, and $(p + q)$ is a four-digit number; the hundreds digit of p and q can have the following possibilities (given that the tens digits of p and q are 7 and 5, respectively, there is a carry from the tens place in addition):

- $p = \mathbf{1}7\#$, $q = \mathbf{9}5\#$
- $p = \mathbf{1}7\#$, $q = \mathbf{8}5\#$
- $p = \mathbf{2}7\#$, $q = \mathbf{8}5\#$
- $p = \mathbf{2}7\#$, $q = \mathbf{7}5\#$
- $p = \mathbf{3}7\#$, $q = \mathbf{7}5\#$
- $p = \mathbf{3}7\#$, $q = \mathbf{6}5\#$
- $p = \mathbf{4}7\#$, $q = \mathbf{6}5\#$
- $p = \mathbf{4}7\#$, $q = \mathbf{5}5\#$

Note: $p = \mathbf{5}7\#$, $q = \mathbf{5}5\#$ is not possible since it would violate the condition that $p < q$

Thus, we can observe that the digit in the hundreds place of q is at least 5.

Thus, Option C is not correct.

The correct answers is option B.

244. Working with the options:

(A) LHS $= m\%mn = \dfrac{m}{mn} - \dfrac{mn}{m} = \dfrac{1}{n} - n$

RHS $= m\,(1\%n) = m\left(\dfrac{1}{n} - \dfrac{n}{1}\right) = \dfrac{m}{n} - mn$

Thus, LHS $\neq$ RHS – Option A is not correct

(B) LHS $= m\%n = \dfrac{m}{n} - \dfrac{n}{m}$

RHS $= -\,(n\%m) = -\left(\dfrac{n}{m} - \dfrac{m}{n}\right) = \dfrac{m}{n} - \dfrac{n}{m}$

Thus, LHS = RHS – Option B is correct

(C) LHS $= \dfrac{1}{m}\%\dfrac{1}{n} = \dfrac{\left(\frac{1}{m}\right)}{\left(\frac{1}{n}\right)} - \dfrac{\left(\frac{1}{n}\right)}{\left(\frac{1}{m}\right)} = \dfrac{n}{m} - \dfrac{m}{n}$

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RHS = $n\%m = \frac{n}{m} - \frac{m}{n}$

Thus, LHS = RHS – Option C is correct

The correct answers are options B and C.

245. Working with the options:

(A) $c \sim (-c) = \frac{1}{c} + \frac{1}{(-c)} = 0$ – Option A is correct

(B) $c \sim \left(\frac{c}{c-1}\right) = \frac{1}{c} + \frac{1}{\left(\frac{c}{c-1}\right)} = \frac{1}{c} + \frac{c-1}{c} = \frac{1+c-1}{c} = 1$ – Option B is correct

(C) $\frac{2}{c} \sim \frac{2}{c} = \frac{1}{\left(\frac{2}{c}\right)} + \frac{1}{\left(\frac{2}{c}\right)} = \frac{c}{2} + \frac{c}{2} = c$ – Option C is correct

The correct answers are options A, B and C.

246. $kx + 15 = 3k$

=> $kx = 3k - 15$

=> $x = \frac{3k - 15}{k}$

=> $x = 3 - \frac{15}{k}$

Since x is a positive integer, k must be factors of 15 (either positive or negative) so that $\left(3 - \frac{15}{k}\right)$ is positive.

Thus, we have:

$3 - \frac{15}{k} > 0$

=> $\frac{3k - 15}{k} > 0$

=> $\frac{3(k-5)}{k} > 0$

=> $\frac{k-5}{k} > 0$

=> $k - 5 > 0$ AND $k > 0$ => $k > 5$

OR

$k - 5 < 0$ AND $k < 0$ => $k < 0$

Since k should be factors of 15, we have:

$k = 15,\ -1,\ -3,\ -5,\ -15$

The answers are B, C, D, E and H.

Alternate approach:

We have: $kx + 15 = 3k$

Working with the options:

(A) $k = -30 \Rightarrow -30x + 15 = -90 \Rightarrow x = 3.5$ – Does not satisfy since x is not an integer
(B) $k = -15 \Rightarrow -15x + 15 = -45 \Rightarrow x = 4$ – Satisfies: x is a positive integer
(C) $k = -5 \Rightarrow -5x + 15 = -15 \Rightarrow x = 6$ – Satisfies: x is a positive integer
(D) $k = -3 \Rightarrow -3x + 15 = -9 \Rightarrow x = 8$ – Satisfies: x is a positive integer
(E) $k = -1 \Rightarrow -x + 15 = -3 \Rightarrow x = 18$ – Satisfies: x is a positive integer
(F) $k = 3 \Rightarrow 3x + 15 = 9 \Rightarrow x = -2$ – Does not satisfy since x is not positive
(G) $k = 5 \Rightarrow 5x + 15 = 15 \Rightarrow x = 0$ – Does not satisfy since x is not positive
(H) $k = 15 \Rightarrow 15x + 15 = 45 \Rightarrow x = 2$ – Satisfies: x is a positive integer

247. $a = 22.567$, rounded to its tenth place

$\Rightarrow a = 22.6$ (since the number to the right of the tenth digit, '5' is greater than 5)

$b = 225.78$, rounded to its unit place

$\Rightarrow b = 226$ (since the number to the right of the decimal, '7' is greater than 5)

$c = 2.45$, rounded to its units place

$\Rightarrow c = 2$ (since the number to the right of the units digit, '2' is less than 5)

(Note: It would be incorrect to round up 2.45 as $2.45 \approx 2.5 \approx 3$)

Thus, working with the options:

(A) $b = 226 = 10 \times 22.6 = 10a$ – Correct
(B) $10c = 10 \times 2 = 20 \ngtr a$ – Incorrect
(C) $b - 10a = 226 - 10 \times 22.6 = 0 < c$ – Correct

The correct answers are options A and C.

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248. Let the middle integer be x.

Since the largest integer is 3 times the middle integer, we have:

The largest integer = $3x$

Since the smallest integer is 23 less than the largest integer, we have:

The smallest integer = $(3x - 23)$

Since the sum of the integers is 40, we have:

$(3x - 23) + x + 3x = 40$

=> $x = 9$

=> The three integers are:

- $3 \times 9 - 23 = 4$
- 9
- $3 \times 9 = 27$

The correct answers are B, D and G.

249. Let the percent change from first quarter to the second quarter be $p\%$

Thus, we have:

Income in second quarter = (Income in first quarter) $\times \left(1 \pm \frac{p}{100}\right)$

=> Income in first quarter = $\frac{\text{Income in second quarter}}{\left(1 \pm \frac{p}{100}\right)}$

In the relation above, (+) is used if the percent change is positive and (–) is used if the percent change is negative.

Thus, based on the above relation, we have:

Sector	Net Income (billion $) in second quarter	Percent change from first quarter	Net Income (billion $) in first quarter
Telecommunications	4.80	−20%	$\frac{4.80}{\left(1-\frac{20}{100}\right)}=\frac{4.80}{0.80}=6.00$ (**Greatest**)
Power	7.20	+20%	$\frac{7.20}{\left(1+\frac{20}{100}\right)}=\frac{7.20}{1.2}=6.00$ (**Greatest**)
Agriculture	5.00	−10%	$\frac{5.00}{\left(1-\frac{10}{100}\right)}=\frac{5.00}{0.90}=5.56$
Services	9.00	+200%	$\frac{9.00}{\left(1+\frac{200}{100}\right)}=\frac{9.00}{3}=3.00$
Information Tech.	2.50	−50%	$\frac{2.50}{\left(1-\frac{50}{100}\right)}=\frac{2.50}{0.5}=5.00$

The correct answers are options A and B.

250. Let us calculate the percent growth in coffee consumption from the previous year for each of the years 2012 to 2015:

Year	Consumption (million kilograms)	Percent growth from previous year
2011	540	–
2012	648	$\frac{648-540}{540}\times 100=20\%$ (**Lowest**)
2013	810	$\frac{810-648}{648}\times 100=25\%$
2014	972	$\frac{972-810}{810}\times 100=20\%$ (**Lowest**)
2015	1,296	$\frac{1,296-972}{972}\times 100=33.3\%$

The correct answers are options A and C.

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4.3 Numeric Entry Questions

251. Since the problem asks for a percent value, we can assume any suitable value of Joe's income in 2001.

Let Joe's income in 2001 be \$100.

Joe's income in 2002

= \${(100 + 10)% of 100}

= \$110

Taxes paid in 2002

= \$(3.4% of 100)

= \$3.4

Thus, percent of income (2012) paid in taxes

$= \frac{3.4}{110} \times 100$

= 3.09%

The correct answer is '3.09.'

252. We know that the number of students in 2001 = 350.

Let number of students in 1981 be n.

Thus, the number of students in 1991

$= (100 + 50)\,\%$ of n

$= \frac{150}{100} \times n$

$= \frac{3n}{2}$

Again, number of students in 2001

$= (100 + 250)\,\%$ of n

$= \frac{350}{100} \times n$

$= \frac{7n}{2}$

Thus, we have:

$\frac{7n}{2} = 350$

$=> n = 100$

=> The number of students in 1991

$= \frac{3 \times 100}{2}$

= 150

Thus, the percent increase in the number of students from 1991 to 2001

$= \frac{350 - 150}{150} \times 100$

= 133.33% = ≈ 133%

The correct answer is '133.'

253. Number of shares of stock X with Mr. John = n.

Dividend earned on the above n shares = \$150.

Thus, dividend earned per share

$= \$\left(\frac{150}{n}\right)$

Number of shares of stock X with Mrs. John = m.

Dividend earned on the above m shares = $\$\left(\frac{4,500}{n}\right)$.

Thus, dividend earned per share

$= \$\frac{\left(\frac{4,500}{n}\right)}{m}$

$= \$\left(\frac{4,500}{mn}\right)$

Since the shares are of the same stock X, the dividend earned per share is the same.

Thus, we have:

$\frac{150}{n} = \frac{4,500}{mn}$

=> $m = 30$

Thus, the number of shares of stock X with Mrs. John = 30.

The correct answer is '30.'

254. It is known that each of the voters voted for at least one of the candidates.

60 percent of the female voters voted for Edith.

Also, 20 percent of the female voters voted for both candidates.

Thus, the above 20 percent of the female voters are included in the 60 percent female voters who voted for Edith.

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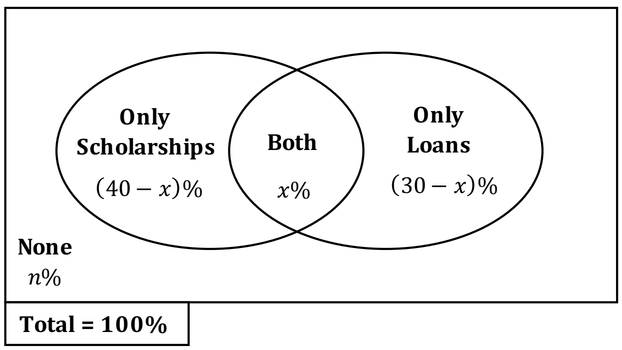
Thus, there are 60 percent of the female voters who voted for only Edith or for both Edith and Jose.

Thus, the remaining (100 – 60) = 40 percent of the voters voted only for Jose.

The data about the total number of voters is redundant.

The correct answer is '40.'

255. Let us draw the corresponding Venn-diagram:



We need to determine the value of n.

Since 25 percent of those surveyed had received scholarships but no loans, we have

$(40 - x)\% = 25\%$

$\Rightarrow x = 15\%$

Thus, the percent of students who received scholarships or loans

$= (40 + 30 - x)\%$

$= 55\%$

Thus, the percent of students who received neither student loans nor scholarships ($n\%$)

$= (100 - 55)\%$

$= 45\%$

The correct answer is '45.'

256. Let the number of sales transactions made by Y in May be x.

Thus, the number of sales transactions made by X in June

$= (100 + 50)\%$ of x

$= \frac{3x}{2}$

Also, the number of sales transactions made by Y in June

$= (100 + 25)\%$ of x

$= \frac{5x}{4}$

Thus, the required ratio

$= \frac{3x}{2} : \frac{5x}{4}$

$= 6 : 5$

The correct answer is '$\frac{6}{5}$.'

Alternate approach:

Salesperson	**May**	**June**
X		(100+50)% of 100 = 150
Y	Say, 100	(100+25)% of 100 = 125

Thus, the required ratio

$= \frac{150}{125} = \frac{6}{5}$

257. Since ■, △ and ∀ represent positive digits, their values can only be from 1 to 9, inclusive.

Since ■ < △, the possible values of ■ and △ are

■ = 6 and △ = 8

OR

■ = 5 and △ = 9

△ is an odd number, we have:

△ = 9

The correct answer is '9.'

258. Let the length of BC = x.

Thus, we have:

AB = $10 - x$

CD = $15 - x$

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We know that:

$\frac{AB}{BC} = \frac{BC}{CD}$

=> $BC^2 = AB \times CD$

=> $x^2 = (10 - x)(15 - x)$

=> $x^2 = 150 - 25x + x^2$

=> $25x = 150$

=> $x = 6$

Thus, we have

AD

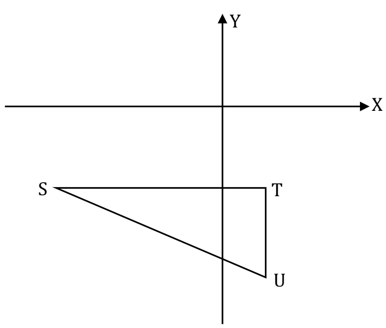
= AC + BD – BC

= 10 + 15 – 6

= 19

The correct answer is '19.'

259. Let us refer to the diagram below:



Let the coordinates of S be $(a, -5)$.

Also, let the coordinates of U be $(1, d)$.

Since ST is parallel to the X-axis, S and T have the same Y-coordinate.

Also, since UT parallel to the Y-axis, U and T have the same X-coordinate.

Thus, the coordinates of T = $(1, -5)$

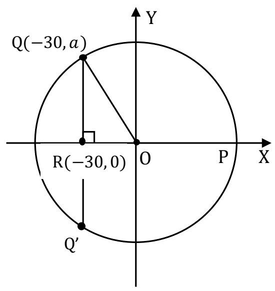
Thus, the sum of the coordinates of point T

= $1 + (-5)$

= -4

The correct answer is '−4.'

260. Let us refer to the diagram below:



We know that the P has coordinates $(50, 0)$.

Thus, the radius of the circle = 50.

Let the coordinates of point Q be $(-30, a)$.

In right angled triangle QRO, we have:

QR = a, RO = 30 and QO = Radius of the circle = 50

From Pythagoras' theorem, we have:

$QR^2 + RO^2 = QO^2$

$=> a^2 + 30^2 = 50^2$

$=> a^2 = 1,600$

$=> a = \pm 40$

Thus, there are two possible positions of Q, as shown in the diagram above: Q $(-30,\ 40)$ and Q' $(-30, -40)$.

Thus, the distance of P from either position of Q

= The length of PQ or PQ'

$= \sqrt{(50 - (-30))^2 + (0 - a)^2}$

$= \sqrt{(50 - (-30))^2 + (0 - (\pm 40))^2}$

$= \sqrt{80^2 + 40^2}$

$= \sqrt{8,000}$

$= 40\sqrt{5}$

$=\approx 40 \times 2.236$

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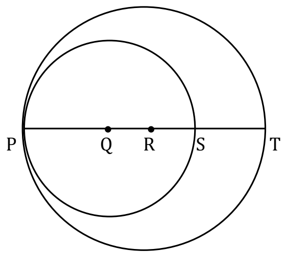
= 89.44

The correct answer is ‘89.44.’

261. Let us refer to the diagram below:



Required area = (Area of bigger circle with center R) – (Area of smaller circle with center Q)

$= \pi \times PR^2 - \pi \times PQ^2$

We know that:

RS = 1 and ST = 4

=> RT = 1 + 4 = 5

=> PR = RT = 5

Also, we have:

PS = PT – ST

= 2 × PR – ST

= 10 – 4 = 6

=> PQ + QS = 6

=> 2 × PQ = 6

=> PQ = 3

Thus, we have:

PR = 5 and PQ = 3

Thus, required area

$= \pi \times PR^2 - \pi \times PQ^2$

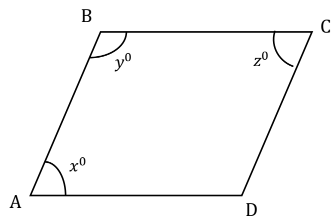
$= \pi\ (5^2 - 3^2)$

$= 16\pi$

$= \approx 50.24$

The correct answer is '50.24.'

262. Let us refer to the diagram below:



In the parallelogram above, we have:

$x = z$ (angles at opposite vertices are equal) . . . (i)

$x + y = y + z = 180$ (angles at adjacent vertices are supplementary) . . . (ii)

We know that:

$y = 3x$

Substituting the above value of y in (ii), we have:

$x + y = 180$

$=> x + 3x = 180$

$=> x = 45^\circ$

The correct answer is '45.'

263. We know that for all n:

$t_{(n+1)} = \frac{t_n}{2}$

Thus, we have:

$t_2 = \frac{t_1}{2},\ t_3 = \frac{t_2}{2}, \ldots$

Thus, we see that every number starting from t_2 is half of the previous number.

Let $t_5 = x$

$=> t_1 = 2 \times t_2 = 2^2 \times t_3 = 2^3 \times t_4 = 2^4 \times t_5$

$=> t_1 = 16x$

$=> t_1 - t_5 = 16x - x = 15x$

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$=> 15x = \frac{15}{16}$

$=> x = t_5 = \frac{1}{16}$

The correct answer is '$\left(\frac{1}{16}\right)$.'

264. We can see that line k passes through the points $(0,\ 0)$ and $\left(\frac{16}{5}, -\frac{12}{5}\right)$.

Thus, slope of line k

$= \left(\frac{-\frac{12}{5} - 0}{\frac{16}{5} - 0}\right)$

$= -\frac{3}{4}$

Since the product of the slopes of lines l and k is -1, slope of line l =

$-\left(\frac{1}{-\frac{3}{4}}\right)$

$= \frac{4}{3}$

The correct answer is '$\left(\frac{4}{3}\right)$.'

265. If line a intersects the X-axis at $(p, 0)$ and the Y-axis at $(0, q)$, the slope of the line

$= \frac{q - 0}{0 - p} = -\frac{q}{p} = -\left(\frac{\text{Y intercept}}{\text{X intercept}}\right)$

Thus, the slope of line a

$= -\left(\frac{-1}{-1}\right) = -1$

Since lines a and b are parallel, slope of line b is -1.

Let line b intersect the Y-axis at $(0, n)$.

We also know that line b passes through $(10,\ 20)$.

Thus, we have:

Slope of line b

$= \frac{20 - n}{10 - 0} = -1$

$=> 20 - n = -10$

$=> n = 30$

Thus, the Y-intercept of line b is 30.

The correct answer is '30.'

266. $4x + by + c = 0$

$=> by = -4x - c$

$=> y = \left(-\frac{4}{b}\right)x - \frac{c}{b}$

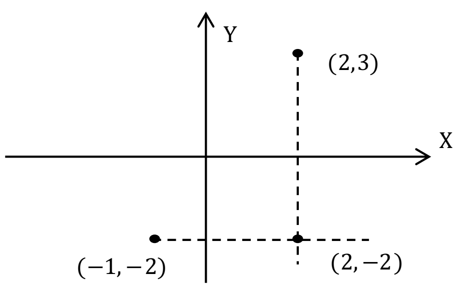
$=>$ The slope of the line $= \left(-\frac{4}{b}\right)$

$=> -\frac{4}{b} = \frac{2}{3}$

$=> b = -6$

The correct answer is '−6.'

267. The figure depicting the two vertices of the rectangle $(-1, -2)$ and $(2,\ 3)$ is shown below:



Since the length and width of the rectangle are parallel to the X and Y axes, the dotted lines shown in the figure above must denote the length and width of the rectangle.

Thus, the third vertex must be the point of intersection of the dotted lines i.e. $(2, -2)$.

Thus, we know that the length of the rectangle is the difference between the X values of the coordinates of the two points (since the length is parallel to the X axis).

Thus, the length of the rectangle

$= 2 - (-1) = 3$

Also, the width of the rectangle is the difference between the Y values of the coordinates of the two points (since the width is parallel to the Y axis).

Thus, the width of the rectangle

$= 3 - (-2) = 5$

Thus, the perimeter of the rectangle

$= 2 \times (\text{Length} + \text{Width})$

$= 2 \times (3 + 5)$

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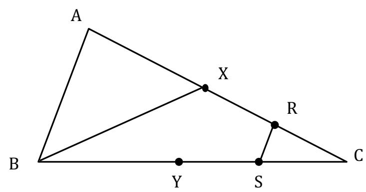
$= 16$

The correct answer is '16.'

268. The figure depicting the information in the problem is shown below:



We know that:

AX = CX, RX = RC, BY = CY, and YS = SC

Thus, we have:

$$\frac{CR}{CA} = \frac{CR}{2 \times CX} = \frac{CR}{2 \times 2 \times CR} = \frac{1}{4}$$

Also, we have:

$$\frac{CS}{CB} = \frac{CS}{2 \times CY} = \frac{CS}{2 \times 2 \times CS} = \frac{1}{4}$$

Thus, triangle CRS is similar to triangle CAB since:

$$\frac{CR}{CA} = \frac{CS}{CB}, \text{ and } \angle RCS = \angle ACB \text{ (included angle)}$$

Thus, ratio of the corresponding sides of the above two similar triangles

$$= \frac{CR}{CA} = \frac{CS}{CB} = \frac{1}{4}$$

Thus, ratio of the areas of the above two similar triangles

$$= \frac{\text{Area of triangle CRS}}{\text{Area of triangle CAB}} = (\text{Ratio of their corresponding sides})^2$$

$$= \left(\frac{1}{4}\right)^2 = \frac{1}{16} \ldots \text{(i)}$$

We know that the area of triangle ABX = 32.

We know that a line drawn from the vertex which divides the base in the ratio 1 : 1, also divides the area in the same ratio i.e. 1 : 1.

Since AX = CX, we have:

Area of triangle BCX = Area of triangle ABX = 32.

Thus, area of triangle CAB = 32 + 32 = 64.

Thus, from (i):

$$\frac{\text{Area of triangle CRS}}{64} = \frac{1}{16}$$

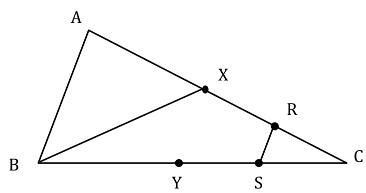
=> Area of triangle CRS = $\frac{64}{16}$ = 4

The correct answer is '4.'

Alternate approach:

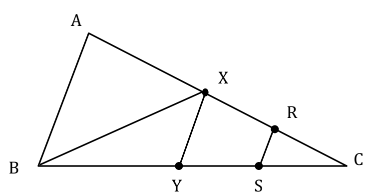
A line segment drawn from a vertex to the midpoint of the opposite side divides the triangle into two equal triangles.

Here, in the triangle ABC, a line segment BX drawn from vertex B to the midpoint of AC at X divides triangle ABC into two equal triangles: triangle ABX and triangle CBX.



Thus, Area of triangle CXB = Area ABX = 32.

Again, in the triangle CBX, a line segment XY drawn from vertex X to the midpoint of CB at Y divides triangle CBX into two equal triangles: triangle CXY and triangle BXY.



Thus, Area of triangle BXY = Area CXY = $\frac{32}{2}$ = 16.

We can visualize that in triangle CXY, line segment RY divides triangle CXY in two equal triangles: triangle RXY and triangle CRY of $\frac{16}{2}$ = 8 units of area.

Again, we can visualize that in triangle CRY, line segment RS divides triangle CRY in two equal triangles: triangle RSY and triangle CRS of $\frac{8}{2}$ = 4 units of area.

269. We know that:

$\angle P = 30^\circ + 2 \times \angle Q$

Also, since PQ = QR, we have:

$\angle R = \angle P = 30^\circ + 2 \times \angle Q$

Also, in triangle PQR:

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$\angle P + \angle Q + \angle R = 180^\circ$

$=> (30^\circ + 2 \times \angle Q) + \angle Q + (30^\circ + 2 \times \angle Q) = 180$

$=> 5 \times \angle Q = 120^\circ$

$=> \angle Q = 24^\circ$

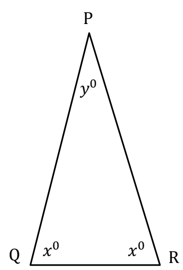
$=> \angle R = 30^\circ + 2 \times \angle Q$

$= 30^\circ + 2 \times 24^\circ$

$= 78^\circ$

The correct answer is '78.'

270. Let us bring out the figure.



We know that the sum of the angles in a triangle is 180°

$=> 2x + y = 180^\circ \ldots$(i)

We know that:

$x + y = 100^\circ$

$=> x = 100 - y$

Thus, from (i), we have:

$=> 2\,(100 - y) + y = 180^\circ$

$=> y = 20^\circ$

The correct answer is '20.'

271. We have:

$2^{\sqrt{x}} = 8$

$=> 2^{\sqrt{x}} = 2^3$

$=> \sqrt{x} = 3$

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$=> x = 3^2 = 9$

$=> 2^x = 2^9 = 512$

Note: It would be WRONG to imagine the answer as: 2^x as the square of $2^{\sqrt{x}}$, i.e. $8^2 = 64$.

The correct answer is '512.'

272. We have:

$|x + 1| = 2\,|x - 1|$

$=> x + 1 = \pm 2\,(x - 1)$

$=> x + 1 = \pm\,(2x - 2)$

Thus, we have:

$x+1= (2x-2)$	$x+1= -(2x-2)$
$=> x = 3$	$=> x = \frac{1}{3}$
$=> \left\lvert x - \frac{5}{3} \right\rvert = \left\lvert 3 - \frac{5}{3} \right\rvert = \frac{4}{3}$	$=> \left\lvert x - \frac{5}{3} \right\rvert = \left\lvert \frac{1}{3} - \frac{5}{3} \right\rvert = \left\lvert -\frac{4}{3} \right\rvert = \frac{4}{3}$

The correct answer is '$\frac{4}{3}$.'

Since this is a question on Student-produced response, even if you come up with a single correct option, your answer would be correct. Had you only considered '+' sign for the absolute equation, you would have got one of the correct answer.

$|x + 1| = 2\,|x - 1|$

$=> x + 1 = 2(x - 2)$

$=> x = 3$

$=> \left\lvert x - \frac{5}{3} \right\rvert = \left\lvert 3 - \frac{5}{3} \right\rvert = \frac{4}{3}$

273. Let the smaller volume of the solution be x liters.

Thus, the larger volume of the solution = $2x$ liters.

We need to find the minimum concentration of milk in any of the two containers so that when mixed they result in 80% milk solution.

Since one container has the minimum milk concentration, the other must have the maximum possible milk concentration, i.e. 100% (this is the limiting case).

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Also, in order to find the minimum concentration in one container, we must have 100% concentration of milk in the container having the larger volume, i.e. $2x$ liters, so that a large quantity of milk is obtained.

Since the entire contents of both containers are mixed to get 30 liters of solution, we have

$x + 2x = 30$

$=> x = 10$

Let the concentration of milk in $x = 10$ liters solution be c%, the minimum concentration of milk in any container.

Thus, we finally have: $2x = 20$ liters of 100% milk and $x = 10$ liters of c% milk.

Thus, on mixing, the concentration of milk

$= \left(\frac{20 \times 100 + 10 \times c}{20 + 10}\right)\%$

Since the mixture has 80% milk, we have

$\frac{20 \times 100 + 10 \times c}{20 + 10} = 80$

$=> 10c = 400$

$=> c = 40$

The correct answer is '40.'

Alternate approach:

We know that total volume of both the containers is 30 liters; thus, the smaller container would have a volume of 10 liters, and the larger container would have a volume of 20 liters.

We also know that 80% of 30 liters = 24 liters is milk, thus total volume of water = 30 – 24 = 6 liters. The situation can be expressed in the following table.

	Smaller Container	**Larger Container**	**Total**
Milk			24
Water			6
Total	10	20	**30**

To assure that the smaller container has the least possible amount of milk, let us assign full amount of Water, i.e., 6 liters; This will leave the possibility of only 10 – 6 = 4 liters of milk, equal to $\frac{4}{10} \times 100\% = 40\%$.

	Smaller Container	Larger Container	Total
Milk	10 – 6 = 4		24
Water	6 (Assigned)		6
Total	10	20	**30**

274. We know that:

From 100 liters of milk in a cask, 20 liters are removed and then 10 liters of water are added. This process is repeated twice.

Thus, in the first cycle, fraction of the total contents of the cask removed = $\frac{20}{100} = \frac{1}{5}$

Thus, fraction of contents left = $1 - \frac{1}{5} = \frac{4}{5}$

Thus, amount of milk left after the first cycle = $100 \times \frac{4}{5}$ = 80 liters.

Now, 10 liters of water are added.

Thus, total contents of the cask = (80 + 10) = 90 liters.

Thus, in the second cycle, fraction of the total contents of the cask removed = $\frac{20}{90} = \frac{2}{9}$

Thus, fraction of contents left = $1 - \frac{2}{9} = \frac{7}{9}$

Thus, amount of milk left after the second cycle = $80 \times \frac{7}{9} = \frac{560}{9}$ liters.

Total volume of cask after removal = (90 − 20) = 70 liters.

Now, 10 liters of water are added.

Thus, total contents of the cask = (70 + 10) = 80 liters.

Thus, required fraction of milk = $\left(\frac{\frac{560}{9}}{80}\right) = \frac{7}{9}$

The correct answer is '$\left(\frac{7}{9}\right)$.'

The process can be understood through a tree-structure diagram.

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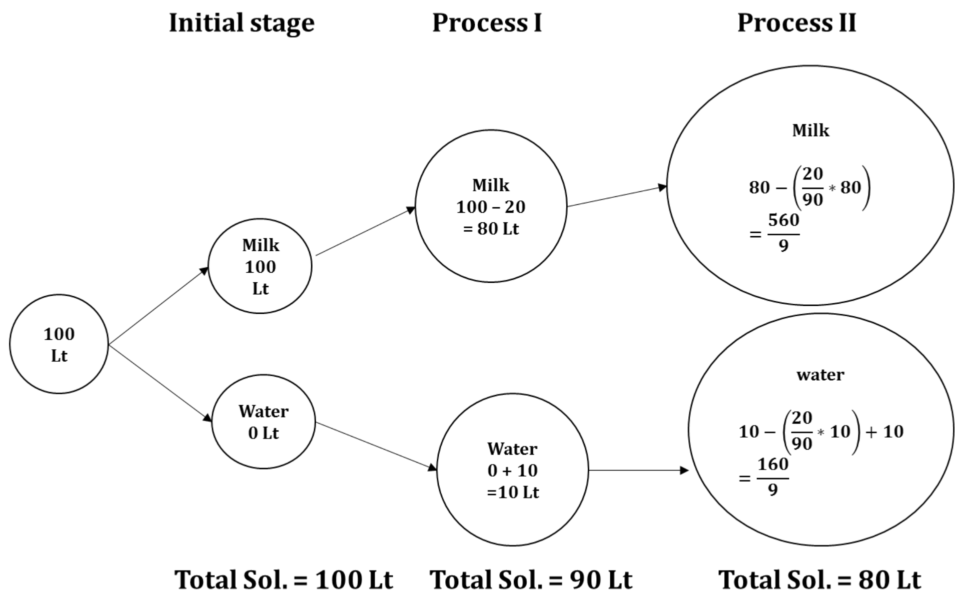


Thus, required fraction of milk = $\left(\frac{\frac{560}{9}}{80}\right) = \frac{7}{9}$.

275. Let the quantity of P and Q be x liters each (since their ratio is 1 : 1).

Let the quantity of R be y liters.

Thus, we have:

$$\frac{80x + 70x + 50y}{x + x + y} = 60$$

=> $150x + 50y = 120x + 60y$

=> $3x = y$

=> $x : y = 1 : 3$

Thus, the ratio of quantity of Q and R = 1 : 3 = $\frac{1}{3}$.

The correct answer is '$\left(\frac{1}{3}\right)$.'

276. Let us understand the concept of the modulus (or the absolute value):

$|x - a|$ refers to the distance of the point x from the point a on either side of a.

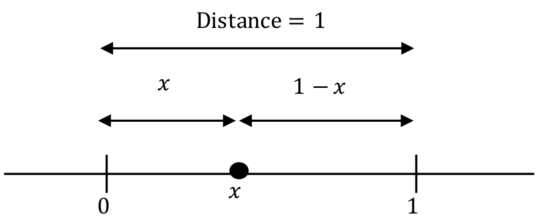
Thus: $|x - a| = b$ implies that the distance of the point x from the point a on either side of a is b units.

Thus, $|x| = |x - 0|$ represents the distance of x from the point 0.

Also, $|x - 1|$ represents the distance of x from the point 1.

We know that $0 \le x \le 1$.

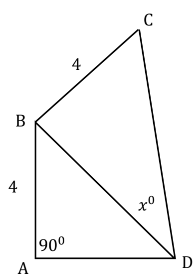
Thus, it can be clearly observed that $(|x| + |x-1|)$ will be 1, as shown in the diagram below:



The correct answer is '1.'

Alternatively, we can pick any value of x and plug-in $(|x| + |x-1|)$. Say $x = \frac{1}{3}$, thus, $\left(\left|\frac{1}{3}\right| + \left|\frac{1}{3} - 1\right|\right) = \frac{1}{3} + \frac{2}{3} = 1$. We may take another value; say $x = \frac{1}{4}$, thus, $\left(\left|\frac{1}{4}\right| + \left|\frac{1}{4} - 1\right|\right) = \frac{1}{4} + \frac{3}{4} = 1$.

277. Let us bring out the figure.



In right angled triangle ABD:

$BD^2 = AB^2 + AD^2$

=> $BD^2 = 16 + 32 = 48$

=> $BD = 4\sqrt{3}$

We have, in triangle BCD:

BC = 4

$BD = 4\sqrt{3}$

CD = 8

We observe that:

$CD^2 = 64$, and

$BD^2 + BC^2 = 48 + 16 = 64$

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=> $CD^2 = BD^2 + BC^2$ (Pythagoras' theorem is satisfied)

=> $\angle CBD = 90^\circ$

Thus, in the right angled triangle BCD, we have the ratio of the sides:

$BC : BD : CD = 1 : \sqrt{3} : 2$

Thus, it is a 30-60-90 triangle.

In a 30-60-90 triangle, the length of the side opposite the 30° is half the length of the hypotenuse (side opposite the 90°).

Here, we have:

CD (hypotenuse) = 8, and

$BC = 4 = \frac{CD}{2}$

=> x° (angle opposite BC) = 30°

=> $x = 30$

The correct answer is '30.'

278. Let the ages of P, Q and R be p, q and r respectively.

We have:

$\frac{p + q + r}{3} = 24$

=> $p + q + r = 72$ ...(i)

Since R is 6 years elder to Q, we have:

$r = q + 6$ ...(ii)

Since the difference between the ages of P and Q is 6 years, and P is the youngest, we have:

$q - p = 6$

=> $p = q - 6$

=> $p + q + r = (q - 6) + q + (q + 6)$ ...Using (ii)

$= 3q$

=> $3q = 72$ ...Using (i)

=> $q = 24$

=> $p = 24 - 6 = 18$

The correct answer is '18.'

Alternate approach:

We know that the difference between the ages of P and Q age is 6 years with P the youngest, and R is 6 years elder to Q. This means that Q's age is at the mid of P's age and R's age, or Q's age is the average (arithmetic mean) of P, Q, and R's ages.

Again, we know that the average age is 24, equal to Q's age, thus P's age = 24 – 6 = 18.

279. Let the amounts with A, B and C be $\$a$, $\$b$ and $\$c$ respectively.

Thus, we have:

$a + b + c = 60 \ldots \text{(i)}$

We also know that:

$a + b = (100 + 40)\,\%$ of c

$=> a + b = \dfrac{140}{100} \times c$

$=> a + b = \dfrac{7c}{5}$

$=> a + b + c = \dfrac{7c}{5} + c = \dfrac{12c}{5}$

Since $a + b + c = 60$, we have:

$=> \dfrac{12c}{5} = 60$

$=> c = 25$

$=> a + b = 60 - c \ldots \text{Using (i)}$

$=> a + b = 35 \ldots \text{(ii)}$

We also know that:

$b = 6 + (a + c)$

$=> b = 6 + (a + 25)$

$=> b = a + 31$

Substituting the value of b is (ii), we have:

$a + (a + 31) = 35$

$=> a = 2$

The correct answer is '2.'

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280. We have:

Given that $z = w - x + 3 \ldots$(i)

Given that $y + z = x - 4$

$=> z = x - y - 4 \ldots$(ii)

Given that $y - w = z - 7$

$=> z = y - w + 7 \ldots$(iii)

Adding (i), (ii) and (iii), we have:

$3z = 6$ (since x, y and w cancel out)

$=> z = 2$

The correct answer is '2.'

281. Oil production by other countries in 1975 = 19 billion barrels

Oil production by Saudi Arabia in 1975 = 8 billion barrels

Percent of oil from Saudi Arabia imported by India = 15%

Thus, oil from Saudi Arabia imported by India

= 15% of 8 billion barrels

= 1.2 billion barrels

Thus, the required percent

$= \frac{1.2}{19} \times 100$

$= \approx 6.32\%$

The correct answer is '6.32.'

282. Total oil production by Saudi Arabia in the five years combined

= 8 + 10 + 8 + 10 + 12

= 48 billion barrels

Oil production by Saudi Arabia in 1974 = 10 billion barrels

Thus, the required angle

$= \frac{10}{48} \times 360^\circ$

= 75°

The correct answer is '75.'

283. Oil production by Saudi Arabia in all years combined

= 8 + 10 + 8 + 10 + 12

= 48 billion barrels

Oil production of other countries in all years combined

= 24 + 20 + 19 + 20 + 21

= 104 billion barrels

Thus, oil production of all countries (including Saudi Arabia) in all years combined

= 104 + 48 = 152 billion barrels

Thus, the required percent

$= \frac{48}{152} \times 100$

= 31.58%

The correct answer is '31.58.'

284. Oil production by Saudi Arabia in 1975 = 8 billion barrels

Percent of oil from Saudi Arabia imported by India = 15%

Thus, oil from Saudi Arabia imported by India

= 15% of 8 billion barrels

= 1.2 billion barrels

Oil production by Saudi Arabia in 1977 = 12 billion barrels

Percent of oil from Saudi Arabia imported by India = 8%

Thus, oil from Saudi Arabia imported by India

= 8% of 12 billion barrels

= 0.96 billion barrels

Thus, the required percent decrease

$= \frac{1.2 - 0.96}{1.2} \times 100\%$

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= 20%

The correct answer is '20.'

285. Total value of tyre sales = $230 mn

Since 20% of the tyre sales were in the East, value of tyre sales in the East

= 20% of $230 mn

= $ 46 mn

Total value of sales in the East = $130 mn

Thus, the value of the sales of other products in the East

= $(130 – 46) mn

= $84 mn

The correct answer is '84.'

286. Total value of sales of all products

= $(200 + 140 + 230 + 270 + 700) mn

= $1540 mn

If the sales of sports goods double, the new value of sales from sports goods next year

= $\$(2 \times 200)$ mn

= $400 mn

Since the sales of other products remain unchanged, the total sales would increase by an amount equal to the increase in the value of sales from sports goods, i.e. $200 mn.

Thus, total value of sales of all products next year

= $(1540 + 200) mn

= $1740 mn

Thus, the required percent

$= \frac{400}{1740} \times 100$

= 22.99%

$= \approx 23\%$

The correct answer is '23.'

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287. Value of sales from FMCG goods = \$700 mn

Total value of sales of all products

= \$(200 + 140 + 230 + 270 + 700) mn

= \$1540 mn

Thus, the required percent

$= \frac{700}{1540} \times 100\%$

= 45.45%

The correct answer is '45.45.'

288. Total revenue generated from printer sales by Company X in 2013

$= \$(23,000 \times 63)$

= \$1,449,000

Since this revenue forms 25% i.e. $\frac{1}{4}$ of the revenue of all four companies, we have:

The total revenue from printers sold by all four companies in 2013

$= \$(1,449,000 \times 4)$

= \$5,796,000

= 5,796 thousand dollars

The correct answer is '5796.'

289. Total revenue generated from printer sales by Company X in 2010

$= \$(14000 \times 50)$

= \$700,000

Total revenue generated from printer sales by Company X in 2011

$= \$(16000 \times 55)$

= \$880,000

Thus, the required percent increase

$= \frac{880,000 - 700,000}{700,000} \times 100\%$

= 25.71%

The correct answer is '25.71.'

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290. We have already calculated the total revenue from printers sold by all four companies in 2013 to be \$5,796,000.

However, we need not calculate the revenue generated by Company W using that data.

Since the revenue share of Company W in 2013 is 25%, the same as that of X in 2013, we have:

Total revenue generated from printer sales by Company W in 2013

= Total revenue generated from printer sales by Company X in 2013

= \$1,449,000

Thus, the increase in revenue generated from printer sales by company W in 2014, following a 20% increase

= 20% of \$1,449,000

= \$289,800

= 289.8 thousand dollars

= ≈ 290 thousand dollars

The correct answer is '290.'

291. Let the weight assigned to each quiz be k.

Thus, the weight assigned to each test = $3k$.

Since Dan scored 81 and 76 on two quizzes and scored 96 on the only test, we have:

Dan's overall grade average = $\frac{(81 \times k + 76 \times k + 96 \times 3k)}{(k + k + 3k)}$

= 89

The correct answer is '89.'

292. Joe's total scores in the first four tests

= 80 + 82 + 79 + 84

= 325

Since Joe should have an average score of 85 in 6 tests, his total score should be

= 85×6

= 510

Thus, his total score in the next two tests

= 510 – 325

= 185

Thus, his average score for the next two tests

$= \frac{185}{2}$

= 92.5

The correct answer is '92.5.'

293. We know that the range is 2.

Since we need four integers at a gap of 2, we must have repetition of the integers.

Let the smallest integer be a.

Thus, the largest integer = $(a + 2)$.

Thus, we can use only a, $(a + 1)$ and $(a + 2)$ as the integers.

Since the average is 3 and all numbers are not the same (else, the range would have been '0'), we must have a number greater than 3 (i.e. 4 or higher) and a number less than 3 (i.e. 2 or lower).

Thus, we may have the possible scenarios:

- The numbers used are: 2, 3, 3 and 4

  Mean = $\frac{2 + 3 + 3 + 4}{4} = 3$

  Range = 4 – 2 = 2

  Thus, all constraints are satisfied.

  Thus, the product of the integers = $2 \times 3 \times 3 \times 4 = 72$

- The numbers used are: 2, 3, 4 and 4

  Here, mean is greater than 3 – Does not satisfy the constraint

- The numbers used are: 2, 2, 3 and 4

  Here, mean is less than 3 – Does not satisfy the constraint

- The numbers used are: 2, 2, 4 and 4

  Mean = $\frac{2 + 2 + 4 + 4}{4} = 3$

  Range = 4 – 2 = 2

  Thus, all constraints are satisfied.

  Thus, the product of the integers = $2 \times 2 \times 4 \times 4 = 64$

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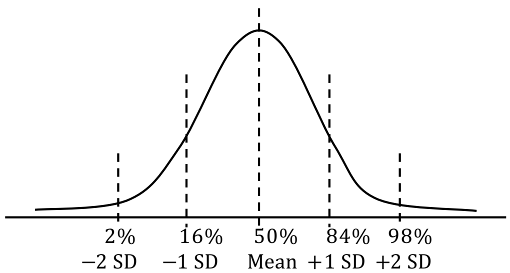
Thus, the maximum value of the product of the numbers is 72.

The correct answer is '72.'

294. A normally distributed data is shown below:

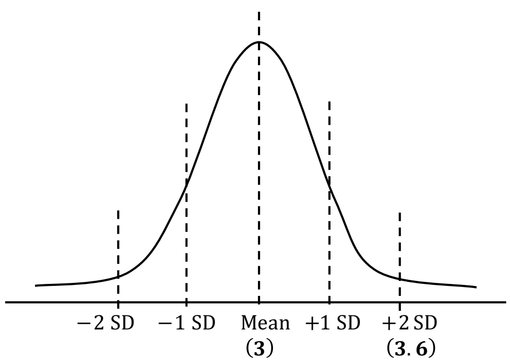


The value of the mean length is 3 cm.

Also, the standard deviation is 0.3 cm.

Thus, 3.6 cm is 3.6 – 3 = 0.6 cm greater than the mean, i.e. $\left(\frac{0.6}{0.3}\right)$ = 2 standard deviations to the right of the mean.

Thus, we have the following diagram:



Thus, percent of the population lying between 3 cm and 3.6 cm, i.e. the 'Mean' and '+2 SD'

= 98% – 50%

= 48%

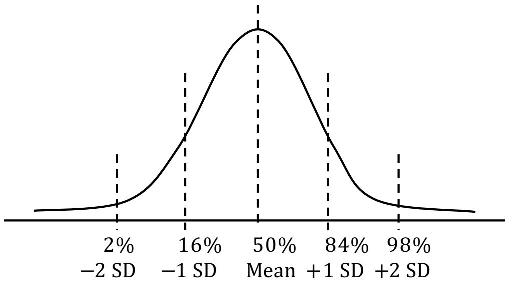
Thus, the required probability

$= \frac{48}{100}$

= 0.48

The correct answer is '0.48.'

295. A normally distributed data is shown below:

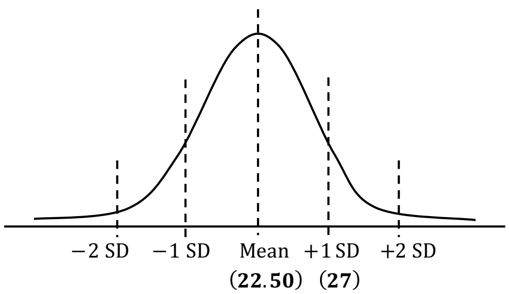


The value of the mean hourly wage is $22.50.

Also, the standard deviation is $4.50

Thus, $27 is 27 – 22.50 = $4.50 greater than the mean, i.e. 1 standard deviation to the right of the mean.

Thus, we have the following diagram:



Thus, percent of the population having hourly wages greater than $27, i.e. the to the right of '+1 SD'

= 100% – 84%

= 16%

The correct answer is '16.'

296. The inter-quartile range (IQR) of a set of data is the distance between Q1 (quartile 1) and Q3 (quartile 3).

Q1 is essentially the median of the first half of the set and Q3 is the median of the second half of the set.

The first 12 positive multiples of 6 are:

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66 and 72

Since there are 12 terms, we have:

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Q1 is the median of the first $\frac{12}{2} = 6$ terms

= Average of the $\left(\frac{6}{2}\right)^{\text{th}}$ term and the $\left(\frac{6}{2}+1\right)^{\text{th}}$ term

= Average of the 3^{rd} term and the 4^{th} term

$= \dfrac{18+24}{2}$

= 21

Q3 is the median of the last $\frac{12}{2} = 6$ terms

= Average of the $\left(6+\frac{6}{2}\right)^{\text{th}}$ term and the $\left(6+\left(\frac{6}{2}+1\right)\right)^{\text{th}}$ term

= Average of the 9^{th} term and the 10^{th} term

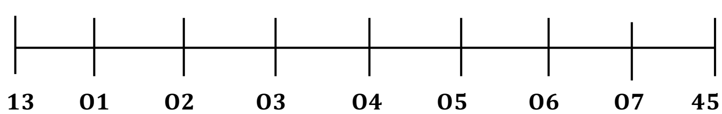
$= \dfrac{54+60}{2} = 57$

Thus, IQR

= 57 – 21 = 36

The correct answer is ‘36.’

297. Since ‘Octiles’ divides a set of data in 8 ordered groups and there are 7 ‘Octiles’, we have the following diagram, where the Octiles are shown as O1, O2, . . . O7:



From the diagram, it is clear that:

- O2 (Octile 2) is the same as Q1 (Quartile 1)
- O4 (Octile 4) is the same as the Median
- O6 (Octile 6) is the same as Q3 (Quartile 3)

Thus, we have to calculate the median (= Octile 4) of the set.

The median of the above set of 33 numbers is the value of $\frac{(33+1)}{2} = 17^{\text{th}}$ number

= 13 + (17 − 1) = 29

The correct answer is ‘29.’

298. The average score obtained by 15 students was 5.

Thus, the total score obtained by the 15 students = $5 \times 15 = 75$.

The possible scores are 0, 1, 2 ... 6 and 7, i.e. 8 possible scores.

Since each score was obtained at least once, the sum of the above 8 scores

$= 1 + 2 + 3 + \ldots + 7$

$= \frac{7 \times 8}{2} = 28$

Thus, the sum of the scores of the remaining 7 students (= 15 – 8)

$= 75 - 28$

$= 47$

In order to find the minimum possible score of the lowest scoring seven students, we assume that the other scores are the maximum possible.

Thus, we have 7 – 1 = 6 students scoring 7 each, giving us a total of $6 \times 7 = 42$.

Thus, the remaining score = 47 – 42 = 5.

Thus, one student scored 5.

However, one student has already scored 5 (since we took each of the 8 possible scores are present at least once).

Thus, the minimum possible score obtained by two students = 5 each

Thus, the scores 0 to 4 are present once each and scores thereafter are present twice each.

Thus, the minimum possible sum of scores of the lowest scoring seven students out of 15 students

$= 0 + 1 + 2 + 3 + 4 + 5 + 5$

$= 20$

The correct answer is '20.'

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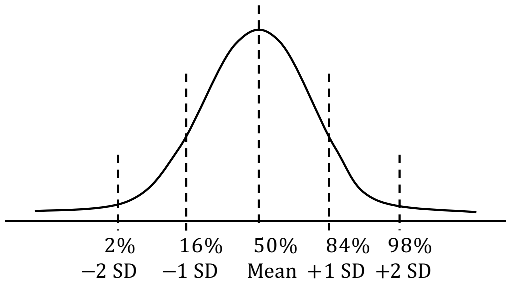
299. A normally distributed data is shown below:



Let the mean value be m grams and the standard deviation be s grams.

Since one standard deviation below the mean is 250 grams, we have

$m - s = 250 \ldots$(i)

Again, since two standard deviations above the mean is 265 grams, we have:

$m + 2s = 265 \ldots$(ii)

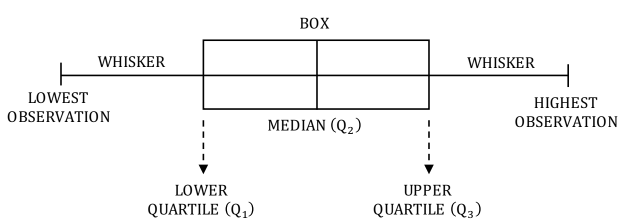
Subtracting (i) from (ii), we have:

$3s = 15$

$=> s = 5$

The correct answer is '5.'

300. A box-and-whisker plot is a convenient means of graphically representing data using quartiles, as shown below:



Thus, we have, from the given plot:

- Q1 (25^{th} percentile value) = 13
- Q2 (50^{th} percentile value) = 14
- Q3 (75^{th} percentile value) = 20

We know that a score of 25 represents the 85^{th} percentile value on the plot above.

Thus, a score between 20 and 25 is represented by values between 75^{th} percentile and 85^{th} percentile, i.e. 85 – 75 = 10% of the total number of students

= 10% of 60

= 6

The correct answer is '6.'

301. Let the length and width of the rectangle be l and w, respectively.

Since the area of the rectangle is 1,200, we have:

$l \times w = 1,200$

New length of the rectangle = $2l$.

New width of the rectangle

$= w + 25\%$ of w

$= w\left(1 + \frac{1}{4}\right)$

$= \frac{5w}{4}$

Thus, the new area

$2l \times \frac{5w}{4}$

$= \frac{5}{2} \times (l \times w)$

$= \frac{5}{2} \times 1,200$

$= 3,000$

The correct answer is '3,000.'

Alternate approach:

We know that area of a rectangle is product of length and width,

Thus,

Area $\propto$ length and Area $\propto$ width

Since the width increased by 25%, or it is now 1.25 times the original, area would also be 1.25 times the original. Similarly, since the length is double, area would also be two times the original.

Thus, the new area = 1.25×2 times the original = $2.5 \times 1,200 = 3,000$.

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302. Total amount = \$2,400.

Total interest earned in 1 year = \$360.

Thus, the average rate of interest

$= \frac{360}{2,400} \times 100\%$

= 15%

Let John had lent \$$x$ at 10% and \$$y$ at 22%.

Since the average rate of interest obtained was 15%, we have:

$\frac{(x \times 10) + (y \times 22)}{(x + y)} = 15$

$=> 10x + 22y = 15x + 15y$

$=> 5x = 7y$

$=> \frac{x}{y} = \frac{7}{5}$

Thus, the larger part is \$$x$ and the smaller part is \$$y$.

Thus, the larger part was lent at 10% rate of interest.

The correct answer is '10.'

Alternate approach:

From above, we know that the average rate of interest = 15%

Had both the parts been equal, the average rate of interest would have been the average of the interests at which they were lent = $\frac{10 + 22}{2}$ = 16%

Since 15% < 16%, and 15% is relatively closer to 10% than to 22%, it implies that the part which is lent @ 10% must have more weight than the other part.

303. We have:

$ad = bc$

$=> \frac{a}{b} = \frac{c}{d}$

Let us assume that:

$\frac{a}{b} = \frac{c}{d} = k$

$=> a = bk$, and $c = dk$

$=> a + c = bk + dk = k\,(b + d)$

$=> \frac{a+c}{b+d} = k$

$=> \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = k$

(The above rule is known as 'Addendo')

Since we know that:

$\frac{a+c}{b+d} = 7$

$=> \frac{a}{b} = 7$

The correct answer is '7.'

304. Let the number of pencils purchased be x and erasers purchased be y.

Cost of pencils = $\$(0.15x)$

Cost of erasers = $\$(0.29y)$

Thus, total Cost of all items = $\$(0.15x + 0.29y)$

Thus, we have:

$0.15x + 0.29y = 4.40$

$=> 15x + 29y = 440$

We know that x and y are positive integers.

Thus, we have:

$15x = 440 - 29y$

$=> x = \frac{440 - 29y}{15}$

Since x is a positive integer, $(440 - 29y)$ must be divisible by 15.

We separate out the part from $(440 - 29y)$ which is divisible by 15. Thus, we have:

$=> x = \frac{(435 - 30y) + (5 + y)}{15}$

$=> x = (29 - 2y) + \left(\frac{5+y}{15}\right)$

Thus, the value of y should be such that $(5 + y)$ is divisible by 15

$=> y = 10,\ 25,\ 40 \ldots$ etc.

Thus, we have: If $y = 10$:

$x = (29 - 2y) + \left(\frac{5+y}{15}\right)$

$= (29 - 2 \times 10) + \left(\frac{5+10}{15}\right)$

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$= 10$

Working with the next value of $y = 25$, we have:

$x = (29 - 2y) + \left(\frac{5 + y}{15}\right) = -19,$ i.e. not possible

Thus, the only solution is:

$x = y = 10$

The correct answer is '10.'

305. Let the number of diaries and notebooks be x and y, respectively.

We know that:

$x > 10$

Thus, the cost of diaries $= 8x > 80$

Since the minimum value of $x = 11$, we have

$=> \ 8x \geq 88 \ldots$(i)

Cost of notebooks = $ $(25y)$

Thus, we have:

$25y \geq 150$

$=> y \geq 6 \ldots$(ii)

Since the total cost of all the items was less than $260, we have:

$=> 8x + 25y < 260$

However, from (i), we know that: $8x \geq 88$

$=> 25y < 260 - 80 = 172$

$=> y < \frac{172}{25}$

$y < 6.48 \ldots$(iii)

Thus, from (ii) and (iii), we have:

$6 \leq y < 6.48$

$=> y = 6$

The correct answer is '6.'

Alternate approach:

Assume that David bought notebooks worth \$150, or $\frac{150}{25}$ = 6 books. Thus, he has 260 – 150 = \$110 to accommodate diaries; we know that he bought more than 10 diaries, the minimum number of diaries would be 11, or he will have to spend 11×8 = \$88 to buy them.

The sum left now would be 110 – 88 = 22; with \$22, he cannot buy one more notebooks, he can buy two more diaries, costing $2 \times 8 = 16 < 22$.

Thus, number of notebooks = 6 and number of diaries ≤ 13.

306. Median of the numbers in the set is the middle number in the set once the numbers are arranged in ascending or descending order.

Since there are five numbers in the list, the median is the middle number = $\left(\frac{5+1}{2}\right)^{th}$number, i.e. the 3rd number.

Since the median of the numbers in the list is 10, one between p and q must be 10.

We know that the given four numbers in ascending order are 6, 10, 12 and 17

Since the median, i.e. the 3rd number is 10, the remaining number must be less than 10.

We also know that one between p and q is 10, with $p < q$.

Thus, the 5th number which is less than 10 must be p and q must be 10.

The correct answer is '10.'

307. Let the number of shirts in the inventory last month = x.

Number of shirts sold = $\frac{3x}{4}$.

Number of shirts remaining unsold = $x - \frac{3x}{4} = \frac{x}{4}$.

Thus, we have:

$\frac{x}{4} = 40$

$=> x = 160$

Thus, total revenue

$= \$\left(\frac{3x}{4} \times 20\right)$

$= \$\left(\frac{3}{4} \times 160 \times 20\right)$

$= \$2,400$

The correct answer is '2,400.'

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308. Let the number of units of item X sold be x.

The number of units of item Y sold = $(1,000 - x)$.

Revenue from the sale of units of item X = $\$(1,500x)$.

Revenue from the sale of units of item Y = $\$\left(2,000 \times (1,000 - x)\right)$.

Since the revenue from the sale of units of items X was 3 times the revenue from the sale of units of item B, we have

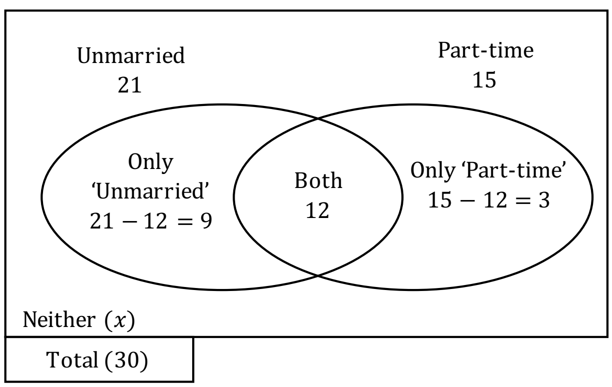
$1,500x = 3 \times (2,000 \times (1,000 - x))$

$=> x = 4,000 - 4x$

$=> x = 800$

The correct answer is '800.'

309. The given data can be represented in a Venn-diagram as shown below:



Thus, we have:

$9 + 12 + 3 + x = 30$

$=> x = 6$

The correct answer is '6.'

310. Amount spent on furnishing is 20% of the total amount that he spent on painting and interior designing.

Thus, we have

$\text{furnishing} = \frac{1}{5} \times (\text{painting} + \text{interior designing})$

$=> 5 \times \text{furnishing} = (\text{painting} + \text{interior designing})$

$=> 5 \times \text{furnishing} + \text{furnishing} = (\text{painting} + \text{interior designing}) + \text{furnishing}$

$=> 6 \times \text{furnishing} = \$12,000$

$=> \text{furnishing} = \$ \left(\frac{1}{6} \times 12,000\right)$

$=> \text{furnishing} = \$2,000$

The correct answer is '2,000.'

311. Time taken by 5 skilled workers to complete the job = 18 hours.

Thus, time taken by 1 skilled worker to complete the job = $18 \times 5 = 90$ hours ... (i)

Since an apprentice works at $\frac{2}{3}$ the rate of a skilled worker, we can say that 1 apprentice is equivalent to $\frac{2}{3}$ of a skilled worker.

Thus, 3 apprentices are equivalent to $\left(3 \times \frac{2}{3}\right) = 2$ skilled workers.

Thus, 4 skilled workers and 3 apprentices are equivalent to $(4 + 2) = 6$ skilled workers.

Thus, from (i), we have:

Time taken by 6 skilled workers to complete the job

$= \frac{90}{6}$

$= 15$ hours

The correct answer is '15.'

312. $P - R = 2,370$

$=> P = 2,370 + R$

Since $1 < R < 9$, we have:

$2,370 + 1 < P < 2,370 + 9$

$=> 2,371 < P < 2,379$

Since P is divisible by 9, we need to find a number divisible by 9 between 2,371 and 2,379.

The only possible value of P is

$P = 2,376$

$=> R = 6$

$=> \frac{P}{R} = \frac{2,376}{6}$

$= 396$

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The correct answer is '396.'

313. Total earnings of Craig in the first 5 months = $\$\ (1,500 \times 5) = \$7,500 \ldots$(i)

Total earnings of Craig in the first 25 months = $\$\ (2,700 \times 25) = \$67,500 \ldots$(ii)

Total earnings of Craig in the last 25 months = $\$\ (3,400 \times 25) = \$85,000 \ldots$(iii)

Thus, from (i) and (iii):

His total earnings in two and half years or 30 months = $\$\ (7,500 + 85,000) = \$92,500 \ldots$(iv)

Thus, his total annual earnings in the last 5 months

= (His total earnings in 30 months) – (His earnings in the first 25 months)

= $\$\ (92,500 - 67,500)$

= $\$25,000$

=> Average annual earnings in the last 5 months

$= \$\left(\frac{25,000}{5}\right)$

= $\$5,000$

The correct answer is '5000.'

314. Each of the initial n bacteria weighed 10^{-12} grams.

Thus, total weight of the initial n bacteria = $10^{-12} \times n$ grams.

Each of the n bacteria gave birth to n more bacteria weighing 10^{-12} grams each.

Thus, total number of bacteria, including new born = n^2

Thus, total weight of the bacteria born = $10^{-12} \times n^2$ grams.

We know that the first n bacteria weighed $\frac{1}{16}$ of the total weight of n^2 bacteria.

Thus, the weight of the new bacteria born was $\left(1 - \frac{1}{16}\right) = \frac{15}{16}$ of the total weight of all bacteria.

Thus, we have:

Ratio of the weight of the initial n bacteria and the new bacteria born

$$= \frac{\left(\frac{1}{16}\right)}{\left(\frac{15}{16}\right)} = \frac{1}{15}$$

$$=> \frac{10^{-12} \times n}{10^{-12} \times n^2} = \frac{1}{15}$$

$=> n = 15$

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The correct answer is '15.'

315. Total number of party A parliamentarians = 1,800.

Total number of party B parliamentarians = 3,000.

Since $\frac{3}{4}$ of the party A parliamentarians voted for the ordinance, we have:

Number of party A parliamentarians who voted for the ordinance = $\dfrac{3}{4} \times 1,800 = 1,350$

Number of female parliamentarians of party A who voted for the ordinance = $\dfrac{1}{3} \times 1,350 = 450$ ...(i)

Since $\frac{2}{3}$ of the party B parliamentarians voted for the ordinance, we have:

Number of party B parliamentarians who voted for the ordinance = $\dfrac{2}{3} \times 3,000 = 2,000$

Number of female parliamentarians of party B who voted for the ordinance = $\dfrac{1}{2} \times 2,000 = 1,000$ ...(ii)

Thus, from (i) and (ii):

Total number of females who voted for the ordinance

= 450 + 1,000

= 1,450

The correct answer is '1,450.'

Alternate approach:

Let us put down the data in a table.

	TOTAL	**FOR**		**AGAINST**
Party A parliamentarians	1,800	$\frac{3}{4} \times 1,800 = 1,350$		
		Female	**Male**	
		$\frac{1}{3} \times 1,350 =$ 450		
Party B parliamentarians	3000	$\frac{2}{3} \times 3,000 = 2,000$		
		Female	**Male**	
		$\frac{1}{2} \times 2,000 =$ 1,000		
		Total Females = 450 + 1,000 = 1,450		

316. Total number of persons on the school = 600.

Number of students = $\dfrac{2}{3} \times 600 = 400$.

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Let the number of boys and the number of girls be b and g, respectively.

Thus, we have:

$b + g = 400 \ldots$(i)

Since $g > 2b$, we have, substituting in (i):

$3b < 400$

$=> b < 133.33$

Since $b > 132$, we have:

$b = 133$

Thus, the number of girls

= 400 – 133 (Note: it is not '600 – 133')

= 267

The correct answer is '267.'

Alternatively, since number of girls & boys = 400 and number of boys > 132, the minimum value of number of boys = 133; this gives number of girls = 400 – 133 = 267. Since $267 > 2 \times 133$, the solution is valid.

317. Number of bottles examined = 6,000

Number of bottles that failed to pass examination

= 25% of 6,000

= 1,500

Number of bottles that were incorrectly tagged

$= \frac{3}{4} \times 1,500$

= 1,125

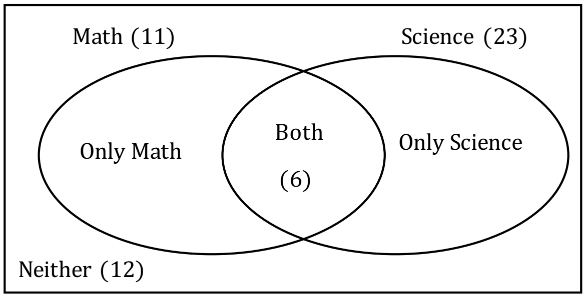
Thus, the number of bottles that had broken seal

= 1,500 – 1,125

= 375

The correct answer is '375.'

318. The above information can be represented in the form of a Venn-diagram as shown below:



Total number of students

= (# Math) + (# Science) – (# Both) + (# Neither)

= 11 + 23 – 6 + 12

= 40

The correct answer is '40.'

319. Number of people who attended the training program = 300

Thus, the number of males

= 70% of 300

= 210

Number of males who were not freelancers = 90.

Thus, the number of males who were not freelancers

= 210 – 90 = 120

Thus, the required percent

$= \frac{120}{300} \times 100\%$

= 40%

The correct answer is '40.'

320. Amount invested at r% simple interest for 1 year = \$25,000

Thus, interest accumulated

$= \$25,000 \times \frac{r}{100}$

$= \$250r$

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Let the amount invested at $2r\%$ simple interest for 1 year be $\$p$.

Thus, interest accumulated

$= \$p \times \frac{2r}{100}$

$= \$\left(\frac{pr}{50}\right)$

Since the interest from the second investment was $\frac{3}{4}$ of the interest from the first, we have:

$\frac{pr}{50} = \frac{3}{4} \times 250r$

$=> p = \frac{3}{4} \times 50 \times 250$

$= \$9,375$

The correct answer is ‘9,375.’

Alternate approach:

Since for same amount invested, the interest on the second investment is double that on the first, the interest accumulated on the second investment would have been double that accumulated on the first.

However, the interest on the second investment is only $\frac{3}{4}$ of that accumulated on the first investment.

Thus, the amount of the second investment must have been $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ of the amount of the first investment, i.e. $25,000 \times \frac{3}{8} = \$9,375$.

321. In order to form a triangle, we need to select three points from the set S such that the selected three points do not all fall on the same line.

Since no three of the points in S are collinear, the number of triangles

$= C_3^5$

$= C_{(5-3)}^5 = C_2^5$

$= \frac{5 \times 4}{2 \times 1}$

$= 10$

The correct answer is ‘10.’

322. A number when is divided by 7, can leave remainders: 0, 1, 2, 3, 4, 5, or 6.

Since the numbers are 7 consecutive integers, the remainders would be all the possible remainders from 0 to 6, i.e. 0, 1, 2, 3, 4, 5 and 6.

For example:

Numbers selected are: 14, 15, 16, 17, 18, 19 and 20.

Thus, remainders obtained on dividing by 7 are: 0, 1, 2, 3, 4, 5, and 6.

Thus, sum of the remainders = 21

The correct answer is '21.'

323. Let the number of LCD TVs and LED TVs be x and y, respectively.

Thus, we have:

$x + y = 260 \ldots \text{(i)}$

The ratio of the number of LED TVs to the number of LCD TVs at the showroom is 7 to 6.

Thus, we have:

$$\frac{y}{x} = \frac{7}{6}$$

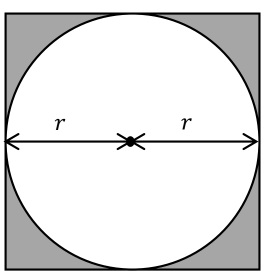
Using (i), we have:

$$y = \left(\frac{7}{7+6}\right) \times 260$$

$= 140$

The correct answer is '140.'

324.



Let the radius of the circle be r.

Thus, each side of the square = $2r$.

Area of the square = $(2r)^2 = 4r^2$

Area of the circle = πr^2

The shaded area is given as $16(4 - \pi)$

The shaded area = Area of the square – Area of the circle

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$= 4r^2 - \pi r^2 = r^2 (4 - \pi)$

Thus, we have:

$r^2 (4 - \pi) = 16(4 - \pi)$

$=> r^2 = 16$

$=> r = 4$

=> Area of the square = $4r^2$

$= 4 \times 4^2$

$= 64$

The correct answer is '64.'

325. Let Jack's monthly salary in both January and February be $\$x$.

Let his sales in January and February be $\$j$ and $\$a$, respectively.

His remuneration in January

$= \$ (x + 4\% \text{ of } (j - 2,000))$

$= \$ \left(x + \frac{4 (j - 2,000)}{100}\right)$

$= \$ \left(x + \frac{j}{25} - 80\right)$

$=> x + \frac{j}{25} - 80 = 3,620$

$=> x + \frac{j}{25} = 3,700 \ldots \text{(i)}$

His remuneration in February

$= \$ (x + 4\% \text{ of } (j - 2,000))$

$= \$ \left(x + \frac{4 (a - 2,000)}{100}\right)$

$= \$ \left(x + \frac{a}{25} - 80\right)$

$=> x + \frac{a}{25} - 80 = 3,580$

$=> x + \frac{a}{25} = 3,660 \ldots \text{(ii)}$

Subtracting (ii) from (i):

$\left(x + \frac{j}{25}\right) - \left(x + \frac{a}{25}\right) = 3,700 - 3,660$

$=> \frac{j - a}{25} = 40$

=> $j - a = 1,000$

The correct answer is ‘1000.’

Alternate approach

(1) January remuneration = 3,580 = Salary + Commission for January

(2) February remuneration = 3,620 = Salary + Commission for February

Since ‘Salary’ is the same for both the months, difference in commission = 3,620 – 3,580 = 40

Since commission is 4%, we have 40 = 4% of (Excess Sales)

Excess Sales = $\frac{40}{4\%} = \$1,000$.

326. $m = d + z$

$n = e + y$

Thus, we have:

$m + n = (d + z) + (e + y)$

$= (d + y) + (e + z)$

$= (-3) + 12$

$= 9$

The correct answer is ‘9.’

327. We know that:

$n = \frac{900}{1 + 2^{-t}c}$

For the second month, we have: $t = 2$

=> $300 = \frac{900}{1 + 2^{-2}c}$

=> $1 + 2^{-2}c = 3$

=> $2^{-2}c = 2$

=> $c = \frac{2}{2^{-2}}$

$= 8$

The correct answer is ‘8.’

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328. We know that the operation '#' represents one of addition, subtraction, or multiplication of integers.

Thus, we have:

- If '#' represents addition: $2\#0 = 2 + 0 = 2$ => LHS = RHS
- If '#' represents subtraction: $2\#0 = 2 - 0 = 2$ => LHS = RHS
- If '#' represents multiplication: $2\#0 = 2 \times 0 \neq 2$ => LHS ≠ RHS

Thus, '#' represents either 'addition' or 'subtraction.'

Thus, we have:

$1\#0 = 1 + 0 = 1$

OR

$1\#0 = 1 - 0 = 1$

Thus, in either case, we have:

$1\#0 = 1$

The correct answer is '1.'

329. Let the unit' digit, the tens digit, and the hundreds digit of the positive integer m be h, t and u, respectively.

Thus, we have:

$h \times t \times u = 96$

We know that:

$h = 8$

Thus, we have:

$u \times t = \frac{96}{8} = 12$

Since m is odd, we have:

Possible values of u = 1, 3, 5, 7 and 9 ...(ii)

Thus, the only possible value of u so that t is an integer less than 10, is:

$u = 3$

The correct answer is '3.'

330. We have:

Women : Men : Children = 5 : 2 : 7

Number of women, men and children = $5k$, $2k$ and $7k$, respectively, where k is a constant of proportionality.

Since the number of men is less than 8, we have:

$2k < 8$

$=> k < 4 \ldots$(i)

The total number of women and children

$= 5k + 7k$

$= 12k$

$= (2^2 \times 3)\, k$

Since $12k$ is a perfect square, K must be a multiple of 3, and $k < 4$, we have:

$k = 3$

Thus, the number of women

$= 5k = 5 \times 3$

= 15

The correct answer is '15.'

331. Since the problem asks for a percent value, we can assume a suitable value for the cost price.

Let the cost price be $100.

Thus, the markup

= 25% of $100

= $25

Thus, the selling price

= Cost price + Mark up

= $125

Thus, the required percent

$= \frac{25}{125} \times 100\%$

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= 20%

The correct answer is '20.'

332. We have:

$x + y = 47$

$x = y + 1$

Thus, we have:

$(y + 1) + y = 47$

$=> y = \frac{47 - 1}{2} = 23$

$=> x = 23 + 1 = 24$

The correct answer is '24.'

333. We have:

$x + y = 77 \ldots$(i)

Let the tens and units digits of x be t and u, respectively

$=> x = 10t + u \ldots$(ii)

Let the tens and units digits of y be t and v, respectively

$=> y = 10t + v \ldots$(iii)

Thus, from (i), we have:

$(10t + u) + (10t + v) = 77$

$=> 20t + (u + v) = 77$

Since t, u and v are single digit numbers, we have the possible cases:

t	$u+v$	u	v	x	y	xy
1	57	Not possible, since maximum value of $(u + v) = 9 + 9 = 18$				
2	37					
3	**17**	**9**	**8**	**39**	**38**	$38 \times 39 = 1482$
		8	**9**	**38**	**39**	$39 \times 38 = 1,482$
4	– 3	Not possible, since u and v are positive integers				

Thus, we have:

$xy = 1{,}482$

The correct answer is '1482.'

Alternate approach:

We know that the sum of the two integers is 77. The unit digit '7' can be achieved in two ways.

(1) Sum of unit digits of the integers = 7

(a) Possible pairs of unit digits are: $\{0,7\}$, $\{1,6\}$, $\{2,5\}$, and $\{3,4\}$

(b) Since the sum of unit digits of the integers = 7, sum of place values of tens digits = 77 – 7 = 70; since it is given that the integers have the same tens digits, thus, place value of each integer = $\frac{70}{2}$ = 35; however, it is not possible as place value of tens digit must end with a '0.' This is not a valid case.

(2) Sum of unit digits of the integers = 17 (It means that the tens digit will get a 'carry-over of 1.'

(a) Possible pair of unit digits is: $\{9,8\}$

(b) Again, since the sum of unit digits of the integers = 17, the sum of place values of tens digits = 77 – 17 = 60; since it is given that the integers have the same tens digits, thus, place value of each integer = $\frac{60}{2}$ = 30: possible value. Thus, the tens digit = = $\frac{30}{10}$ = 3.

The integers are: 38 & 39, and the product = 1,482.

334.

	Number of students interested in arts	Number of students not interested in arts	Total
Number of boys			36
Number of girls			$2x$
Total	52	Let the number of students not interested in arts be x	

Thus, we have the total number of students:

$52 + x = 36 + 2x$

=> $x = 16$

=> The total number of students

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$= 52 + x = 52 + 16$

$= 68$

The correct answer is '68.'

335. Let us understand the meaning of the condition: "the number of respondents who did not respond "Like" for either brand was 40."

It means that "the number of respondents who responded either "Dislike" or "Not sure" for both brands was 40."

The new table can be like this.

	Like	**Dislike + Not sure**
Brand X	40	20 + 40 = 60
Brand Y	30	35 + 35 = 70

Say, p = number of respondents who responded either "Dislike" or "Not sure" for either brand X or Y

Thus, we have:

Number of respondents who responded either "Dislike" or "Not sure" for either brand = (Number of respondents who responded either "Dislike" or "Not sure" for X) + (Number of respondents who responded either "Dislike" or "Not sure" for Y) – (Number of respondents who responded either "Dislike" or "Not sure" for both X and Y)

$=> p = 60 + 70 - 40 => p = 90$

Thus, the number of respondents who responded "Like" for both brands = (Total number of respondents) – (Number of respondents who responded either "Dislike" or "Not sure" for either brand)

= 100 – 90 = 10.

The correct answer is '10.'

336. Amount paid by each of the x persons = \$23.

Thus, the total bill of the party

$= \$23x \ldots$ (i)

Amount paid by each of the $(x + 1)$ persons = \$22.

Thus, the total bill of the party

$= \$22\,(x + 1) \ldots$ (ii)

From (i) and (ii), we have:

$23x = 22\,(x + 1)$

$=> x = 22$

Thus, the total bill of the party

$= \$23x$

$= \$\,(23 \times 22)$

= \$506

The correct answer is '506.'

337. Number of seats in each row = 30

Average number of seats unoccupied per row for the front 12 rows = 13

Thus, the average number of seats occupied per row for the front 12 rows

= 30 – 13

= 17

Thus, total number of seats occupied in the front 12 rows

$= 17 \times 12$

= 204

Average number of seats occupied per row for the back 23 rows = 10

Thus, total number of seats occupied in the back 23 rows

$= 10 \times 23$

= 230

Thus, total number of seats occupied

= 204 + 230

= 434

The correct answer is '434.'

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338. Let the number of additional pencils and erasers be a and b, respectively.

Thus, the total number of pencils and erasers are $(a + 2)$ and $(b + 5)$, respectively.

Thus, we have:

$$\frac{a+2}{b+5} = \frac{1}{2}$$

$$=> 2a = b + 1 \ldots \text{(i)}$$

Since the number of pencils added was $\frac{2}{3}$ the number of erasers added, we have:

$$a = \frac{2}{3}b$$

$$=> b = \frac{3}{2}a$$

Thus, from (i), we have:

$$2a = \frac{3}{2}a + 1$$

$$=> a = 2$$

The correct answer is '2.'

339. Let the number of male teachers and female teachers be m and f, respectively.

Since there are 105 more male teachers than female teachers, we have: $m = 105 + f \ldots \text{(i)}$

We know that, if 14 females were hired, the ratio of the number of male teachers to the number of female teachers would be $16 : 9$.

Thus, we have:

$$\frac{m}{f+14} = \frac{16}{9} \ldots \text{(ii)}$$

Thus, from (i) and (ii), we have:

$$\frac{105+f}{f+14} = \frac{16}{9}$$

$$=> 945 + 9f = 16f + 224$$

$$=> 7f = 721$$

$$=> f = 103$$

The correct answer is '103.'

340. Let the tens' and units' digits of x be t and u, respectively

$$=> x = 10t + u$$

$$x = 10t + u$$

$= 9t + (t + u)$

Thus, the remainder when x is divided by 9 is the same as the remainder obtained when $(t + u)$ is divided by 9.

Thus, the remainder when $(t + u)$is divided by 9, the remainder is 5

=> $(t + u) = 9m + 5$, where m is a non-negative integer

We know that $x = 9t + (t + u)$

Thus, $x = 9t + 9m + 5$

=> $x = 9(t + m) + 5$

=> $x = 3(3t + 3m + 1) + 2$

Thus, when x is divided by 3, the remainder is 2.

The correct answer is '2.'

Alternate approach:

Say, the two-digit positive number $x = 9 \times 1 + 5 = 14$, where '1' is quotient

Thus, $x = 14$, when divided by '3', will leave a remainder of '2.'

341. Since the average of w, x, y and z is n, we have:

$$\frac{w + x + y + z}{4} = n$$

=> $w + x + y + z = 4n$

=> $(n - w) + (n - x) + (n - y) + (n - z) = 4n - (w + x + y + z)$

$= 4n - 4n$

$= 0$

The correct answer is '0.'

Note that the sum of deviations of all the elements in a set w.r.t. its mean is 0.

342. We have:

$(|x| + 1)(x + 2) = 0 \ldots$(i)

We know that:

$|x| \geq 0$

=> $|x| + 1 \geq 1$

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Thus, in (i), since $(|x| + 1) \neq 0$, we have:

$x + 2 = 0$

$=> x = -2$

Thus, there is only one possible value of $x = -2$

Thus, the sum of all possible values of x is also '−2.'

The correct answer is '−2.'

343. We have:

$|x + 2| \leq 4$

$=> -4 \leq x + 2 \leq 4$

$=> -4 - 2 \leq x \leq 4 - 2$

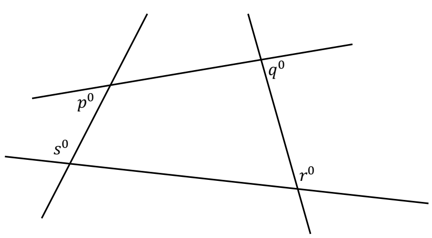
$=> -6 \leq x \leq 2$

Thus, possible values of x are (in descending order): 2, 1, 0, −1, −2, −3, −4, −5 and −6.

Since there are 9 possible values, the median will be the $\left(\frac{9+1}{2}\right)^{\text{th}}$ value, i.e. the 5^{th} value = −2

The correct answer is '−2.'

344. In the diagram, s°, p°, q°, r° represent the exterior angles of the quadrilateral.



The sum of exterior angles of any polygon is 360°.

Thus, we have:

$s + p + q + r = 360^\circ$

$=> p + q = 360 - (s + r)$

$= 360 - (105 + 145)$

$= 110^\circ$

The correct answer is '110.'

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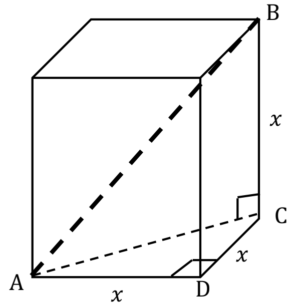
345. Let the length of each edge of the cube be x.



The longest diagonal is the line AB.

In right angled triangle ADC:

$AC^2 = AD^2 + DC^2$

$=> AC^2 = x^2 + x^2 = 2x^2$

In right angled triangle ACB:

$AB^2 = AC^2 + BC^2$

$=> AB^2 = 2x^2 + x^2 = 3x^2$

$=> AB = x\sqrt{3}$

Thus, we have:

$x\sqrt{3} = 10\sqrt{3}$

$=> x = 10$

Thus, the volume of the cube

$= x^3$

$= 1,000$

The correct answer is '1000.'

346. Let the selling price of the appliance be $\$x$

Profit made on the appliance

= 20% of $\$x$

$= \$\left(\frac{x}{5}\right)$

Since the selling price is $50 more than the cost price, we have:

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$\frac{x}{5} = 50$

$=> x = 250$

The correct answer is '250.'

347. Let the capacity of the tank be x gallons.

After 200 gallons are removed, volume of the chemical remaining was $\frac{3}{7}$ of the tank's capacity, which also was 1,600 gallons less than the tank's capacity.

Thus, we have:

$x - 1,600 = \frac{3x}{7}$

$=> \frac{4x}{7} = 1,600$

$=> x = 2,800$

Note: The value of 200 gallons is redundant. This value would have been helped us find the initial volume of Chemical X present in the tank.

The correct answer is '2800.'

348. Let the lengths of the three pieces, in cm, be a, b and c, where $a > b > c$.

(Since the lengths are distinct integers, we can arrange them in a pre-defined order)

Thus, we have:

$a + b + c = 8 \ldots \text{(i)}$

We know that:

$a = b + c$

Thus, from (i):

$a + (b + c) = 8$

$=> a + a = 8$

$=> a = 4$

$=> b + c = 4$

$=> b = 3, c = 1$ (since $b > c$, and they are integers)

Thus, the product of the lengths

$= a \times b \times c$

$= 4 \times 3 \times 1$

$= 12$

The correct answer is '12.'

349. We have:

$ab = 2 => b = \frac{2}{a}$

$ac = 2 => c = \frac{2}{a}$

Since $a + b + c = 5$, we have:

$a + \frac{2}{a} + \frac{2}{a} = 5$

$=> a + \frac{4}{a} = 5$

$=> a^2 - 5a + 4 = 0$

$=> (a - 1)(a - 4) = 0$

$=> a = 1$ or 4

Thus, we have:

$a = 1,\ b = \frac{2}{1} = 2\ \&\ c = \frac{2}{1} = 2$

OR

$a = 4,\ b = \frac{2}{4} = \frac{1}{2}\ \&\ c = \frac{2}{4} = \frac{1}{2}$

Thus, the largest value of any of the three numbers is $a = 4$.

The correct answer is '4.'

350. For the first 9 pages (1 to 9), a total of 9 digits are used.

Thus, remaining digits = 97 – 9 = 88

From 10 onwards, each number has 2 digits.

Thus, the number of two-digit numbers used = $\frac{88}{2} = 44$

Thus, the number of pages = 9 + 44 = 53

The correct answer is '53.'

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351. We have:

$x = 2$ is a root of $x^2 + (k^2 - 2)\, x = 0$

Thus, $x = 2$ should satisfy the quadratic equation, i.e. for $x = 2$, the LHS should be equal to the RHS of the quadratic equation.

Thus, we have:

$2^2 + (k^2 - 2) \times 2 = 0$

$=> 2\,(k^2 - 2) = -4$

$=> k^2 - 2 = -2$

$=> k^2 = 0$

$=> k = 0$

The correct answer is '0.'

352. The boy was supposed to find the average of the consecutive integers 1, 2, 3, $\ldots n$.

The average of the first n positive integers 1, 2, 3, $\ldots n$

$= \frac{\text{(First term)} + \text{(Last term)}}{2}$

$= \frac{1 + n}{2}$

Thus the sum of the n positive integers

$= n \times \left(\frac{1 + n}{2}\right)$

$= \frac{n\,(1 + n)}{2}$

Since the number the student missed was the largest, it must be n.

Thus, the sum of the numbers $= \frac{n\,(1 + n)}{2} - n$

Thus, the average of the $(n - 1)$ numbers $= \frac{\left(\frac{n(1+n)}{2} - n\right)}{n - 1}$

Since the average he obtained was 3.2, we have

$\frac{\left(\frac{n(1+n)}{2} - n\right)}{n - 1} = 3$

$=> n + n^2 - 2n = 6n - 6$

$=> n^2 - 7n + 6 = 0$

$=> (n - 6)\,(n - 1) = 0$

$=> n = 1$ or 6

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It is clear from the problem that n must be greater than 1

=> $n = 6$

The correct answer is ‘6.’

Alternate approach:

Since the largest number was missed and the average came to 3, it must be that the average of the original set of numbers must be greater than 1.

We know that the average of the first 5 numbers (starting with 1), i.e. 1, 2, 3, 4 and 5 is ‘3.’

Thus, to increase the average to a value above 3, we need to take at least 6 integers.

Trying with 6 integers, i.e. 1, 2, 3, 4, 5 and 6, we see that if the largest number ‘6’ is left out, the average of the remaining numbers is ‘3.’

Thus, the required number is ‘6.’

353. Let the number of rows = number of columns in the initial square formation = m

Thus, total students = m^2

When the students are changed to a rectangle formation, the number of columns = $(m + 3)$

Thus, the number of rows = $\left(\frac{m^2}{m+3}\right)$

Since the number of rows must be an integer, we have: m^2 is divisible by $(m + 3)$

=> $m^2 = (m^2 - 9) + 9$ is divisible by $(m + 3)$

=> $(m + 3)(m - 3) + 9$ is divisible by $(m + 3)$

Since $(m + 3)(m - 3)$ is divisible by $(m + 3)$, we have:

9 is divisible by $(m + 3)$

Factors of 9 are: 1, 3 and 9.

Thus, we have:

- $m + 3 = 1 \Rightarrow m = -2$: Not possible
- $m + 3 = 3 \Rightarrow m = 0$: Not possible
- $m + 3 = 9 \Rightarrow m = 6$: Possible

Thus, the number of students = $m^2 = 6^2 = 36$

The correct answer is ‘36.’

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354. Let the price of 1 apple, 1 orange and 1 lemon be a, r and l respectively.

We know that:

$5a + 4r + 3l = 13 \ldots$(i)

$3a + 4r + 5l = 11 \ldots$(ii)

After combining both statements, we cannot determine the individual values of a, r and l.

However, we need to check, if by combining both equations, we can find the value of $(a + r + l)$.

We can see that if we add (i) and (ii):

$8a + 8r + 8l = 24$

$=> a + r + l = \frac{24}{8} = 3$

The correct answer is '3.'

355. Let the age of B, x years ago be b years.

Thus, the present age of B = $(x + b)$ years.

Since A, at present, is 2 times as old as B was x years ago, we have:

A's present age = $2b$ years.

Thus, A's age x years ago = $(2b - x)$ years.

Since B, at present, has the same age as A had x years ago, we have:

$x + b = 2b - x$

$=> b = 2x$

Thus, the present age of B = $(x + b)$ years = $3x$ years.

Also, the present age of A = $2b$ years = $4x$ years.

Since the sum of the present ages of A and B is 70 years, we have:

$3x + 4x = 70$

$=> x = 10$

Thus, A's present age = $4x$ years = 40 years.

The correct answer is '40.'

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356. The quadratic equation $x^2+qx+p = 0$ has the same roots as the quadratic equation $x^2+px+q = 0$.

Thus, both should be the same quadratic equation.

Hence, comparing the coefficient of x (or, comparing the constant terms), we have:

$p = q$

Thus, we have:

$p^2 + p^2 = 9$

$=> p^2 = \frac{9}{2}$

The correct answer is '$\frac{9}{2}$.'

357. $a^3 + b^3 + c^3 = 3$, where a, b & c are non-negative integers.

If any among a, b or c is zero, then the others would be irrational in order to have $a^3+b^3+c^3 = 3$. For example:

- $a = b = 0 => c^3 = 3 => c = \sqrt[3]{3}$, which is not an integer
- $a = 0$: There are no two integers whose cubes add up to 3
  - If $a = 0$ & $b = 1$: $c^3 = 3 - 1 = 2 => c = \sqrt[3]{2}$, which is not an integer
  - If $a = 0$ & $b = 2$: $c^3 = 3 - 8 = -5 => c$ is not a positive integer

Also, if any of a, b & c is greater than 1, i.e. has a value of 2 or greater, the others have to be negative in order to have $a^3 + b^3 + c^3 = 3$. For example:

- $a = 2 => 2^3 + b^3 + c^3 = 3 => b^3 + c^3 = -5$

Thus, the only set of values of a, b & c which satisfies the above equation is:

$a = b = c = 1$

Thus, we have:

$(a + b)(b + c)(c + a)$

$= 2 \times 2 \times 2$

$= 8$

The correct answer is '8.'

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Alternate approach:

Since the question does not put any restriction on the values of a, b, & c, we can assume that $a = b = c$.

Thus,

$a^3 + b^3 + c^3 = 3$

$=> 3a^3 = 3$

$=> a = 1$

$=> a = b = c = 1$

$=> (a + b)(b + c)(c + a) = (1 + 1) \times (1 + 1) \times (1 + 1)$

$=> 2 \times 2 \times 2 = 8$

358. We have:

$xy = r$

$xz = r^2$

$yz = r^3$

Multiplying the above three equations, we have:

$x^2y^2z^2 = r^6$

$=> xyz = r^3$ (since x, y, z and r are positive, we ignore the negative square root)

Thus, we have:

$x = \frac{xyz}{yz} = \frac{r^3}{r^3} = 1$

$y = \frac{xyz}{xz} = \frac{r^3}{r^2} = r$

$z = \frac{xyz}{xy} = \frac{r^3}{r} = r^2$

We know that:

$x^2 + y^2 + z^2 = 91$

$=> 1 + r^2 + r^4 = 91$

$=> r^4 + r^2 - 90 = 0$

$=> (r^2 - 9)(r^2 + 10) = 0$

$=> r^2 = -10$ OR 9

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However, r^2 cannot be negative

=> $r^2 = 9$

=> $r = 3$ OR -3

Since r is positive, we have

$r = 3$

The correct answer is '3.'

359. Since one-third of the beans, each costing $6, are of the cheaper variety, let the number of beans of the cheaper variety and that of the costlier variety be x and $2x$, respectively.

Since the price of each bean of the costlier variety is thrice that of each bean of the cheaper variety, we have:

Price of each bean of the costlier variety

= $(3 × 6)

=$18

Total price of all beans of the cheaper variety = $ ($6x$)

Total price of all beans of the costlier variety = $ ($2x \times 18$) = $ ($36x$)

Thus, total price of all beans = $ ($6x + 36x$) = $ ($42x$)

Thus, the required percent

$= \left(\frac{36x}{42x}\right) \times 100\%$

$= \frac{600}{7}\%$

= 85.71%

The correct answer is '85.71.'

360. Total number of students in the first class = 20.

Number of students with a GPA lower than that of Joe

= 60% of 20 = 12

Thus, the number of students with a GPA HIGHER than OR EQUAL to that of Joe

= (Total # of students) – (# of students with a lower GPA than Joe) – 1 (Joe himself)

= 20 – 12 – 1 = 7

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Number of students with a GPA equal to that of Joe

= 10% of 20 = 2

Thus, the number of students in the first class who have GPA higher than that of Joe

= (# of students with a GPA higher than or equal to that of Joe) – (# of students with a GPA equal to that of Joe)

= 7 – 2

= 5 ... (i)

Number of students in the second class = 10.

80% of the new students have a higher GPA than Joe has.

Thus, the number of students in the second class who have GPA higher than that of Joe

= 80% of 10

= 8 ... (ii)

Thus, from (i) and (ii), we have:

The total number of students with a GPA higher than that of Joe

= (# of students in the first class with a GPA higher than that of Joe) + (# of students in the second class with a GPA higher than that of Joe)

= 5 + 8

= 13

The correct answer is '13.'

361. Total length of the race = 1,000 meters.

Speed of A = 10 meters per second.

Thus, the time taken by A to complete the race = $\frac{\text{Distance}}{\text{Speed}}$

$= \frac{1,000}{10}$

= 100 seconds

We know that had A allowed B to start the race from a point 100 meters ahead of him, A would have still managed to beat B by 20 seconds.

Since A beats B by 20 seconds, time taken by B to complete the race of 900 meters (after a start of 100 meters allowed by A)

= 100 + 20

= 120 seconds

Thus, B ran 900 meters in 120 seconds to complete the race.

Thus, speed of B

$= \frac{900}{120}$

= 7.5 meters per second

The correct answer is '7.5.'

362. It is given that $(m - 2)$ and the quotient when m is divided by 2have the same integer value.

Let the quotient when m is divided by 2 be q and the corresponding remainder be r.

Thus, we have

$m = 2q + r \ldots$(i)

Also, we have

$m - 2 = q \ldots$(ii)

Substituting the value of m from (ii) in (i), we have

$m = 2\,(m - 2) + r$

=> $m = 2m - 4 + r$

=> $m = 4 - r$

Since the possible values of r (the remainder when m is divided by 2) are either 0 or 1, we have

- $r = 0$:

  => $m = 4 - r = 4 - 0 = 4$

  Let us verify the given statement:

  $m - 2 = 4 - 2 = 2$

  The quotient when $m = 4$ is divided by 2 is 2.

  Thus, $m = 4$ satisfies the given condition.

- $r = 1$:

  => $m = 4 - r = 4 - 1 = 3$ (a prime number)

  However, we know that m is not a prime number.

  Hence, $m \neq 3$.

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Thus, the only value of $m = 4$.

The correct answer is '4.'

363. We know that:

$3 < x < 4$

We also know that:

$x + 0.005 > 4$

$=> x > 3.995$

Possible values of x (less than 4) are 3.996, 3.997, 3.998, 3.999, 3.9991, and so on.

Thus, the hundredth digit of the decimal representation of x is '9.'

The correct answer is '9.'

364. Let $3^x = k$

$=> 9^x = (3^2)^x = 3^{2x} = (3^x)^2 = k^2$

Thus, we have:

$k^2 = 4k - 3$

$=> k^2 - 4k + 3 = 0$

$=> (k - 1)(k - 3) = 0$

$=> k = 1$ OR 3

$=> 3^x = 1$ OR 3

$=> x = 0$ OR 1

Since x is a positive integer, the only possible value of $x = 1$.

The correct answer is '1.'

Alternate approach:

We see that compared to the RHS, $4(3^x) - 3$, the LHS, 9^x, the larger exponent of $9^x = 4(3^x) - 3$, will increase in a larger proportion when the value of x increases; thus, for the equality, we must search for smaller values of x.

The minimum eligible value of x is $x = 1$. At $x = 1$, RHS $= 9^x = 9^1 = 9$.

At $x = 1$, LHS $= 4(3^x) - 3 = 4 \times 3^1 - 3 = 9$. Since LHS = RHS, $x = 1$ is the correct answer.

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If you try plugging-in larger values of $x = 2,\ 3,\ 4\ldots,$ we find that $9^x \gg 4\left(3^x\right) - 3$

365. We have:

$pq + qr + rp = 3$, where p, q & r are positive integers

The minimum possible values possible for p, q & r are $p = q = r = 1$

If we take: $p = q = r = 1$, we have:

$pq + qr + rp = 1 + 1 + 1 = 3$

Since the given equation is satisfied by the minimum possible values of p, q & r, for any other (higher) value of p, q & r, the above equation will not be satisfied.

Thus, we have:

$\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$

$= \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$

$= 3$

The correct answer is '3.'

366. Since ab is a two-digit number, we can represent it as $(10a + b)$.

Since the two-digit number ab is 11 more than the product of the digits a and b, we have:

$10a + b = ab + 11$

$=> 10a - 11 = b\left(a - 1\right)$

$=> b = \frac{10a - 11}{a - 1}$

$=> b = \frac{10\left(a - 1\right) - 1}{a - 1}$

$=> b = 10 - \frac{1}{a - 1}$

Since b is an integer, $(a - 1)$ must be a factor of 1

Thus, we have:

$a - 1 = 1 => a = 2$

$=> b = 10 - \frac{1}{1} = 9$

The correct answer is '9.'

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367. $(n+3)^n = 216$

Since n is a positive integer, let us take some values of n:

- $n = 1$

  => $(n+3)^n = (1+3)^1 = 4 < 216$

- $n = 2$

  => $(n+3)^n = (2+3)^2 = 25 < 216$

- $n = 3$

  => $(n+3)^n = (3+3)^3 = 216$

- $n = 4$

  => $(4+3)^4 = 7^4 = 2401 > 216$

Thus, we see that:

- For $n < 3$, LHS < RHS
- For $n = 3$, LHS = RHS
- For $n > 3$, LHS > RHS

Thus, the only possible integer solution for $n = 3$

The correct answer is '3.'

Alternate approach:

By factoring 216, we get $216 = 2^3 \times 3^3$

Since the LHS, $(n+3)^n$ of $(n+3)^n = 216$ is an exponent with a single base, we must transform $2^3 \times 3^3$ to make it a single base exponent, thus $216 = 2^3 \times 3^3 = 6^3$

Thus, $(n+3)^n = 6^3$

=> $n = 3$.

368. We know that:

$f(x) = 3x + 2$

Since $f(\sqrt{c}) = 8$, we have:

$3\sqrt{c} + 2 = 8$

=> $3\sqrt{c} = 6$

$=> \sqrt{c} = 2$

$=> c = 4$

$=> f(c) = f(4)$

$= 3 \times 4 + 2$

$= 14$

The correct answer is '14.'

Note: The value of c is not the required answer. It might happen that by mistake you respond '4' as the answer. Problems should be read carefully before putting down answers.

369. We know that:

$f(x) = x^{\left(\frac{3}{4}\right)} - 7$

Since $f(k^2) = 57$, we have:

$(k^2)^{\left(\frac{3}{4}\right)} - 7 = 57$

$=> (k^2)^{\left(\frac{3}{4}\right)} = 64$

$=> k^{\left(\frac{3}{2}\right)} = 64$

$=> k = 64^{\left(\frac{2}{3}\right)}$

$= (4^3)^{\left(\frac{2}{3}\right)}$

$= 4^2$

$= 16$

The correct answer is '16.'

370. We know that:

$f(p^2) = p^4 - p^2 + 1$

$=> f(p^2) = (p^2)^2 - (p^2) + 1$

$=> f(x) = x^2 - x + 1$

Since $f(5) = p$, we have:

$p = f(5)$

$= 5^2 - 5 + 1$

$= 21$

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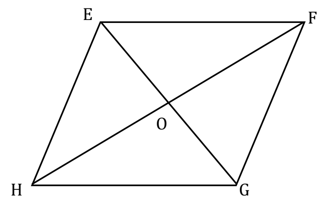
The correct answer is ‘21.’

371.



In a rhombus, the diagonals are perpendicular to one another.

Thus, in right-angled triangle EOF:

$\angle OEF = 2x$

$\angle EFO = (x + 3)$

$\angle EOF = 90^\circ$

Thus, we have:

$\angle OEF + \angle EFO + \angle EOF = 180^\circ$

$=> 2x + (x + 3) + 90^\circ = 180^\circ$

$=> x = 29^\circ$

Thus, we have:

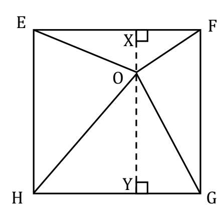
$\angle GFE = 2\angle EFO$

$= 2\,(x + 3) = 2\,(29 + 3)$

$= 64^\circ$

The correct answer is ‘64.’

372.



We draw a line through O, perpendicular to EF and GH.

Thus, we have:

The sum of areas of triangle EOF and triangle HOG

$= \frac{1}{2} \times EF \times OX + \frac{1}{2} \times HG \times OY$

$= \frac{1}{2} \times EF \times OX + \frac{1}{2} \times EF \times OY$ (since HG = EF)

$= \frac{1}{2} \times EF \times (OX + OY) = \frac{1}{2} \times EF \times XY$

$= \frac{1}{2} \times EF \times EF$ (since XY = EF)

$= \frac{1}{2} \times (EF)^2$

$= \frac{1}{2} \times$ (Area of the square EFGH)

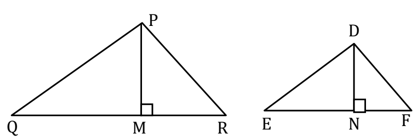
$= \frac{1}{2} \times 16 = 8$

The correct answer is '8.'

373. Since XY and WZ are drawn parallel to BC, we have:

Triangle AXY is similar to triangle AWZ, which is similar to triangle ABC.

For any two similar triangles PQR and DEF, we have:



Let the ratio of sides of the two triangles be k.

Thus, we have:

$$\frac{PQ}{DE} = \frac{PR}{DF} = \frac{QR}{EF} = \frac{PM}{DN} = k$$

$$\frac{\text{(Area of triangle PQR)}}{\text{(Area of triangle DEF)}} = \left(\frac{\frac{1}{2} \times QR \times PM}{\frac{1}{2} \times EF \times DN}\right) = \left(\frac{QR \times PM}{EF \times DN}\right) = \left(\frac{QR}{EF}\right) \times \left(\frac{PM}{DN}\right) = k \times k = k^2$$

Thus, for two similar triangles, the ratio of their area equals the square of the ratio of their corresponding sides.

Thus, we have:

$$\frac{\text{(Area of triangle AXY)}}{\text{(Area of triangle ABC)}} = \left(\frac{AX}{AB}\right)^2$$

Since AX : XW : WB = 2 : 1 : 2, we have:

$$\frac{\text{(Area of triangle AXY)}}{\text{(Area of triangle ABC)}} = \left(\frac{AX}{AB}\right)^2 = \left(\frac{2}{2+1+2}\right)^2 = \frac{4}{25}$$

$$=> \frac{\text{(Area of triangle AXY)}}{25} = \frac{4}{25}$$

=> Area of triangle AXY = 4

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The correct answer is ‘4.’

374. Let the number of rounds of the game played = n

Since A lost 3 rounds, he must have won the remaining $(n - 3)$ rounds.

Since B lost 4 rounds, he must have won the remaining $(n - 4)$ rounds.

Since C lost 5 rounds, he must have won the remaining $(n - 5)$ rounds.

Total number of rounds won by A, B and C together

$= (n - 3) + (n - 4) + (n - 5)$

$= 3n - 12$

Since there is only one winner in each round, the total number of rounds won by A, B and C together is the same as the total number of rounds played.

Thus, we have:

$3n - 12 = n$

$=> n = 6$

The correct answer is ‘6.’

375. We know that:

$x^2 = y$ and $z = y + 1$

Thus, the possibilities are

- $x = 1,\ y = 1$ – Not possible, since x and y must be different digits
- $x = 2,\ y = 4\ =>\ z = 4 + 1 = 5$
- $x = 3,\ y = 9\ =>\ z = 9 + 1 = 10$ – Not possible, since z, a digit, cannot be 10

Thus, the only possible solution is

$x = 2, y = 4\ \&\ z = 5$

$=> x + y + z = 11$

The correct answer is ‘11.’

4.4 Quantitative Comparison Questions

376.

<table>
<tr><th><u>Quantity A</u></th><th><u>Quantity B</u></th></tr>
<tr><td>Percent increase in the number of males from 2002 to 2003</td><td>Percent increase in the number of females from 2002 to 2003</td></tr>
<tr><td colspan="2">We know that

The ratio of the number of male and female workers in 2002 = 3 : 4.

Let the number of male and female workers in 2002 be $3k$ and $4k$ respectively, where k is a constant of proportionality.

The ratio of the number of male and female workers in 2003 = 10 : 7.

Let the number of male and female workers in 2003 be $10l$ and $7l$, respectively, where l is another constant of proportionality (not necessarily same as k)</td></tr>
<tr><td>Percent increase in the number of male workers from 2002 to 2003 = P_m

$= \left(\frac{10l - 3k}{3k}\right) \times 100$
$= \left(\frac{10l}{3k} - 1\right) \times 100$</td><td>Percent increase in the number of female workers from 2002 to 2003 = P_f

$= \left(\frac{7l - 4k}{4k}\right) \times 100$
$= \left(\frac{7l}{4k} - 1\right) \times 100$</td></tr>
<tr><td colspan="2">Comparing the ratios, we see that

$\left(\frac{10l}{3k}\right) = \frac{10}{3} \times \frac{l}{k} = 3.33 \times \frac{l}{k}$
$\left(\frac{7l}{4k}\right) = \frac{7}{4} \times \frac{l}{k} = 1.75 \times \frac{l}{k}$
Thus, we can say that:

$\left(\frac{10l}{3k}\right) > \left(\frac{7l}{4k}\right)$

$\Rightarrow P_m > P_f$</td></tr>
<tr><td colspan="2">Thus, Quantity A is greater than Quantity B.

The correct answer is option A.</td></tr>
</table>

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Alternate approach:

Say, on a ratio scale, the number of male and female workers in 2002 = 3 & 4, respectively.

Again, say, on the ratio scale, the number of male and female workers in 2003 = x & y, respectively.

Thus, $\frac{x}{y} = \frac{10}{7}$

=> $x = \frac{10y}{7}$

Percent increase in the number of male workers from 2002 to 2003	Percent increase in the number of female workers from 2002 to 2003
$= \frac{x-3}{3}$ $= \frac{\frac{10y}{7}-3}{3}$ $= \frac{10y-21}{21}$ $= \frac{y}{2.1} - 1$	$= \frac{y-4}{4}$ $= \frac{y}{4} - 1$

Comparing the ratios, we see that:

$\frac{y}{2.1} - 1 > \frac{y}{4} - 1$ since for Quantity A, relatively smaller number, 2.1, divides y, compared to relatively larger number, 4.

377.

Quantity A	**Quantity B**
Fraction of the members who are mechanical engineers in the club	$\frac{1}{3}$

We know that

Percent of female members who are mechanical engineers

$= \frac{1}{3} \times 75\% = 25\% = \frac{1}{4}$

Percent of male members who are engineers = 30%

Since only engineers can be mechanical engineers, the percent of male members who are mechanical engineers $\leq 30\% = \frac{3}{10}$.

Since for both male and female members, the fraction of mechanical engineers among them is less than $\frac{1}{3}$, among all members, the fraction of mechanical engineers in the club is definitely less than $\frac{1}{3}$.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Note: Had you hurriedly analyzed the questions and based on the information "Exactly 30 percent of the male members are engineers," concluded that since the information about the fraction of male mechanical engineers is not given, the finite answer to this question is indeterminable, it would have been incorrect.

In most QC questions, we must think of 'To what extent the maximum or the minimum value of Quantity A or Quantity B can be?'; based on that we can answer many seemingly indeterminable questions.

Remember that you need not always get an exact answer to correctly answer a QC question.

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378.

Quantity A	**Quantity B**
Dollar expense per person	\$0.60

We have the lower limit and upper limit for the number of persons who can turn up for the gathering and those for the expense.

We have to calculate the ratio of expense to number of persons.

Since the range of number of persons and that of expense are given, we must find out the greatest value and the smallest value of the ratio and then compare them with Quantity B.

The greatest value of the ratio of expense per person

$$= \frac{\text{Maximum expense}}{\text{Minimum number of persons}} = \frac{75,000}{100,000} = \$0.75$$

$= 0.75 > .60$

=> Quantity A is greater than Quantity B

Let us calculate the smallest value of the ratio of expense person

The smallest value of the ratio of expense per person

$$= \frac{\text{Minimum expense}}{\text{Greatest number of persons}} = \frac{50,000}{120,000} = \$0.417$$

$= 0.417 < 0.60$

=> Quantity B is greater than Quantity A

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

379.

Quantity A	**Quantity B**
t	1

Let the actual distance be d kilometers.

Thus, Steve's estimate of the distance ranged from $(d + 4)$ kilometers to $(d - 4)$ kilometers.

Let Steve's actual average speed be s kilometers/hour.

Thus, Steve's estimate of his speed ranged from $(s + 8)$ kilometers to $(s - 8)$ kilometers/hour.

Thus, actual time = $\frac{d}{s}$ hours.

Maximum value of the estimated time

$= \frac{\text{(Maximum distance)}}{\text{(Minimum speed)}} = \left(\frac{d+4}{s-8}\right)$ hours

Minimum value of the estimated time

$= \frac{\text{(Minimum distance)}}{\text{(Maximum speed)}} = \left(\frac{d-4}{s+8}\right)$ hours

Let us take a value:

If $d = 30$ & $s = 20$ (value of d is much greater than s):

- $t = \left(\frac{d+4}{s-8}\right) - \frac{d}{s} = \frac{34}{12} - \frac{30}{20} = 2.83 - 1.5 = 1.33 > 1$

Quantity A is greater than Quantity B.

- $t = \frac{d}{s} - \left(\frac{d-4}{s+8}\right) = \frac{30}{20} - \frac{26}{28} = 1.50 - 0.93 = 0.57 < 1$

Quantity B is greater than Quantity A.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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In a nutshell, we are interested in predicting the maximum and the minimum values of $\frac{d \pm 4}{s \pm 8}$. If Steve drove from one planet to another planet at a snail's pace, $\frac{d \pm 4}{s \pm 8}$ would be too high; whereas if Steve drove from one bloc to another bloc at a supersonic speed, $\frac{d \pm 4}{s \pm 8}$ would be too low, thus the value of $h = \frac{d \pm 4}{s \pm 8}$ cannot be determined.

380.

Quantity A	**Quantity B**
The tenth digit in the decimal representation of x	0

Since $0 < x < 1$ and $8x$ is an integer, we must have:

$x = \left(\frac{\text{An integer less than 8}}{8}\right)$

If $x = \frac{1}{8}$ (smallest possible value of x):

$x = 0.125$

=> The tenth digit is '1'

Since for the smallest possible value of x, the tenth digit is '1', the tenth digit will always be greater than or equal to '1' for all higher values of x.

Other possible values of x are $x = \frac{1}{2} = 0.50$ & $x = \frac{1}{4} = 0.25$. We see that the tenth digit is rather greater than '1'.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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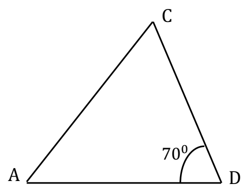
381.

Quantity A	**Quantity B**
Length of AC	Length of AD



We know that in a triangle, the longest side is opposite to the largest angle and the shortest side is opposite to the smallest angle.

$\angle CDA = 70^\circ$

Thus, the sum of the other two angles in triangle ACD

$= 180^\circ - 70^\circ = 110^\circ$

If AC is the shortest side, then $\angle CDA$ should be the smallest angle.

It follows that each of the other two angles of the triangle ACD must be more than 70°.

In such an event, the sum of those two angles should be more than 140°, which is not possible.

Thus, $\angle CDA$ is not the smallest angle

=> AC is not the shortest side.

However, since the value of the other two angles in triangle ACD is not known, we cannot determine the shortest side.

Thus, we can have the following scenarios:

- Let $\angle DAC = 50^\circ$ and $\angle DCA = 60^\circ$ => $AC > AD > CD$

=> Quantity A is greater than Quantity B

- Let $\angle DAC = 30^\circ$ and $\angle DCA = 80^\circ$ => $AD > AC > CD$

=> Quantity A is less than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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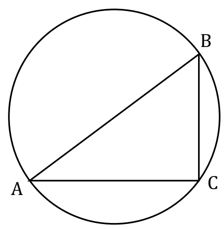
382.

Quantity A	**Quantity B**
Circumference of the circle	20π



Let the length of the sides BC, AC and AB, respectively be $3x, 4x$ and $5x$, where x is a constant of proportionality.

We see that: $(5x)^2 = (3x^2) + (4x)^2$

=> Triangle ABC is right-angled at C.

=> AB is the diameter of the circle
(since the diameter subtends 90° at the circumference)

Sum of the three sides of the triangle

$= 3x + 4x + 5x = 12x$.

Since the perimeter of the triangle is 48, we have

$12x = 48$

=> $x = 4$

=> Diameter of the circle = AB

$= 5x$

$= 20$

=> Radius of the circle

$= \frac{20}{2}$

$= 10$

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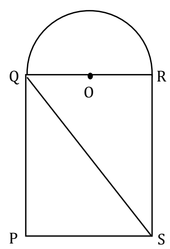
=> Circumference of the circle $= 2\pi \times 10$ $= 20\pi$
Thus, Quantity A is equal to Quantity B. The correct answer is option C.

383.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
Area of the semi-circular region with center O and diameter QR	3π



We know that

QR : PQ :: 5 : 12

=> Let QR = $5x$ and PQ = $12x$, where x is a constant of proportionality.

We also know that QS = 13.

Thus, applying Pythagoras theorem in triangle SPQ, we have

$PQ^2 + PS^2 = QS^2$

=> $PQ^2 + QR^2 = 13^2$

=> $(12x)^2 + (5x)^2 = 13^2$

=> $144x^2 + 25x^2 = 169$

=> $x^2 = 1$

=> $x = 1$ (x cannot be -1 since lengths must be positive)

=> QR = $5x = 5$

=> Radius of the semi-circle = $\frac{5}{2}$

Thus, the area of the semi-circle

$= \frac{1}{2} \times \pi \times \left(\frac{5}{2}\right)^2 = 3.125\pi > 3\pi$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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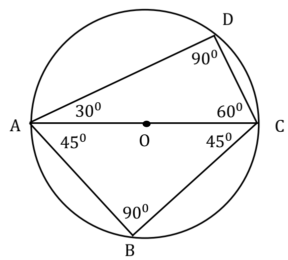
384.

Quantity A	**Quantity B**
$a + b + c$	7

The diagram corresponding to the above information is shown below:



We know that AC is the diameter of the circle (centered at O).

Since the radius of the circle is 1, we have AC = 2

Since the diameter always subtends a right angle at the circumference, we have

$\angle ABC = \angle ADC = 90^\circ$

Also, we know that $\angle DAC$ is 30° and $\angle BAC$ is 45°

Thus, in triangle ADC, we have $\angle DCA = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

Also, in triangle ABC, we have $\angle BCA = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

Thus, triangle ADC is a 30-90-60 triangle => AD : DC : AC = $\sqrt{3} : 1 : 2$

Since AC = 2, we have

AD = $\sqrt{3}$, and DC = 1

=> Area of triangle ADC

$= \frac{1}{2} \times AD \times DC$

$= \frac{1}{2} \times \sqrt{3} \times 1$

$= \frac{\sqrt{3}}{2} \ldots (i)$

Also, we have triangle ABC to be a 45-90-45 triangle => AB : BC : AC = $1 : 1 : \sqrt{2}$

Since AC = 2, we have

$AB = BC = \frac{2}{\sqrt{2}} = \sqrt{2}$

=> Area of triangle ABC

$= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times \sqrt{2} \times \sqrt{2}$

$= 1 \ldots \text{(ii)}$ Thus, from (i) and (ii), we have

Area of ABCD = Area of triangle ABC + Area of triangle ADC

$= \left(1 + \frac{\sqrt{3}}{2}\right)$

$= \frac{2 + \sqrt{3}}{2}$

Thus, we have

$\frac{a + \sqrt{b}}{c} = \frac{2 + \sqrt{3}}{2}$

=> $a = 2,\ b = 3$ & $c = 2$

=> $a + b + c = 7$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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385.

Quantity A	**Quantity B**
b	$\frac{3}{2}$

Since b is 3 more than a, and c is 9 more than b, we have

$b = a + 3 \ldots$ (i)

$c = b + 9$

Using (i):

$c = a + 12 \ldots$ (ii)

Also, we have

$\frac{a}{b} = \frac{b}{c}$

$=> \frac{a}{a+3} = \frac{a+3}{a+12}$

$=> a^2 + 12a = a^2 + 6a + 9$

$=> a = \frac{3}{2}$

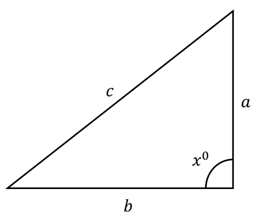
$=> b = 3 + \frac{3}{2} = \frac{9}{2}$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

386.

Quantity A	**Quantity B**
x°	90°



We know that

- If $x = 90$, we have $c^2 = a^2 + b^2$
- If $x < 90$, we have $c^2 < a^2 + b^2$
- If $x > 90$, we have $c^2 > a^2 + b^2$

We know that

$a^2 + b^2 < 15$

Also, we know that

$c = 4$

$=> c^2 = 16$

Thus, we see that:

$c^2 > a^2 + b^2$

$=> x^\circ > 90^\circ$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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387.

Quantity A	**Quantity B**
Distance of the point (r, s) from the origin	Distance of the point (u, v) from the origin
$= \sqrt{(r-0)^2 + (s-0)^2}$ $= \sqrt{r^2 + s^2}$	$= \sqrt{(u-0)^2 + (v-0)^2}$ $= \sqrt{u^2 + v^2}$

We know that

$u = 1 - r$ and $v = 1 - s$

$=> u^2 + v^2$

$= (1-r)^2 + (1-s)^2$

$= 2 + r^2 + s^2 - 2(r+s) \ldots$(i)

Also, we know that

$r = 1 - s$

$=> r + s = 1 \ldots$(ii)

Thus, from (i) and (ii), we have

$u^2 + v^2$

$= 2 + r^2 + s^2 - 2 \times 1$

$= r^2 + s^2$

Thus, we have

$u^2 + v^2 = r^2 + s^2$

$=> \sqrt{u^2 + v^2} = \sqrt{r^2 + s^2}$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

Alternate approach:

We can assume smart and convenient values for (r, s) and (u, v).

From the relations $r = v = 1 - s$, let us assume that $s = 0 => r = v = 1 - 0 = 1$

Thus, $u = 1 - r = 1 - 1 = 0$

So the two points are: (1, 0) and (0, 1). Each of these two points lie on X-axis and Y-axis, respectively and thus are equidistant from the origin; they are 1 unit away from the origin.

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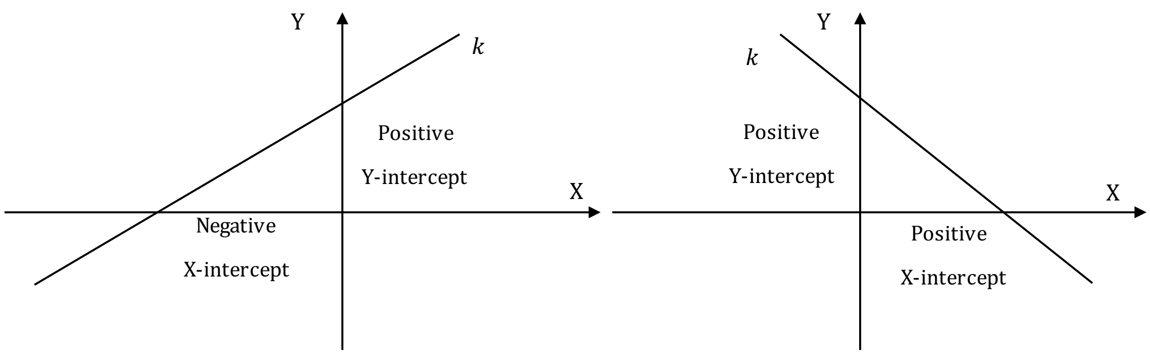
388.

Quantity A	**Quantity B**
Slope of the line k	0

We know that the line k makes a positive intercept on the Y-axis.

Thus, the possible orientations of the line are shown below:



If any line intersects the X-axis at $(p, 0)$ and the Y-axis at $(0, q)$, the slope of the line

$$= \frac{q - 0}{0 - p} = -\frac{q}{p} = -\left(\frac{\text{Y intercept}}{\text{X intercept}}\right)$$

Thus, we see that:

- In the left diagram, the slope

$$= -\frac{(\text{Positive Y} - \text{intercept})}{(\text{Negative X} - \text{intercept})} = \text{A positive value}$$

=> Quantity A is greater than Quantity B

- In the right diagram, the slope

$$= -\frac{(\text{Positive Y} - \text{intercept})}{(\text{Positive X} - \text{intercept})} = \text{A negative value}$$

=> Quantity A is less than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

389.

Quantity A	**Quantity B**
b	0

Slope (m) of the line k passing through $(0,\ 0)$ and (a, b) is:

$m = \frac{b-0}{a-0} = \frac{b}{a}$

Since the above slope is negative, we have

$\frac{b}{a} < 0$

Thus, the possible cases are:

Case (a): $a > 0$ and $b < 0$

Case (b): $a < 0$ and $b > 0$

We also know that $a < b$.

This is satisfied only by Case (b) above

=> $a < 0$ and $b > 0$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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390.

Quantity A	**Quantity B**
The average amount spent by all the friends	\$12

We know that there are 20 friends in all.

Let the average amount spent by each friend = \$$a$.

Thus, the total amount spent by the 20 friends = \$20a.

The amount spent by the first five friends = \$ (5×21) = \$105.

The average amount spent by the remaining $(20 - 5) = 15$ friends = \$ $(a - 3)$.

Thus, the total amount spent by the 15 friends = \$ $(15 \times (a - 3))$.

Thus, the total amount spent by all the friends = \$ $\{15 \times (a - 3) + 105\}$

Thus, we have

$20a = 15 \times (a - 3) + 105$

$=> 5a = 60$

$=> a = 12$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

391.

Quantity A	**Quantity B**
The year in which Jack was born	1989

We know that in 2010, Suzy was 24 years old.

Since Suzy is 5 years elder to Jack, in 2010, Jack was 24 – 5 = 19 years old.

Thus, the year in which Jack was born

= 2010 – 19

= 1991

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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392.

Quantity A	**Quantity B**
x	y

We have

$2x + 3y < 6 \ldots$(i)

$3x + 2y = 6$

$=> 3x = 6 - 2y \ldots$(ii)

Multiplying (i) with $\frac{3}{2}$:

$\frac{3}{2} \times 2x + \frac{3}{2} \times 3y < 6 \times \frac{3}{2}$

$=> 3x + \frac{9y}{2} < 9$

Substituting the value of $3x$ above:

$6 - 2y + \frac{9y}{2} < 9$

$=> \frac{5y}{2} < 3$

$=> y < \frac{6}{5} \ldots$(iii)

Thus, from (ii) and (iii):

$3x > 6 - 2 \times \frac{6}{5}$

$=> 3x > \frac{18}{5}$

$=> x > \frac{6}{5} \ldots$(iv)

Thus, from (iii) and (iv), we have

$x > y$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach 1:

$2x + 3y < 6$ and $3x + 2y = 6$

$3x + 2y = 6$

$=> 2x + 2y = 6 - x$

Rewriting in the inequality $2x + 3y < 6$ as $(2x + 2y) + y < 6$ and by plugging-in the value of $(2x + 2y) = 6 - x$ in it, we get,

$=> 6 - x + y < 6$

$=> -x + y < 0$

$=> y < x$

Alternate approach 2:

Observing the inequality $2x + 3y < 6$, we find that two xs and three ys make a sum, which is less than 6.

Again, observing the equation $3x + 2y = 6$, we find that if one x is replaced with one y in the inequality $2x + 3y < 6$, the sum increases and its value reaches 6. It implies that x is a bigger contributor than y.

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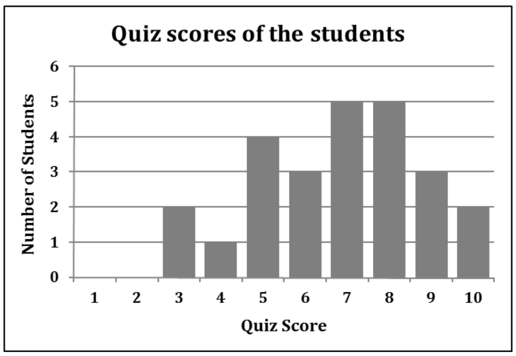
393.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The new mean quiz score of the students, considering score for question number '2'	7



Total number of students

$= 2 + 1 + 4 + 3 + 5 + 5 + 3 + 2$

$= 25$

Total score of all the students before giving full credit for question number '2'

$= (2 \times 3) + (1 \times 4) + (4 \times 5) + (3 \times 6) + (5 \times 7) + (5 \times 8) + (3 \times 9) + (2 \times 10)$

$= 170$

Total score of all the students after giving full credit for question number '2'

$= 170 + 1 \times 25 = 195$

Thus, the new mean score

$= \frac{195}{25} = 7.8$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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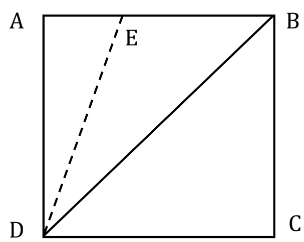
394.

Quantity A	**Quantity B**
The length of AE	0.5

The diagram of the square is shown below:



We know that, the square, when folded along the line DE, side AD coincides with diagonal BD.

This is possible only if

$\angle ADE = \angle BDE$

Thus, in triangle ABD, DE is the internal angle bisector of $\angle ADB$.

Thus, we have

$\frac{AE}{BE} = \frac{AD}{BD}$ (internal angle bisector theorem) ... (i)

In right angled triangle ABD, we have

AD = AB = 1 (since it is given that the side of the square is 1)

Thus, from Pythagoras' theorem, we have

$BD^2 = AD^2 + AB^2$

$= 1 + 1 = 2$

$\Rightarrow BD = \sqrt{2}$

Thus, from (i), we have

$\frac{AE}{BE} = \frac{1}{\sqrt{2}}$

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Since AB = 1, we have

$AE = \frac{1}{1+\sqrt{2}} \times AB$

=> $AE = \frac{1}{1+\sqrt{2}}$

=≈ $\frac{1}{1+1.41}$

= 0.415

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Alternate approach:

Let us focus on triangle ADB.

We know that DE is the internal angle bisector of $\angle ADB$.

In any equilateral triangle, the angle bisector also bisects the side opposite to the angle it bisects.

Also, in an isosceles triangle, the angle bisector of the 'unequal angle' also bisects the side opposite to that angle, i.e. it bisects the 'unequal side'.

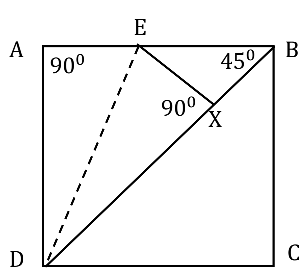
In this case, though triangle ADB is isosceles, the angle bisector bisects one of the equal angles (since $\angle ADB = \angle ABD = 45°$).

Thus, the angle bisector in this case, i.e. DE, does not bisect the side opposite to the concerned angle.

Thus, DE does not bisect AB

=> $AE \neq BE$

Let us draw EX perpendicular to BD.



It is clear that triangles ADE and XDE are congruent to one another, since
DE is a common side, $\angle$DAE =$\angle$DXE = 90°, and $\angle$ADE =$\angle$XDE

Thus, AE = EX.

In triangle EXB, EB is the hypotenuse, and hence, the longest side.

Thus, we have

EB > EX

=> EB > AE

Thus, AE must be less than half of AB

=> AE < 0.5

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395.

Quantity A	**Quantity B**
Percent discount offered by the store	60%

Since the problem asks for a percent value, we can assume any suitable price per square foot of carpet.

Let the price per square foot be \$1.

Thus, the price, before discount, of a 10-feet by 12-feet carpet

$= \$(10 \times 12 \times 1)$

$= \$120$

Since the discounted price of the above carpet was equal to the price, before discount, of a 6-feet by 8-feet carpet, we have

The final price, after discount, of the 10-feet by 12-feet carpet

$= \$(6 \times 8 \times 1)$

$= \$48$

Thus, discount

$= 120 - 48$

$= \$72$

Thus, percent discount

$$= \frac{\text{(Discount)}}{\text{(Price before discount)}} \times 100\%$$

$$= \frac{72}{120} \times 100$$

$= 60\%$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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396.

<table>
<tr><th><u>Quantity A</u></th><th><u>Quantity B</u></th></tr>
<tr><td>$\sqrt{(x-5)^2}$</td><td>$(x-5)$</td></tr>
<tr><td colspan="2">We know that the radical sign i.e. $\sqrt{k}$ takes only the positive square root of k.

Thus, we can have the following cases:

• $\sqrt{(x-5)^2} = (x-5)$ – Condition: If $x - 5 \geq 0 => x \geq 5$
• $\sqrt{(x-5)^2} = \sqrt{(5-x)^2} = (5-x)$ – Condition: If $5 - x > 0 => x < 5$

We have

$-x\,|x| > 0$

We know that $|x| \geq 0$ for all values of x

$=> -x > 0$

$=> x < 0$

Thus, it satisfies the condition that $x < 5$

Thus, we have

$\sqrt{(x-5)^2} = (5-x)$</td></tr>
<tr><td>Since $x < 5$, we have

$5 - x > 0$</td><td>Since $x < 5$, we have

$x - 5 < 0$</td></tr>
<tr><td colspan="2">Thus, Quantity A is greater than Quantity B.

The correct answer is option A.</td></tr>
</table>

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397.

Quantity A	**Quantity B**
$\frac{x}{y}$	xy

We have

$y < -1$ => y is negative

Since $xy > 0$

=> $x < 0$

Thus, both x and y are negative

=> Both $\frac{x}{y}$ and xy are positive

Thus, we compare the absolute values of $\frac{x}{y}$ and xy.

$\left\lvert\frac{x}{y}\right\rvert = \lvert x\rvert \times \left\lvert\frac{1}{y}\right\rvert$ Since $y < -1$ => $1 < \lvert y\rvert < \infty$ => $0 < \left\lvert\frac{1}{y}\right\rvert < 1$ Thus, $\left\lvert\frac{x}{y}\right\rvert$ is effectively $\lvert x\rvert$multiplied with a fraction between 0 and 1 => $\left\lvert\frac{x}{y}\right\rvert < \lvert x\rvert$ Since $\frac{x}{y}$ is positive, we have $0 < \frac{x}{y} < \lvert x\rvert$	$\lvert xy\rvert = \lvert x\rvert \times \lvert y\rvert$ Since $y < -1$ => $1 < \lvert y\rvert < \infty$ Thus, $\lvert xy\rvert$ is effectively $\lvert x\rvert$multiplied with a number greater than 1 => $\lvert xy\rvert > \lvert x\rvert$ Since xy is positive, we have $xy > \lvert x\rvert$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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Alternate approach:

From the above approach, we know that x & y both are negative. Since in Quantity A and in Quantity B, x is in the numerator, thus, the deciding variable would be y. So essentially, we have to compare $\frac{1}{|y|}$ & $|y|$.

Since $y < -1$, for all values of y, $\frac{1}{|y|} < |y|$.

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398.

Quantity A	**Quantity B**
$\lvert x-y \rvert$	$\lvert x-z \rvert$

We have

$|y| > |z|$

Since there is nothing mentioned about whether x, y and z are positive or negative, it is best to take some sample values of x, y and z, and than proceed in an analytical way.

Thus, we have

- Let $y = 3,\ z = 2$ and $x = 1$

$|x-y| = |1-3| = 2$

$|x-z| = |1-2| = 1$

$=> |x-y| > |x-z|$

Thus, Quantity A is greater than Quantity B

- Let $y = 3,\ z = -2$ and $x = 1$

$|x-y| = |1-3| = 2$

$|x-z| = |1-(-2)| = 3$

$=> |x-y| < |x-z|$

Thus, Quantity A is less than Quantity B.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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399.

Quantity A	**Quantity B**
$\frac{m^2}{n}$	-1

We have

$m^2 + n < 0$

$=> m^2 < -n$

Since $mn \neq 0$, we have

$m \neq 0$ and $n \neq 0$

Thus, m^2 is always positive

$=> -n > m^2 > 0$

$=> -n > 0$

$=> n < 0$

Thus, we have

$\frac{m^2}{n} = \frac{\text{(A positive quantity)}}{\text{(A negative quantity)}}$ = A negative quantity

$=> \frac{m^2}{n} < 0$

Also, the absolute value of n must be greater than m^2 (since $m^2 + n < 0$)

Thus, $\frac{m^2}{n}$ must be a fractional value between 0 and -1.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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Alternate approach:

We know that $n < 0$.

Thus, we have

$m^2 < -n$

Dividing by n and reversing the inequality, we have

$0 > \frac{m^2}{n} > -1$

400.

Quantity A	**Quantity B**
Twice the number of clerks working in the factory	The numerical value of the average weekly salary, in dollars, of the clerks

Let the number of clerks be c.

Since the total number of clerks and supervisors is 60, we have

Number of supervisors = $(60 - c)$.

Since the average weekly salary of 60 people is \$120, we have

Total weekly salary of 60 people = $\$ (60 \times 120) = \$7,200$.

Let the average weekly salary of clerks be $\$x$.

Thus, the average weekly salary of the supervisors = $\$ (x + 120)$

Thus, total salary of clerks and supervisors

$= \$ (x \times c + (x + 120) (60 - c))$

Thus, we have

$x \times c + (x + 120) (60 - c) = 7,200$

$=> 60x - 120c = 0$

$=> 2c = x$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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401.

Quantity A	**Quantity B**
Percent of the total employees who attended the meeting	33%

Since the problem asks for a percent value, we can choose a suitable value of the total number of employees.

Let the total number of employees in the company = 100.

Thus, the number of men = 35% of 100 = 35

The number of women = 100 – 35 = 65

Number of men in the company who attended the meeting

$= 20\% \text{ of } 35 = \frac{20}{100} \times 35$

$= 7$

Number of women in the company who attended the meeting

$= 40\% \text{ of } 65 = \frac{40}{100} \times 65$

$= 26$

Total number of employees who attended the meeting

$= 7 + 26$

$= 33$

Thus, the percent of the total employees who attended the meeting

$= \frac{33}{100} \times 100$

$= 33\%$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

402.

Quantity A	**Quantity B**
Percent of total toffees with the two friends who have the least number of toffees	50%

Since the problem asks for a percent value, we can choose a suitable value of the total number of employees.

Let the total number of toffees = 100.

Let the number of toffees with R, Y and M be r, y and m respectively.

Thus, we have

$r + y + m = 100$

Since M has $\frac{1}{4}$ = 25% of the total number of toffees, we have

m = 25% of 100 = 25

$=> y + r = 100 - 25 = 75 \ldots$(i)

The difference between the toffees of Y and R is $\frac{1}{10}$ of the total number of toffees, i.e. 10 toffees. However, we have no information regarding who, between Y and R, has the lesser number of toffees.

However, this does NOT necessarily mean that the value of Quantity A cannot be uniquely determined, leading to the answer as 'D'.

If we look at Quantity A, we simply need to find the total number of toffees with the two friends having the least number of toffees.

We need not find the respective number of toffees with the three friends.

Thus, we have two friends having a total of 75 toffees, with one friend having 10 toffees more than the other.

Thus, the number of toffees with the friend having the higher amount = $\frac{75 + 10}{2}$ = 42.5 toffees

Thus, the number of toffees with the friend having the lesser amount = $\frac{75 - 10}{2}$ = 32.5 toffees

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Thus, the number of toffees with the two friends who have the least number of toffees = 25 + 32.5 = 57.5, which is 57.5% of the total
Thus, we have The number of toffees with the friends: 25, 32.5 and 42.5. (The above represent the number of toffees out of a total 100 toffees, i.e. the percent shares, hence can be decimal values)
Thus, Quantity A is greater than Quantity B. The correct answer is option A.

403.

Quantity A	**Quantity B**
The area of the floor of the actual kitchen, in square meters, expressed as an integer to the nearest unit	37 square meters

The dimensions of the room on the plan are 6 cm by 2.7 cm.

Since 1 centimeter represents 1.5 meters, the actual dimensions of the room are:

$6 \times 1.5 = 9$ meters and $2.7 \times 1.5 = 4.05$ meters

Thus, the actual area of the room

$= 9 \times 4.05$

$= 36.45$ square meters

Expressing the above value as an integer to the nearest unit, we have

36 square meters
(Note: It would be INCORRECT to approximate as: $36.45 = \approx 36.5 = \approx 37$)

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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404.

Quantity A	**Quantity B**
The positive difference between the area of the square and that of the circle, expressed as a decimal to the nearest tenth	14

Length of the diagonal of the square = 7 inches.

Thus, area of the square

$= \frac{1}{2} \times (\text{Length of diagonal})^2$

$= \frac{1}{2} \times 7^2$

= 24.5 square inches

Length of the diameter of the circle = 7 inches.

Thus, the area of the circle

$= \pi \times \left(\frac{\text{Length of diameter}}{2}\right)^2$

$= \pi \times \left(\frac{7}{2}\right)^2$

= 38.477 square inches

Thus, the positive difference in areas 38.47

= 38.477 – 24.5

= 13.977 square inches

Expressing the above value as a decimal to the nearest tenth, we have

14 square inches
(Since the digit after ‘9’ is ‘7’, we need to increase ‘3’ by ‘1’ to make it ‘4.’)

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

405.

Quantity A	**Quantity B**
Time taken, in minutes, for Joe and Carl to run a combined distance of 99 laps	222.75 minutes

Joe runs around the track at a rate of 30 laps per 75 minutes.

Thus, fraction of a lap covered by Joe in 1 minute

$= \frac{30}{75}$

$= \frac{2}{5}$

Carl runs around the track at a rate of 20 laps per 40 minutes.

Thus, fraction of a lap covered by Carl in 1 minute

$= \frac{20}{40}$

$= \frac{1}{2}$

Thus, total part of a lap covered by both in 1 minute

$= \frac{2}{5} + \frac{1}{2}$

$= \frac{9}{10}$

Thus, time taken to cover a combined distance of 1 lap

$= \frac{1}{\left(\frac{9}{10}\right)}$

$= \frac{10}{9}$ minutes

Thus, time taken to cover a combined distance of 99 laps

$= \frac{10}{9} \times 99$

= 110 minutes

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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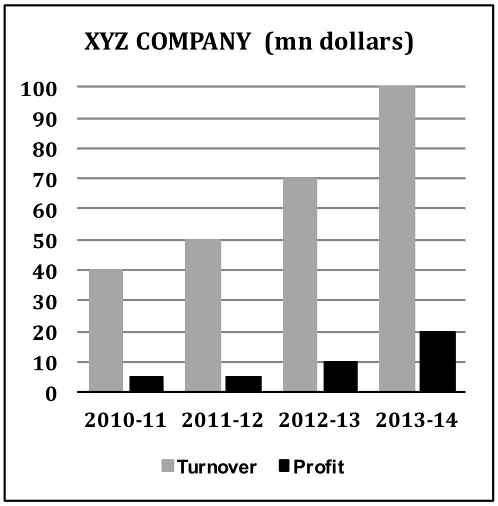
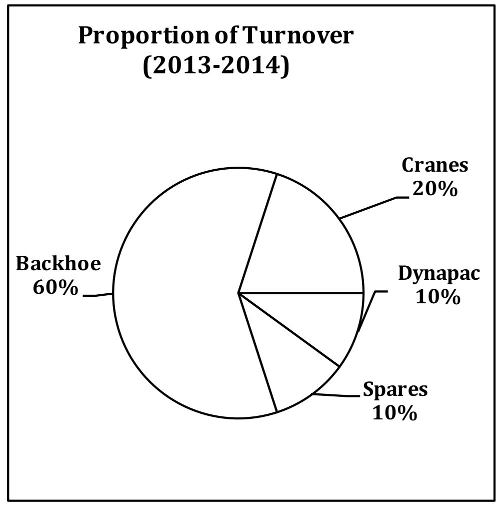
The graphs and the table for the following four questions.





Profile of Customer Segments – 2013-2014					
BACKHOE		CRANES		DYNAPAC	
Mining	10%	Engineering Industries	40%	Contractors	60%
Process Industries	10%	Steel Industries	10%	Government	20%
Plant Hirers	10%	Plant Hirers	10%	Plant Hirers	10%
Government	15%	Granite Quarries	30%	Mining	10%
Steel Industries	55%	Mining	10%		

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406.

Quantity A	**Quantity B**
Contribution to XYZ's turnover in 2013-2014 from the purchases of the three products by the government	\$10 mn

Total turnover in 2013-2014 of XYZ = \$100 mn

Turnover from the sale of Backhoe = 60% of \$100 mn = \$60 mn

Turnover from the sale of Backhoe by purchase from Government

= 15% of \$60 mn

= \$9 mn

Turnover from the sale of Cranes is not to be considered since there was no purchase made by the Government.

Turnover from the sale of Dynapac = 10% of \$100 mn = \$10 mn

Turnover from the sale of Dynapac by purchase from Government

= 20% of \$10 mn

= \$2 mn

Thus, total turnover as a contribution from Government

= \$(9 + 2) mn

= \$11 mn

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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407.

Quantity A	**Quantity B**
The ratio of the profits from Cranes to the sale of Cranes	$\frac{2}{3}$

Total profit in 2013-2014 = \$20 mn

Since profit from the sale of Cranes was twice the combined profit from the sale of Backhoe, Dynapac and spares, we have

Profit from sales of Cranes

$$= \frac{2}{1+2} \times \$20 \text{ mn}$$
$$= \$\left(\frac{40}{3}\right) \text{ mn}$$

Turnover from the sales of Cranes in 2013-14

= 20% of \$100 mn

= \$20 mn

Thus, the ratio of profits from Cranes to the turnover from the sale of Cranes

$$= \frac{\left(\frac{40}{3}\right)}{20}$$
$$= \frac{2}{3}$$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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408.

Quantity A	**Quantity B**
The greatest ratio of XYZ's profit to turnover across the four year periods	$\frac{1}{4}$

We need to calculate the ratio of profit to turnover for each of the four year periods.

Thus, we have

- 2010-2011: Ratio $= \frac{5}{40} = \frac{1}{8}$
- 2011-2012: Ratio $= \frac{5}{50} = \frac{1}{10}$
- 2012-2013: Ratio $= \frac{10}{70} = \frac{1}{7}$
- 2013-2014: Ratio $= \frac{20}{100} = \frac{1}{5}$ (Greatest)

Thus, the greatest ratio

$= \frac{1}{5} < \frac{1}{4}$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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409.

Quantity A	**Quantity B**
$\frac{a}{b}$	$\frac{2}{1}$

We have previously calculated the turnover in 2013-2014 from purchase made by the government to be \$11 mn = a.

To find the contribution to turnover from contractors, we only need to consider only the sale of Dynapac since there is no purchase of Backhoe and Cranes by contractors.

Total turnover in 2013-2014 of XYZ = \$100 mn

Thus, turnover from the sale of Dynapac = 10% of \$100 mn = \$10 mn

Turnover from the sale of Dynapac by purchase from contractors

= 60% of \$10 mn

= \$6 mn = b

Thus, the required ratio

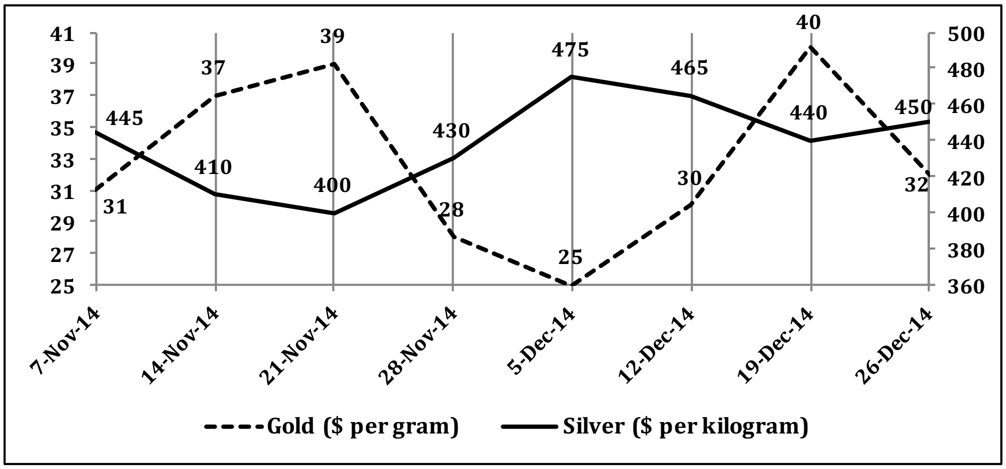
$= \frac{a}{b} = \frac{11}{6} < 2$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

410.

Quantity A	**Quantity B**
Positive difference of the combined price of 40 gram gold and 1.5 kilogram silver on 21st November and their combined price on 12th December	\$300



From the line-graph, we have

	Price of gold per gram	**Price of 40 gram gold**	**Price of silver per kilogram**	**Price of 1.5 kilogram silver**	**Total price**
21st Nov	\$39	\$(39 × 40) = \$1,560	\$400	\$(400 × 1.5) = \$600	\$(1,560 + 600) = **\$2,160**
12th Dec	\$30	\$(30 × 40) = \$1,200	\$465	\$(465 × 1.5) = \$697.50	\$(1,200 + 697.50) = **\$1,897.50**

Thus, the difference in price

= \$(2,160 – 1,897.50) = \$262.50

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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411.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$\frac{A}{B}$	0.10

The highest price of gold per gram = \$40

The lowest price of gold per gram = \$25

Thus, difference between the highest and the lowest price of gold per gram

= \$(40 – 25)

= \$15

Thus, we have

$A = 15$

The highest price of silver per kilogram = \$475

The lowest price of silver per kilogram = \$400

Thus, difference between the highest and the lowest price of silver per kilogram

= \$(475 – 400)

= \$75

Thus, we have

$B = 75$

Thus, we have

$\frac{A}{B} = \frac{15}{75} = \frac{1}{5} = 0.20$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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412.

Quantity A	**Quantity B**
The ratio of the median price of gold per gram and the median price of silver per kilogram on the 8-day period	$\frac{1}{20}$

Arranging, in ascending order, the prices of gold per gram of the 8 days given above, we have

25, 28, 30, 31, 32, 37, 39 and 40

Thus, the median is the average of the 4^{th} and 5^{th} values

$= \frac{31 + 32}{2}$

= 31.5

Arranging, in ascending order, the prices of silver per kilogram of the 8 days given above, we have

400, 410, 430, 440, 445, 450, 465 and 475

Thus, the median is the average of the 4^{th} and 5^{th} values

$= \frac{440 + 445}{2}$

= 442.5

Thus, the required ratio

$= \frac{31.5}{442.5}$

$= \approx 0.071 > \frac{1}{20} = 0.05$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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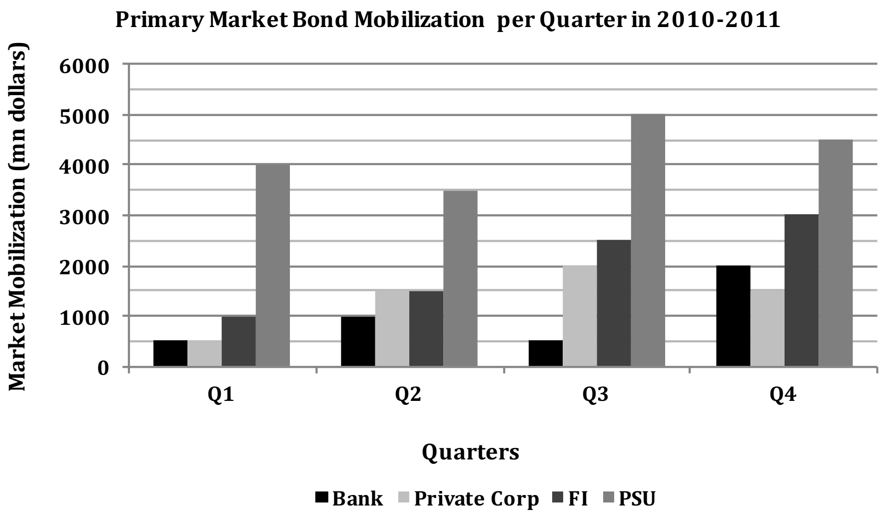
413.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The difference between the total values, in million dollars, of bonds mobilized in primary market by the four sectors in Q1 and in Q2	$1,500 mn



Total primary market mobilization of bonds in Q1 by the four sectors

= $(500 + 500 + 1,000 + 4,000) mn

= $6,000 mn

Total primary market mobilization of bonds in Q2 by the four sectors

= $(1,000 + 1,500 + 1,500 + 3,500) mn

= $7,500 mn

Thus, the difference between the total values of bonds mobilized in primary market by the four sectors in Q1 and in Q2

= $(7,500 – 6,000) mn

= $1,500 mn

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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414.

Quantity A	**Quantity B**
The percent decrease in the value of bonds mobilized in primary market by PSU in Q4 over Q3 in 2010-2011	11.11%

The value of bonds mobilized in primary market by PSU in Q3 = \$5,000 mn

The value of bonds mobilized in primary market by PSU in Q4 = \$4,500 mn

Thus, the required percent decrease

$= \frac{5,000 - 4,500}{5,000} \times 100\%$

= 10%

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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415.

Quantity A	**Quantity B**
The simple quarterly growth rate in the value of bonds mobilized in primary market by FI from Q1 to Q4 in 2010-2011	50%

The value of bonds mobilized in primary market by FI in Q1 = $1,000 mn

The value of bonds mobilized in primary market by FI in Q4 = $3,000 mn

Thus, the percent growth

$$= \frac{3,000 - 1,000}{1,000} \times 100\%$$

= 200%

This growth occurs over a gap of three quarters (Q1 to Q2, Q2 to Q3 and Q3 to Q4).

Thus, the simple quarterly growth rate

$$= \frac{200}{3}\%$$

= 66.66%

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

416.

Quantity A	**Quantity B**
The number of boys	36

Let the number of boys be x.

Thus, we have

Number of girls = $\frac{1}{2}x - 1$.

Total amount distributed = \$600.

Amount received on average by each = \$12.

Thus, total number of boys and girls = $\frac{600}{12} = 50$.

Thus, we have

$x + \left(\frac{1}{2}x - 1\right) = 50$

=> $\frac{3x}{2} = 51$

=> $x = 34$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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417.

Quantity A	**Quantity B**
The ratio in which wheat varieties A and B are mixed	$\frac{1}{2}$

Let the quantities of varieties A and B used are a pounds and b pounds respectively.

Price per pound of variety A = \$18.

Since we know that one variety is priced at \$3 per pound greater than the other, the price of variety B per pound can be:

- \$18 + \$3 = \$21

OR

- \$18 – \$3 = \$15

However, since the average price of the mixture is \$16 per pound and variety A is priced higher than the average price, the price of variety B must be lower than the average price.

Thus, the average price of variety B = \$15 per pound.

Thus, we have

$$\frac{18a + 15b}{a + b} = 16$$

$=> 18a + 15b = 16a + 16b$

$=> 2a = b$

$=> \frac{a}{b} = \frac{1}{2}$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

418.

Quantity A	**Quantity B**
Twice the number of students who joined the class	The number of students initially present in the class

Let the number of students in the class before the students join = x.

Let the number of students who join the class = y.

The initial average age of the class = 20 years.

The average age of the students who join the class is 17 years.

The average age of all the students after the new students join = 19 years.

Thus, we have

$$\frac{(x \times 20 + y \times 17)}{(x + y)} = 19$$

=> $20x + 17y = 19x + 19y$

=> $2y = x$

=> Twice the # of students who joined = The # of students initially present

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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419.

Quantity A	**Quantity B**
The actual price at which the article was sold	\$2,000
Discount offered = 5% of Listed price. Since the reduction in profit equals the reduction in selling price as a result of the discount, we have 5% of Listed price = \$120 => Listed price = $\$\left(120 \times \frac{100}{5}\right) = \$2,400$ Thus, actual selling price $= \$\,(2,400 - 120)$ $= \$2,280$	
Thus, Quantity A is greater than Quantity B. The correct answer is option A.	

420.

Quantity A	**Quantity B**
Measure of the largest interior angle of quadrilateral ABCD	120°

We know that two of the angles are 90° each.

Since the sum of the angles of a quadrilateral is 360°, the sum of the other two angles

$= 360^\circ - (90^\circ + 90^\circ) = 180^\circ$

We know that

$\angle ABC = 2 \times \angle BCD$

Let us consider all possible cases for the relation $\angle ABC = 2\angle BCD$:

- $\angle BCD = 90^\circ \Rightarrow \angle ABC = 2 \times \angle BCD = 180^\circ$ – Not possible in a quadrilateral
- $\angle ABC = 90^\circ \Rightarrow \angle BCD = \frac{90^\circ}{2} = 45^\circ$

Since there is one more 90° angle, the measure of the fourth angle

$= 360^\circ - (90^\circ + 90^\circ + 45^\circ) = 135^\circ$

Thus, the largest interior angle is $135^\circ > 120^\circ$

Thus, Quantity A is greater than Quantity B

- Neither $\angle ABC$ nor $\angle BCD$ is equal to 90°

Thus, we have

$\angle ABC + \angle BCD = 360^\circ - (90^\circ + 90^\circ) = 180^\circ$

Since one angle between $\angle ABC$ and $\angle BCD$ is twice the other, we have

Smaller angle $= \left(\frac{1}{1+2}\right) \times 180^\circ = 60^\circ$

Larger angle $= 180^\circ - 60^\circ = 120^\circ$

Thus, the largest interior angle is 120° Thus, Quantity A is equal to Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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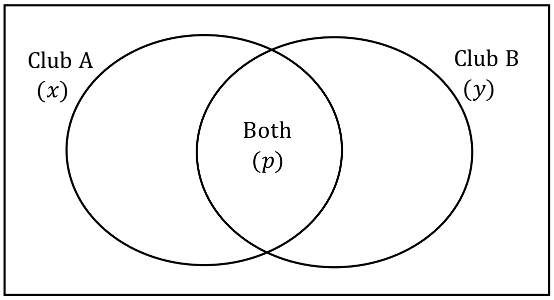
421.

Quantity A	**Quantity B**
Number of patrons of Club A	Number of patrons of Club B

Let the number of patrons in both Club A and Club B be p.



Thus, we have

25% of Club A patrons = p

40% of Club B patrons = p

Thus, we have

25% of Club A patrons = 40% of Club B patrons

$$=> \frac{\text{(Number of Club A patrons)}}{\text{(Number of Club B patrons)}} = \frac{40}{25} = \frac{8}{5} > 1$$

=> Number of Club A patrons > Number of Club B patrons

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

422.

Quantity A	**Quantity B**
Number of seconds required to travel d miles at r yards per second	Number of seconds required to travel d kilometers at r meters per second
We know that 1 mile = 5,280 feet 1 yard equals 3 feet => 1 feet equals $\frac{1}{3}$ yard Thus, distance to be traveled $= d$ miles $= 5,280d$ feet $= \frac{5,280d}{3}$ yards $= 1,760d$ yards Thus, time taken $= \frac{1,760d}{r}$ seconds	We know that 1 kilometer = 1,000 meters Thus, distance to be traveled $= d$ kilometers $= 1,000d$ meters Thus, time taken $= \frac{1,000d}{r}$ seconds
Thus, Quantity A is greater than Quantity B. The correct answer is option A.	

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423.

Quantity A	**Quantity B**
Bob's average marks in the last two tests	80

Let the number of tests taken by Bob before taking the last two tests = x.

Total marks obtained by Bob before taking the last two tests = $85x$.

Total marks obtained by Bob after taking the last two tests = $81(x + 2)$.

Thus, total marks scored by Bob in the last two tests

$= 81\ (x + 2) - 85x$

$= 162 - 4x$

Since x (the number of tests taken) is a positive integer, we have $x \geq 1$

$=> 162 - 4x \leq 158$

Thus, Bob scored at most 158 marks in his last 2 tests.

Thus, Bob's average marks in his last 2 tests is at most $\frac{158}{2} = 79$.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

424.

Quantity A	**Quantity B**
Standard deviation of the scores of Class P's students	Standard deviation of the scores of Class Q's students

Standard deviation (SD) is a measure of deviation of items in a set with respect to their arithmetic mean (average). Closer are the items to the mean value, lesser is the value of SD, and vice versa; this follows that if a set has all equal items, its SD = 0.

We know that the average score of Class Q's students is greater than the average score of Class P's students.

Also, the median score of Class Q's students is greater than the median score of Class P's students.

However, we have no information about the deviation of the actual scores of the students about the mean.

We can take the following examples (assuming 3 students in each of Class P and Class Q):

	Three scores of Class Q	**Three scores of Class P**	**Mean Q**	**Mean P**	**Median Q**	**Median P**	**SD Q**	**SD P**
1.	20, 20, 20	10, 10, 10	20	10	20	10	0	0
	Since there is no deviation of the values about the mean for either Class Q or Class P, the standard deviation of each is '0'. Thus, Quantity A is equal to Quantity B.							
2.	20, 22, 24	10, 11, 12	22	11	22	11	–	–
	Since deviation of the values about the mean is greater for Class Q than for Class P, the standard deviation of Class Q will be greater than that of Class P. Thus, Quantity A is less than Quantity B.							

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	Three scores of Class Q	Three scores of Class P	Mean Q	Mean P	Median Q	Median P	SD Q	SD P
3.	20, 21, 22	10, 12, 14	21	12	21	12	–	–
	Since deviation of the values about the mean is greater for Class P than for Class Q, the standard deviation of Class Q will be less than that of Class P. Thus, Quantity A is greater than Quantity B.							

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

425.

Quantity A	**Quantity B**
The remainder if n is divided by 5	2
Let the quotient when $3n$ is divided by 15 be q. Thus, we have $3n = 15q + 6$, where q is a non-negative integer => $n = 5k + 2$ Thus, the remainder when n is divided by 5 is 2.	
Thus, Quantity A is equal to Quantity B. The correct answer is option C.	
Alternatively, you can assume a convenient value for $3n$. Say, $3n = 15 + 6 = 21$ => $n = \frac{21}{3} = 7$ => $\frac{n}{5} = \frac{7}{5}$ => remainder = 2	

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426.

Quantity A	**Quantity B**
Average age of the group of people initially	20

Let the average of the group of people be x years.

Let the number of people in the group initially be n.

Total age of the people in the group initially = nx.

When 4 people with average age 40 years join, the new total age of the group of $(n+4)$ people

$= nx + 40 \times 4$

$= (nx + 160)$ years

Thus, average age of this group

$= \left(\frac{nx + 160}{n + 4}\right)$ years

This new average is double the initial average. Thus, we have

$\left(\frac{nx + 160}{n + 4}\right) = 2x$

$=> nx + 160 = 2nx + 8x$

$=> nx + 8x = 160$

$=> x\,(8 + n) = 160$

$=> x = \frac{160}{8 + n}$

Since $\frac{160}{8} = 20$, and n is a positive integer, we must have

$\frac{160}{8 + n} < 20$

$=> x < 20$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Alternate approach:

The average age of the 4 people who join is 40 years.

On joining, the average age of the group increases.

Thus, the average age of the group initially must be less than 40.

Since the average age of the group doubles, we can conclude that twice the average age of the initial group is less than 40.

Thus, the average age of the initial group was less than $\frac{40}{2} = 20$.

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427.

Quantity A	**Quantity B**
Present age of B	Double the present age of A

Let the present ages of A and B be a years and b years, respectively.

Four years ago, A's age = $(a - 4)$ years.

Four years ago, B's age = $(b - 4)$ years.

Thus, we have

$b - 4 = 2\,(a - 4)$

$=> b = 2a - 4$

$=> b < 2a$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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428.

Quantity A	**Quantity B**
x	k

We know that

$(2^x)(2^k) = 4$

$=> 2^{x+k} = 2^2$

$=> x + k = 2 \ldots (i)$

We also know that:

$(9^x)(3^k) = 81$

$=> 3^{2x} \times 3^k = 3^4$

$=> 3^{2x+k} = 3^4$

$=> 2x + k = 4 \ldots (ii)$

Thus, from (i) and (ii):

$x = 2, k = 0$

$=> x > k$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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429.

Quantity A	**Quantity B**
x	y

We know that

$x - y^2 > 0$

=> $x > y^2$ & $y^2 > 0$

=> $x > 0$

Also, we know that

$xy < 0$

Since $x > 0$, we have

$y < 0$

Thus, x is positive, while y is negative

=> $x > y$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

430.

Quantity A	**Quantity B**
ab	27

We know that

$9 < a < 15$, and

$5b > 12$

$=> b > \frac{12}{5} = 2.4$

Thus, we may have the following scenarios:

- If $b = 2.5$ and $a = 10$

$=> ab = 25 < 27$

=> Quantity A is less than Quantity B

- If $b = 3$ and $a = 10$

$=> ab = 30 > 27$

=> Quantity A is greater than Quantity B

Note: One must be careful while choosing the value of b. Since it is not mentioned that b is an integer, wrongly choosing the smallest integer value of b would give $b = 3$ and in that case, $ab > 27$. The answer, then, would be wrongly determined as Option A.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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431.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$y - x$	0

We know that

$x = 1 - y$

$=> -x = y - 1$

Adding y to both sides:

$y - x = 2y - 1 \ldots$(i)

We also know that:

$y > 0$

$=> 2y > 0$

$=> 2y - 1 > -1 \ldots$(ii)

Thus, from (i) and (ii):

$y - x > -1$

Thus, there exist possibilities like:

- $y - x = -0.5 < 0$

=> Quantity A is less than Quantity B

- $y - x = 0$

=> Quantity A is equal to Quantity B

- $y - x = 1 > 0$

=> Quantity A is greater than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

Alternate approach:

Let us take a few values

- $y = \frac{1}{2} => x = 1 - y = \frac{1}{2}$

$=> y - x = 0$

- $y = \frac{1}{3} => x = 1 - y = \frac{2}{3}$

$=> y - x = -\frac{1}{3} < 0$

- $y = 2 => x = 1 - y = -1$

$=> y - x = 2 - (-1) = 3 > 0$

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432.

Quantity A	**Quantity B**
Total cost of the TV from Showroom X	Total cost of the TV from Showroom Y

We know that:

From Showroom X, the price is p and the sales tax is $t\%$.

From Showroom Y, the price is P and the sales tax is $T\%$.

Also, we have

$PT > pt$

Let us take a couple of values

- $p = \$100, t = 10\%, P = \$150$ & $T = 12\%$

Here, the condition $PT > pt$ is satisfied.

Total cost from Showroom X = \$ (100 + 10% of 100) = \$110

Total cost from Showroom Y = \$ (150 + 12% of 150) = \$168

=> Quantity A is less than Quantity B.

- $p = \$100, t = 10\%, P = \$80$ & $T = 20\%$

Here, also, the condition $PT > pt$ is satisfied.

Total cost from Showroom X = \$ (100 + 10% of 100) = \$110

Total cost from Showroom Y = \$ (80 + 20% of 80) = \$96

=> Quantity A is greater than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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433.

Quantity A	**Quantity B**
The median price of the three books	\$1.50

Median of the numbers in the set is the middle number in the set once the numbers are arranged in ascending or descending order.

Since there are three things, the middle value is the $\left(\frac{3+1}{2}\right)^{\text{th}}$ value, i.e. the 2^{nd} value.

The average price of all three books = \$1.50

Since one of the books is priced at \$1.50 as well, the average price of the other two books must be the same i.e. \$1.50

Since the average price of the other two books is \$1.50, we have the following situations:

- Both books are priced at \$1.50 each

=> Median price = \$1.50

- One book is priced above \$1.50 and the other is priced below \$1.50

=> Median price = \$1.50

Thus, the median price is \$1.50.

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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434.

Quantity A	**Quantity B**
The amount spent by Joe if he buys 90 toffees in boxes of 5 toffees	Twice the amount spent by Joe if he buys 90 toffees in boxes of 15 toffees
If Joe wants to buy boxes containing 5 toffees: Number of boxes of toffees required $= \frac{90}{5}$ = 18 Cost of each such box = \$4.50 Hence, his total cost $= \$4.50 \times 18$ = \$81.00	If Joe wants to buy boxes containing 15 toffees: Number of boxes of toffees required $= \frac{90}{15}$ = 6 Cost of each such box = \$8.50 Hence, his total cost $= \$8.50 \times 6$ = \$51.00 Thus, twice the amount spent by Joe if he buys 90 toffees in boxes of 15 toffees $= 2 \times \$51.00$ = \$102.00
Thus, Quantity B is greater than Quantity A. The correct answer is option B.	

435.

Quantity A	**Quantity B**
The percent reduction in coffee consumption	20%

Since we need to find a percent value for Quantity A, we can assume a suitable value of the initial quantity of coffee consumed and the initial price of coffee.

Let the initial quantity of coffee consumed be 10 grams and the price per gram of coffee be \$10 per gram.

Thus, initial expenditure on coffee = $\$(10 \times 10) = \$100$

New price of coffee per gram

= \$(10 + 20% of 10)

= \$12

Since the expenditure on coffee remains constant at \$100, the quantity of coffee consumed finally

$= \frac{100}{12}$

= 8.33 grams

Thus, percent reduction in coffee consumption

$= \frac{10 - 8.33}{10} \times 100\%$

= 16.67%

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Note:

- If Amount = Price × Consumption, and price increases by $a\%$, then to maintain the same amount, consumption must be decreased by $b\%$. You will observe that $b\% < a\%$.

- If Amount = Price × Consumption, and price decreases by $a\%$, then to maintain the same amount, consumption must be increased by $b\%$. You will observe that $b\% > a\%$.

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436.

Quantity A	**Quantity B**
Efficiency of doing work of a man	Efficiency of doing work of a woman

Time taken by 5 men to complete the job = 20 hours.

Thus, the number of men required to complete the job in 1 hour = $5 \times 20 = 100 \ldots$ (i)

Time taken by 8 men and 3 women to complete the job = 10 hours.

Thus, in order to complete the job in 1 hour, the number of people required

= 8×10 men and 3×10 women

= 80 men and 30 women

Thus, from (i), we have

80 men and 30 women ≡ 100 men

=> 30 women ≡ 20 men

=> 1 woman ≡ $\frac{20}{30} = \frac{2}{3}$ man

Thus, the efficiency of a woman is $\frac{2}{3}$ of that of a man.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

437.

Quantity A	**Quantity B**
Median of the integers in X	Average (arithmetic mean) of the integers in Y

We know that the sum of the integers in Set X is greater than the sum of the integers in Set Y

Since the sets are evenly spaced, the average of the integers in Set X is greater than the average of the integers in Set Y

If the elements in a set are consecutive even or consecutive odd integers, the median and average (arithmetic mean) have the same value.

Thus, the average of the integers in Set X is same as the median of the integers in Set X

Thus, we have

The median of the integers in X is greater than the average of the integers in Y.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach:

You can assume few values for the sets.

Say X= $\{2, 4, 6\}$ and Y= $\{1, 3, 5\}$, assuring that $2 + 4 + 6 > 1 + 3 + 6$.

We see that median of set X = 4 (Quantity A) and average of set Y = 3 (Quantity B, thus, Quantity A is greater than Quantity B.

Even if you assume even number of terms in each set, you will get the same result.

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438.

Quantity A	**Quantity B**
Number of pencils	Number of erasers

Since Dave purchased at least one of each item, the amount he spent = 12 + 20 = 32 cents

Thus, the amount left

= 108 – 32 = 76

Let the additional number of pencils and erasers be x and y, respectively.

Since the total value of the above items was 76 cents, we have

$12x + 20y = 76$

=> $3x + 5y = 19$

Trying with some integer values, the only possible solution obtained is:

$x = 3$ & $y = 2$

Thus, we have

The number of pencils purchased

$= x + 1 = 4$

The number of erasers purchased

$= y + 1 = 3$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach:

Let the number of pencils and erasers be p and q, respectively.

=> $12p + 20q = 108$

=> $3p + 5q = 27$

=> $p = \frac{27 - 5q}{3} = 9 - \frac{5q}{3}$

Since p is an integer, $\frac{5q}{3}$ must be an integer; thus, $q = 0$, 3 or 6. $q = 0$ is not possible since Dave bought at least one of each item. Also at $q = 6$, $p < 0$, which is also not possible. Thus, $q = 3$ & $p = 4$ => $p > q$.

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439.

Quantity A	**Quantity B**
x	y

Since 10 kilograms of alloy K consists of x kilograms of aluminum and y kilograms of copper, we have

$x + y = 10 \ldots$ (i)

Cost of x kilograms of aluminum = \$$2x$.

Cost of y kilograms of copper = \$$4y$.

Thus, the total cost of 10 kilograms of alloy K, which consists of x kilograms of aluminum and y kilograms of copper = \$ $(2x + 4y)$

Thus, we have

$2x + 4y < 30$

$=> 2(x + y) + 2y < 30$

$=> 2 \times 10 + 2y < 30 \ldots$ from (i)

$=> 2y < 10$

$=> y < 5$

$=> x > 10 - 5 = 5 \ldots$ from (i)

$=> x > y$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

440.

Quantity A	**Quantity B**
Number of shirts	2

Let the number of garments of worth \$50 and \$200 be x and y, respectively.

Total worth of all garments = \$ $(50x + 200y)$

Thus, we have

$50x + 200y = 1,250$

=> $x + 4y = 25$

=> $x = 25 - 4y \ldots$(i)

Since Kevin has fewer than 4 shirts, we have

$x \leq 3$ (since x is an integer)

=> $25 - 4y \leq 3 \ldots$using (i)

=> $4y \geq 22$

=> $y \geq 5.5$

From (i), we have

The maximum value of y (when x is '0') is:

$y = \frac{25}{4} = 6.25$

Thus, we have

$5.5 \leq y \leq 6.25$

Since y is an integer, we have

$y = 6$

=> $x = 25 - 4 \times 6 = 1$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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Alternate approach:

Since number of shirts < 4, possible values of shirts = 1, 2, or 3.

Assume that Kevin has only 1 shirt, then Kevin has $1,250 - 1 \times 50 = 1,200$ worth of jackets or 1,200/200 = 6 jackets. (A possible scenario); it seems that the answer is A; however, we must check other values too (unlike Numeric Entry questions, where even if only one value is feasible, the solution ends, in Quantitative Comparison questions, we must be sure that the solution holds true for all the possible scenarios).

Again, if Kevin has only 2 shirts, then then Kevin has $1,250 - 2 \times 50 = 1,150$ worth of jackets or 1,150/200 = 5.75 jackets. (Not a possible scenario as number of jackets must be an integer.)

Similarly, if Kevin has 3 shirts, then then Kevin has $1,250 - 3 \times 50 = 1,100$ worth of jackets or 1,100/200 = 5.50 coats. (Not a possible scenario as number of coats must be an integer.)

So, number of shirts = 1.

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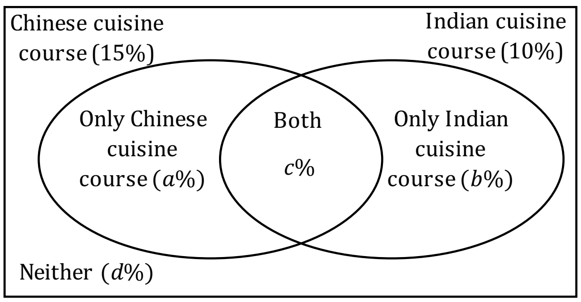
441.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
Percent of the students in the class who joined neither of the two courses	85%

The above information can be represented in the form of a Venn-diagram as shown below:



Since $\frac{2}{3}$ of the students who joined Chinese cuisine course also joined Indian cuisine course, we have

$c = \dfrac{2}{3}$ of 15% of total students

=> c = 10% of total students

=> $b = (10 - 10)\,\% = 0$

Thus, number of students who joined Chinese cuisine course or Indian cuisine course

$= a + b + c$

$= \{(a + c) + b\}$

= {15 + 0}% of the total students

= 15% of the total students

=> d = (100 − 15) = 85% of the total students

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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442.

Quantity A	**Quantity B**
Mary's average speed for the entire trip	56

Distance travelled at p miles per hour for 2 hours = $2p$ miles.

Distance travelled at q miles per hour for $5 - 2 = 3$ hours = $3q$ miles.

Thus, total distance travelled = $(2p + 3q)$ miles.

Total time duration of travel = 2 + 3 = 5 hours.

We know that

$p + \frac{3}{2}q = 140$

$=> 2p + 3q = 280$

Distance = 280 miles

Thus, average speed

$= \frac{280}{5}$

= 56 miles per hour

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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443.

Quantity A	**Quantity B**
The average score on the test for the 100 candidates	55 points

Average score of the 60 candidates = 68.

Thus, total score of the 60 people

$= 60 \times 68$

= 4,080

Average score of the 40 candidates

$= \frac{1}{2} \times 68$

= 34

Thus, total score of the 40 people

$= 40 \times 34$

= 1,360

Thus, total score of the 100 people

= 4,080 + 1,360

= 5,440

Thus, the average score

$= \frac{5,440}{100}$

= 54.4

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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444.

Quantity A	**Quantity B**
Steve's average speed for the entire 50 miles	56 miles per hour

Time taken to drive 30 miles at 60 miles per hour

$= \frac{30}{60}$

$= \frac{1}{2}$ hours . . . (i)

Let her speed for the remaining 50 – 30 = 20 miles be x miles per hour.

Time taken to drive 50 miles at x miles per hour

$= \frac{50}{x}$ hours

Since the time taken would have been 1 hour, we have

$\frac{50}{x} = 1$

=> $x = 50$ miles per hour

Thus, time taken to drive 20 miles at $x = 50$ miles per hour

$= \frac{20}{50}$

$= \frac{2}{5}$ hours . . . (ii)

Thus, total time taken for the trip

$= \frac{1}{2} + \frac{2}{5}$

$= \frac{9}{10}$ hours

Thus, average speed for the entire trip

$= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{50}{\left(\frac{9}{10}\right)}$

$= \frac{500}{9}$

= 55.6 miles per hour

(Note: we should not approximate 55.6 to 56 since the problem does not ask us to approximate to the nearest integer)
Thus, Quantity B is greater than Quantity A. The correct answer is option B.

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445.

Quantity A	**Quantity B**
The number of cans of brand X sold	The number of cans of brand Y sold
The number of cans of brand X in stock = 90.	The number of cans of brand Y in stock = 60. Number of cans of brand Y sold $= \frac{2}{3} \times 60 = 40$
Total number of cans of juice sold $= \frac{1}{2}(90 + 60)$ $= 75$ Thus, the number of cans of brand X sold $= 75 - 40 = 35$	
Thus, Quantity B is greater than Quantity A. The correct answer is option B.	

446.

Quantity A	Quantity B
p	0.8

$C = 65p + 85q \ldots$(i)

$p + q = 1 \ldots$(ii)

Given that $C \geq 73$

$=> 65p + 85q \geq 73$

$=> 65p + 85q \geq 73$

$=> 85p + 85q - 20p \geq 73$

$=> 85\,(p + q) - 20p \geq 73$

$=> 85 - 20p \geq 73$ (since from (ii), we have $p + q = 1$)

$=> 20p \leq 12$

$=> p \leq 0.6 => p < 0.8$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Alternate approach:

For the equation $C = 65p + 85q$, for which $p + q = 1$, C is the weighted mean of p & q. If $p = q = 0.5$, $C = \frac{65+85}{2} = 75$. If $C < 75$, it implies that weight of p is more than that of q => ($p > 0.5$ & $p < 0.5$); similarly, if $C > 75$, it implies that weight of q is more than that of p => ($p < 0.5$ & $q > 0.5$).

Since it is given that $C \geq 73$, we are interested in both: $C < 75$ & $C > 75$. If $C > 75$, $p < 0.5$ => Quantity B > Quantity A.

However, if $75 > C \geq 73$, we have a problem in hand since now $p > 0.5$. But can the value of $p \geq 0.8$?

Let us plug-in $p = 0.8$ in $C = 65p + 85q$ => $C = 65 \times 0.8 + 85 \times 0.2 = 69$. Since at $C = 69 < 73$ => $p < 0.8$. Thus, Quantity B is greater than Quantity A.

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447.

Quantity A	**Quantity B**
Maximum score for class A	Maximum score for class B

For class B, number of distinct integer scores possible from 76 to 100 (since 100 is the maximum possible score)

= (100 – 76) + 1

= 25

Since there are 25 students, and 25 possible scores, each student obtained an integer score from 76 to 100.

Thus, maximum score for class B = 100.

Since no student in class A can obtain a score greater than 100, maximum score for class A will always be less than or equal to the maximum score for class B.

Thus, Quantity A is either 'less than' or 'equal to' Quantity B.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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448.

Quantity A	**Quantity B**
Rate at which Machine C manufactures parts	Rate at which Machine A manufactures parts

Let the constant rate of assembling parts for the three Machines A, B, and C be a, b and c, respectively.

Thus, we have

$r_p = \frac{a}{b}$

$r_q = \frac{b}{c}$

We know that

$r_p < r_q$

$=> \frac{a}{b} < \frac{b}{c} \ldots$(i)

$=> b^2 > ac$

Again, we have

$r_q < 1$

$=> \frac{b}{c} < 1 \ldots$(ii)

$=> c > b$

From (i) and (ii), we have

$\frac{a}{b} < \frac{b}{c} < 1$

$=> \frac{a}{b} < 1$

$=> a < b$

Thus, we have

$c > b > a$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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Alternate approach:

We know that,

$r_p = \frac{a}{b} < 1 => a < b \ldots (i)$

Similarly,

$r_q = \frac{b}{c} < 1 => b < c \ldots (ii)$

From (i) & (ii), we get $a < c$

449.

Quantity A	**Quantity B**
Distance left for Bus A to cover to reach its destination	Distance left for Bus B to cover to reach its destination

Time after which the two buses passed each other = 3 hours.

Average speed of Bus A till the time the buses passed each other = 75 miles per hour.

Thus, distance covered by Bus A = 75×3 = 225 miles.

Thus, distance left till the destination for Bus A

= 420 – 225

= 195 miles

The 195 miles must be the distance covered by Bus B.

Also, the distance left till the destination for Bus B

= Distance covered by Bus A

= 225 miles

Thus, Bus A was nearer to its destination than Bus B.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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450.

Quantity A	**Quantity B**
Annual rent collected by the firm from the property in 2009	Annual rent collected by the firm from the property in 2007

Let the rent collected in 2007 = $\$r$

Thus, the rent collected in 2008

$= (100 + p)\%$ of $\$r$

$= \$\left(\frac{r\,(100+p)}{100}\right)$

$= \$\left(r\left(1+\frac{p}{100}\right)\right)$

Thus, the rent collected in 2009

$= (100 - q)\%$ of $\$\left(r\left(1+\frac{p}{100}\right)\right)$

$= \$\left(\frac{r\,(100+p)}{100} \times \frac{(100-q)}{100}\right)$

$= \$\left(r\left(1+\frac{p}{100}\right)\left(1-\frac{q}{100}\right)\right)$

$= \$\left(r\left(1+\frac{p}{100}-\frac{q}{100}-\frac{pq}{10000}\right)\right)$

$= \$\left\{r+\frac{r}{100}\left(p-q-\frac{pq}{100}\right)\right\}$

We know that

$p - q > \frac{pq}{100}$

$=> p - q - \frac{pq}{100} > 0$

$=> \frac{r}{100}\left(p-q-\frac{pq}{100}\right) > 0$

Thus, we have

$\$\left\{r+\frac{r}{100}\left(p-q-\frac{pq}{100}\right)\right\} > \$r$

Thus, the rent collected in 2009 is greater than the rent collected in 2007.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach:

Let the rent collected in 2007 = \$100

Let the rent collected in 2008 is more than that in 2007 by 10%, thus, $p = 10\%$

=> Rent in 2008 = 100 + 10 = 110.

Since $p - q > \frac{pq}{100}$, by plugging-in the value of $p = 10$, we get

=> $10 - q > \frac{10q}{100}$

=> $p < \frac{100}{11} < 9.09\%$

Since it is given that the rent in 2009 is $q\%$ less than that in 2008, let us assume that $q = 9\%$

Thus, rent in 2009 = Rent in 2008 $\times$ $(1 - 9\%) = 110 \times \left(1 - \frac{9}{100}\right) = 110 \times 0.91 = 100.10 >$ rent in 2007 (100).

Thus, the rent collected in 2009 is greater than the rent collected in 2007.

If you assume much lesser value for q, the value of rent in 2009 would rather increase.

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451.

Quantity A	**Quantity B**
Twice the perimeter of the square	Perimeter of the rectangle

Let the length and width of the rectangle be l and w, respectively.

Let each side of the square be s.

Thus, we have

$lw = s^2 \ldots$(i)

We know that one of the sides of the rectangle is thrice the length of a side of the square.

Taking the length of the rectangle as the longer side, we have

$l = 3s$

Thus, from (i), we have

$w = \frac{s^2}{3s}$

$= \frac{s}{3}$

Twice the perimeter of the square	Perimeter of the rectangle
$= 2 \times 4s = 8s$	$= 2\,(l + w)$ $= 2\left(3s + \frac{s}{3}\right)$ $= \frac{20s}{3}$ $= 6.67s$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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Alternate approach:

Let us assume that the area of the square and the rectangle is 9 units, each.

Thus, side of the square = $\sqrt{9} = 3$

Perimeter of the square = $4 \times 3 = 12$

Say, the length and width of the rectangle be l and w, respectively.

Thus, the area of the rectangle = $l \times w = 9$

We know that $l = 3 \times$ side of the square = $3 \times 3 = 9$

Thus, $9 = 9 \times w \Rightarrow w = 1$

Thus, the perimeter of the rectangle = $2(l + w) = 2(9 + 1) = 20$

Thus, the perimeter of the rectangle (20) < Twice the perimeter of the square ($2 \times 12 = 24$)

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452.

Quantity A	**Quantity B**
The average (arithmetic mean) of a list of 8 numbers as a percent of the sum of the numbers	12.5%

Number of terms in the list = 8

Let the sum of the 8 terms = S

Thus, the average of the 8 terms = $\frac{S}{8}$

Thus, the required percent value

$$= \frac{\left(\frac{S}{8}\right)}{S} \times 100\%$$

$$= \frac{100}{8}\%$$

= 12.5%

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

453.

Quantity A	**Quantity B**
r	10%

The amount accumulated, \$$A$, on a deposit of \$$P$, compounded annually at r% for t years is given by:

$$A = P\left(1 + \frac{r}{100}\right)^t$$

Thus, the amount corresponding to the deposit of \$1,000 for 2 years is given by:

$$A = 1,000\left(1 + \frac{r}{100}\right)^2$$

Thus, we have

$$1,000\left(1 + \frac{r}{100}\right)^2 > 1,210$$

$$=> \left(1 + \frac{r}{100}\right)^2 > \frac{121}{100}$$

Taking square root on both sides:

$$\left(1 + \frac{r}{100}\right) > \frac{11}{10}$$

$$=> \frac{r}{100} > \frac{11}{10} - 1$$

$$=> \frac{r}{100} > \frac{1}{10}$$

$$=> r > 10$$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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Alternate approach:

We can conclude on this question by assuming that $r = 10\%$.

If the amount $\leq$ 1,210, it implies that we need to increase the value of r to a value greater than 10% in order to make the amount greater than 1,210; hence, the answer would be Option A (Quantity A > Quantity B).

However, if the amount > 1,210, it implies that, for a slightly smaller value of r, the amount would still remain slightly greater than 1,210 (as for both the rates, i.e., r% & 10%, the amount is greater than 1,210); hence, the answer would be Option D.

You would notice that in this question, the answer would only be either be A or D. (Your probability of getting the correct answer is 0.5)

Amount @ 10% rate = $1,000\left(1 + \frac{10}{100}\right)^2 = 1,000 \times (1.1)^2 = 1,210$.

Thus, Quantity A is greater than Quantity B.

454.

Quantity A	**Quantity B**
x	1

We know that

$\lceil x \rceil$ denotes the least integer greater than or equal to x

For example:

- $\lceil 2.3 \rceil = 3$
- $\lceil 2 \rceil = 2$
- $\lceil -2.3 \rceil = -2$

We also know that:

$0 < x + \lceil x \rceil < 2$

Let us take a few numbers and check:

- If x is negative or '0', $\lceil x \rceil$ is also either negative or '0'

=> $x + \lceil x \rceil < 0$ – Does not satisfy the given condition

- If $0 < x < 1$

=> $\lceil x \rceil = 1$

=> $1 < x + \lceil x \rceil < 2$ – Satisfies the given condition

- If $x = 1$

=> $\lceil x \rceil = 1$

=> $x + \lceil x \rceil = 2$ – Does not satisfy the given condition

Thus, for even higher values of x, the value of $(x + \lceil x \rceil)$ would be even higher, hence, such values of x are not possible.

Thus, we have

$0 < x < 1$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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455.

Quantity A	**Quantity B**
$f(3)$	11

We know that

$f(x) = 2x + f(x - 1)$

$f(1) = 1$

Thus, we have

- For $x = 2$:

$f(2) = 2 \times 2 + f(2 - 1)$

$=> f(2) = 4 + f(1)$

$=> f(2) = 4 + 1 = 5$

- For $x = 3$:

$f(3) = 2 \times 3 + f(3 - 1)$

$=> f(3) = 6 + f(2)$

$=> f(3) = 6 + 5 = 11$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

456.

Quantity A	**Quantity B**
Unit digit of A	4

Let the units digit of A be u.

Thus, we have

$x = 10 \times 4 + u$

=> $A = 40 + u$

=> $2A = 80 + 2u$

Since the tens digit of $2A$ is 9, it implies that there is a carry from the unit's place to the tens place in the multiplication

=> $2u \geq 10$

=> $u \geq 5$

=> $u > 4$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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457.

Quantity A	**Quantity B**
Total amount to rent a taxi from Taxi X for one day and drive it for m miles	\$22.5

Total amount to rent a taxi from Taxi X

= \$ $(10 + 0.25m)$

Total amount to rent a taxi from Taxi Y

= \$ $(20 + 0.1m)$

The total amount to rent a taxi from Taxi Y for one day and drive it for m miles is less than \$25, we have

$20 + 0.1m < 25$

=> $0.1m < 5$

=> $0.25m < 12.5$; multiplying both the sides by 2.5

=> $10 + 0.25m < 12.5 + 10$; adding 10 to both the sides

=> $10 + 0.25m < 22.5$

Total amount to rent a taxi from Taxi X < 22.5

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

458.

Quantity A	**Quantity B**
Weight of one ball of type P	Weight of one ball of type Q

Let the weight of each ball of type P and type Q be x and y, respectively.

Since the total weight of 6 balls of type P and 5 balls of type Q is less than the total weight of 5 balls of type P and 6 balls of type Q, we have

$6x + 5y < 5x + 6y$

=> $x < y$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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459.

Quantity A	**Quantity B**
The median price of the three cars	$30,000

Price of Suzy's car = $30,000.

Since the average price is $30,000, the price of the other two cars can be:

- Car-1: $30,000, Suzy's car: $30,000, Car-2: $30,000; i.e. all three cars are priced at $30,000 each.

Average price = $\$\left(\frac{30,000 + 30,000 + 30,000}{3}\right)$ = $30,000

Median price = $30,000

- Car-1: $\$\left(30,000 - x\right)$, Suzy's car: $30,000, Car-2: $\$\left(30,000 + x\right)$, where x is any integer less than 30,000; i.e. one car is priced below $30,000 and the other is priced above $30,000.

Average price = $\$\left(\frac{(30,000 - x) + 30,000 + (30,000 + x)}{3}\right)$ = $30,000

Median price = $30,000

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

460.

Quantity A	**Quantity B**
Amount of gas used by truck X in an hour	Amount of gas used by truck Y in an hour
Speed of truck X = 60 miles per hour. Gas consumed by truck X for every 25 miles = 1 gallon. Thus, the amount of gas consumed by truck X for 60 miles $= \frac{1}{25} \times 60$ = 2.4 gallons Thus, the amount of gas consumed by truck X per hour = 2.4 gallons	Speed of truck Y = 40 miles per hour. Gas consumed by truck Y for every 35 miles = 1 gallon. Thus, the amount of gas consumed by truck Y for 40 miles $= \frac{1}{35} \times 45$ = 1.3 gallons Thus, the amount of gas consumed by truck Y per hour = 1.3 gallons
Thus, Quantity A is greater than Quantity B. The correct answer is option A. Note: Since the distance is not mentioned, one might tend to think that the relation cannot be determined, and hence, the answer should be 'Option D'. However, it is clear that it is not the case.	

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461.

Quantity A	**Quantity B**
Smith's yearly remuneration	Twice of Jack's yearly remuneration

Let the remunerations of Smith and Jack be \$$l$ and \$$j$, respectively.

We know that

$l = j + 20,000 \ldots$(i)

We also have:

$l < 40,000$

$=> l = 40,000 - k$, where k is a positive number $\ldots$(ii)

Thus, from (i), we have

$40,000 - k = j + 20,000$

$=> j = 20,000 - k \ldots$(iii)

Thus, from (ii) and (iii), we have

$$\frac{l}{j} = \frac{40,000 - k}{20,000 - k}$$
$$= \frac{2\,(20,000 - k) + k}{20,000 - k}$$
$$= 2 + \left(\frac{k}{20000-k}\right), \text{ where } \left(\frac{k}{20,000-k}\right) \text{ is a positive quantity since } k \text{ is positive}$$

$=> \frac{l}{j} > 2$

$=> l > 2j$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach:

We can assume a couple of extreme values to test the comparison between Quantity A and quantity B.

Say Jack's salary is as low as $1,000, thus Smith's salary = $20,000 + 1,000 = 21,000 < 40,000$.

Thus, $2\times1,000 = 2,000 < 21,000$. (Quantity A is greater than Quantity B).

Say Jack's salary is as high as $19,000, thus Smith's salary = $20,000 + 19,000 = 39,000 < 40,000$.

Thus, $2\times19,000 = 38,000 < 39,000$. (Quantity A is greater than Quantity B).

So, whatever be the salaries of Jack and Smith, Quantity A is greater than Quantity B.

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462.

Quantity A	**Quantity B**
Average (arithmetic mean) weight of all mangoes in the box	100 grams

Average weight of the mangoes

$= \frac{\text{Total weight of the mangoes}}{\text{Number of mangoes}}$

Total weight of the mangoes is greater than 2,020 grams.

Number of mangoes is less than 21.

Let us work with the total weight as 2,021 (least value) and the number of mangoes as 20 (greatest value):

Least average weight of the mangoes

$=\approx \frac{2,021}{20}$

$=\approx$ 101 grams, i.e. greater than 100 grams.

Since the total weight is greater than 2,020 and the number of mangoes is less than 21, the average weight would increase.

Thus, the average weight is greater than 100 grams.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Note: While calculating the average weight, if the number of mangoes are taken as 21 by mistake, then the average would come to $\frac{2,020}{21} = 96.19 < 100$. Since for fewer mangoes, the average would also exceed 100, the answer would erroneously be concluded as 'Option D.'

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463.

Quantity A	**Quantity B**
Greatest common divisor of j and k	2

We know that

$j = k + 1$

Thus, j and k are consecutive integers.

Any two consecutive integers are mutually co-prime, i.e. they have no common factors other than '1', for example: (3, 4), (11, 12), (15, 16), and so on.

Thus, the Greatest common divisor, GCD, of j and k is '1'.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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464.

Quantity A	**Quantity B**
The median number of vehicles per family in the residential society	4

We know that

30% of the families have 5 or more vehicles per family.

40% of the families have 3 or fewer vehicles per family.

Thus, the remaining 100 – (30 + 40) = 30% of the families must have exactly 4 vehicles per family.

The above information is represented in the table below:

Number of vehicles per family	**Number of families**	**Cumulative number of families**	**Median value**
3 of fewer	40%	40%	
4	**30%**	**70%**	**50th** family
5 or more	30%	100%	

Thus, the median number of vehicles correspond to the $\left(\frac{100}{2}\right)^{th}$ = 50th family, i.e. 4 vehicles per family.

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

465.

Quantity A	**Quantity B**
The remainder when x is divided by 8	7

Since x, when divided by 12, leaves remainder 5, we have

$x = 12k + 5$, where k is the quotient when x is divided by 12.

We can see that, if k is an even number, then $12k$ is divisible by 8.

Thus, let us take two cases:

- k is even, i.e. $k = 2m$, where m is a non-negative integer:

=> $x = 12 \times 2m + 5$

=> $x = 24m + 5$

=> $x = 8 \times 3m + 5$

Thus, x, when divided by 8, leaves remainder 5.

- k is odd, i.e. $k = 2m + 1$, where m is a non-negative integer:

=> $x = 12 \times (2m + 1) + 5$

=> $x = 24m + 17$

=> $x = 8\,(3m + 2) + 1$

Thus, x, when divided by 8, leaves remainder 1.

Thus, the remainder is either 1 or 5.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

Note: It might appear that the remainder cannot be uniquely determined, and hence, the answer should be 'Option D'. However, it is not the case.

Thus, it is advised that one should check the values of the answers (multiple answer situations) before deciding upon the answer.

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Alternate approach:

You can assume a couple of values to check what is the remainder.

Say, $x = 5$, thus the remainder, when '5' divided by '8', is '5'.

Now let's say, $x = 12 \times 1 + 5 = 17$, thus the remainder, when '17' divided by '8', is '1'.

Even for higher values of x, for example: 29, 41, 53, 65, ..., the remainder, when x divided by '8', is either '1' or '5'.

466.

Quantity A	**Quantity B**
Number of articles sold by A	Twice the number of articles sold by B

We know that

(Selling price of each article with A) : (Selling price of each article with B) = 4 : 5

Let the number of articles sold by A and B be a and b, respectively.

Thus, ratio of sales revenue of A and B = $4a : 5b$

We also know that:

Sales revenue of B was less than $\frac{5}{14}$ of the total sales revenue of A and B combined

=> $\frac{5b}{4a + 5b} < \frac{5}{14}$

=> $14b < 4a + 5b$

=> $4a > 9b$

=> $\frac{a}{b} > \frac{9}{4}$

=> $\frac{a}{b} > 2$

=> $a > 2b$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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467.

Quantity A	**Quantity B**
Percent increase in volume of the cylinder	40%

Since the problem asks for a percent value, we may take any suitable value of the radius and height of the cylinder.

Let both the radius and the height be 10 each.

In a right solid cylinder having base radius r and height h, the volume is given by $\pi r^2 h$

Thus, the initial volume of the cylinder

$= \pi \times 10^2 \times 10$

$= 1,000\pi$

New value of the radius

$= (100 + 10)\,\%$ of 10

$= 11$

New value of the height

$= (100 + 20)\,\%$ of 10

$= 12$

Thus, the new volume of the cylinder

$= \pi \times 11^2 \times 12$

$= 1,452\pi$

Thus, percent increase in volume

$= \frac{1,452 - 1,000}{1000} \times 100\%$

$= 45.2\%$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

Alternate approach:

We know that to calculate the volume, r is multiplied twice and the height is multiplied once.

Since r increases by 10% and h increases by 20%, the sum of the percent increase values of r (taken twice) and h is 10% + 10% + 20% = 40%.

Thus, the actual percent increase would be greater than 40%.

New volume would be $(1.1)^2 \times 1.2 = 1.452$ times the original volume, thus change in volume = 45.2%.

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468.

Quantity A	**Quantity B**
$\sqrt[6]{6!}$	$\sqrt[4]{4!}$

The values of x and y are modified as shown below in order to compare their values:

Raise both $\sqrt[6]{6!}$ and $\sqrt[4]{4!}$ to the exponent 12 (LCM of 6 and 4 is 12)

$(\sqrt[6]{6!})^{12} = (6!)^2$ and $\left(\sqrt[4]{4!}\right)^{12} = (4!)^3$

Simplify each term

- $(6!)^2 = (6 \times 5 \times 4 \times 3 \times 2 \times 1)^2$

$= (24 \times 30)^2 = 24^2 \times 30 \times 30$

- $(4!)^3 = (4 \times 3 \times 2 \times 1)^3$

$= 24^3 = 24^2 \times 24$

Thus, we have

$(6!)^2 > (4!)^3$

$=> \sqrt[6]{6!} > \sqrt[4]{4!}$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

469.

Quantity A	**Quantity B**
Probability that the number of mangoes in the first bowl will increase	$\frac{1}{8}$

The number of mangoes in the first bowl will increase if an orange from the first bowl is replaced with a mango from the second bowl.

Since mangoes and oranges in the first bowl are in the ratio 3 : 1, let there are $3n$ mangoes and n oranges.

Thus, the probability of selecting an orange from the first bowl

$$= \frac{C_1^n}{C_1^{(3n+n)}} = \left(\frac{n}{4n}\right)$$
$$= \frac{1}{4}$$

The second bowl contains 2 mangoes and 2 oranges.

Thus, the probability of selecting a mango from the second bowl

$$= \frac{C_1^2}{C_1^4} = \frac{2}{4}$$
$$= \frac{1}{2}$$

Thus, probability that the number of mangoes in the first bowl will increase

$$= \frac{1}{4} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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470.

Quantity A	**Quantity B**
Number of apples	22

Total number of fruits = 30.

We also know that:

$$\frac{\text{Number of oranges}}{\text{Total number of fruits}} < \frac{1}{3}$$

$$=> \frac{\text{Number of oranges}}{30} < \frac{1}{3}$$

=> Number of oranges < 10

Since both the number of apples and the number of oranges are prime numbers, we have

Number of oranges = 7 and number of apples = 23

Note: If # of oranges are 5 or 3 or 2, the corresponding # of apples are 25 or 27 or 28, respectively, which are not prime numbers.

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

471.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
Area of the triangle	32

For a given perimeter, the area of a triangle is the maximum if the triangle is equilateral, the most regular triangle.

Thus, if the triangle were equilateral, each side of the triangle = $\frac{24}{3} = 8$

Thus, area of the equilateral triangle

$= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 8^2$

$= 16\sqrt{3} = 27.12$

Thus, the area of the triangle is less than 32 square units.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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472.

Quantity A	**Quantity B**
Product of the digits of the two-digit number	15

Let the number be $10a + b$, where a and b are the tens and units digit, respectively.

Since the number is equal to the sum of 7 times the tens' digit and 6 times the units' digit, we have

$10a + b = 7a + 6b$

$=> 3a = 5b$

$=> \frac{a}{b} = \frac{5}{3}$

Since a and b are single digit numbers, there is only one possible value of a and b:

$a = 5, \ b = 3$

Thus, the number is 53.

Thus, the product of the digits

$= 5 \times 3$

$= 15$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

473.

Quantity A	**Quantity B**
The two-digit number	83

Let the number be $10a + b$, where a and b are the tens and units digit, respectively.

Since the number is equal to the sum of the digits and thrice the product of the digits, we have

$10a + b = a + b + 3ab$

=> $9a = 3ab$

=> $b = 3$

However, the value of a cannot be determined, so a can take any value from 1 to 9.

Thus, the possible values of the two-digit number ab are 13, 23, 33, 43, 53, 63, 73, 83 and 93.

Thus, Quantity A can be 'less than,' 'equal to' or 'greater than' Quantity B.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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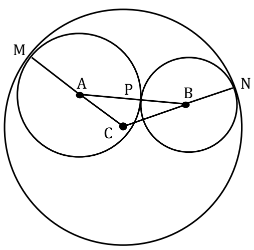
474.

Quantity A	**Quantity B**
Perimeter of triangle ABC	20

Let the radius of the circles with centers A and B be x and y, respectively.



Let us extend the lines CA and CB to intersect the point where the circles touch at M and N, respectively.

Thus, we have

AC = MC – MA = $10 - x$

BC = CN – BN = $10 - y$

AB = AP + BP = $x + y$

Thus, perimeter of triangle ABC

= AC + BC + AB

= $(10 - x) + (10 - y) + (x + y)$

= 20

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

475.

Quantity A	**Quantity B**
k	-3

We know that

$(k-1)^2 - 5 < 4$

$=> (k-1)^2 < 9$

$=> -3 < (k-1) < 3$

$=> -3 + 1 < k < 3 + 1$

$=> -2 < k < 4$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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476.

Quantity A	**Quantity B**
Greatest salary among the five friends	Salary of E

Let the salaries of A, B, C, D and E be a, b, c, d and e.

Since A's salary is greater than C's salary by the same amount as C's salary is greater than B's salary, we have

$a - c = c - b$

$=> 2c = a + b$

$=> c = \frac{a + b}{2}$

$=> a > c > b \ldots$(i)

Since E's salary is more than the individual salaries of A and D, we have

$e > a \ldots$(ii)

$e > d \ldots$(iii)

From (i) and (ii), we have

$e > a > c > b$

Also, from (iii), we have $e > d$

Thus, E has the greatest salary among A, B, C, D and E.

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

477.

Quantity A	**Quantity B**
The number of garments sold on Thursday	4

We know that the number of garments sold on any day, starting Tuesday, is greater than the number of garments sold on the previous day.

Starting with the least number of garments for Monday, i.e. 1, we have

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total
1	2	3	4	5	6	7	28

However, the minimum number of garments sold is only 28 (< 30). Since the total number of garments sold in 30, a couple of possible ways of achieving the result are

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total
1	2	3	4	5	6	7 + 2 = 9	30
1	2	3	4	5	6 + 1 = 7	7 + 1 = 8	30

In either situation, the number of garments sold on Thursday is 4.

If we try to make the number of garments sold on Thursday greater than 4, we have

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total
1	2	3	5	6	7	8	32 ≠ 30

Thus, the number of garments sold on Thursday does not exceed 4.

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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478.

Quantity A	**Quantity B**
B	4

We have

$AB^2 = DCB$

Thus, the units digit of AB and that of DCB are the same.

This is possible only if:

- $B = 0$, for example: $(AB)^2 = 1\mathbf{0}^2 = 10\mathbf{0}$ – Quantity A is less than Quantity B
- $B = 1$, for example: $(AB)^2 = 1\mathbf{1}^2 = 12\mathbf{1}$ – Quantity A is less than Quantity B
- $B = 5$, for example: $(AB)^2 = 1\mathbf{5}^2 = 22\mathbf{5}$ – Quantity A is greater than Quantity B
- $B = 6$, for example: $(AB)^2 = 1\mathbf{6}^2 = 25\mathbf{6}$ – Quantity A is greater than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

479.

Quantity A	**Quantity B**
$\|x\|$	2

We have

$|x-1| = 3\,|x-3|$

$\Rightarrow x - 1 = \pm 3\,(x-3)$

$\boldsymbol{x - 1 = 3(x - 3)}$	$\boldsymbol{x - 1 = -3(x - 3)}$
$\Rightarrow x - 1 = 3x - 9$	$\Rightarrow x - 1 = -3x + 9$
$\Rightarrow x = 4$	$\Rightarrow x = \frac{5}{2}$
$\Rightarrow \|x\| > 2$	$\Rightarrow \|x\| > 2$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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480.

Quantity A	**Quantity B**
a^2	b^2

We have

$a < -b$

$a < 0$

A few possible situations are:

- $a = -1, -b = 3$: $a^2 = 1, b^2 = 9$ – Quantity A is less then Quantity B
- $a = -3, -b = -2$: $a^2 = 9, b^2 = 4$ – Quantity A is greater than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

481.

Quantity A	**Quantity B**
x	6^{12}

Quantity B: $6^{12} = (2 \times 3)^{12}$

$= 2^{12} \times 3^{12} \ldots$(i)

We have

$x > 3^{20}$

$=> x > 3^8 \times 3^{12} \ldots$Separated 3^{12} in order to express in the form shown in (i)

$=> x > (3^2)^4 \times 3^{12}$

$=> x > 9^4 \times 3^{12}$

$=> x > 8^4 \times 3^{12} \ldots$Replaced 9 with 8 since $9 > 8 => 9^{12} > 8^{12}$

$=> x > (2^3)^4 \times 3^{12}$

$=> x > 2^{12} \times 3^{12}$

$=> x > 6^{12}$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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482.

Quantity A	**Quantity B**
Price of a ticket in multiplex A after price revision	Price of a ticket in multiplex B after price revision
Let the initial price of a ticket in multiplex B be $\$100x$ Thus, the initial price of a ticket in multiplex A $= (100 + 20)\%$ of $\$100x$ $= \$120x$	
Final price of a ticket in multiplex A $= ((100 - 20)\,\%$ of $\$120x) + \$5$ $= \$\,(96x + 5)$	Final price of a ticket in multiplex B $= ((100 + 10)\,\%$ of $\$100x) + \$10$ $= \$\,(110x + 10)$
Thus, it is obvious that the price of a ticket in multiplex B is greater than that in multiplex A.	
Thus, Quantity B is greater than Quantity A. The correct answer is option B.	

483.

Quantity A	**Quantity B**
Final price of the stock after the increase	Initial price of the stock before the decrease

Let the initial price of the stock be \$100.

Thus, price of the stock after 20% decrease

= (100 – 20)% of \$100

= \$80

Price of the stock after x% increase

$= (100 + x)\,\%$ of \$80

$= \$\left(\frac{80\,(100 + x)}{100}\right)$

$= \$\left(\frac{4\,(100 + x)}{5}\right)$

Let us take $x = 25$:

Price of the stock after the increase

$= \$\left(\frac{4\,(100 + 25)}{5}\right)$

= \$100

Thus, if $x = 25$, then the final and initial prices are the same.

Since $x < 25$, the final price after x% increase is less than the initial price.

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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484.

Quantity A	**Quantity B**
The amount B received	\$5,000

Let the amounts received by A, B and C be \$$a$, \$$b$ and \$$c$, respectively.

We know that

$a + b + c = 15,000$ ...(i)

Since A received more than B, the same amount, as C received less than B, we have

$a - b = b - c$

=> $a + c = 2b$

Adding b to both sides:

$a + b + c = 3b$

=> $3b = 15,000$ ...From (i)

=> $b = 5,000$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

485.

Quantity A	**Quantity B**
The number of additional people required to complete the work 3 days before time	18

Number of days in which the work is to be completed = 18.

One man alone can complete the work in 270 days.

Thus, number of men required to complete the work in 18 days

$= \frac{270}{18}$

= 15

Number of men required to complete the work in 15 days

$= \frac{270}{15}$

= 18

Thus, the number of additional men required

= 18 – 15

= 3

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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486.

Quantity A	**Quantity B**
xyz	0

We know that

$xy < 0$

=> $x > 0$ and $y < 0$

OR

$x < 0$ and $y > 0$

We also know that:

$xz < 0$

=> $x > 0$ and $z < 0$

OR

$x < 0$ and $z > 0$

Thus, we have the following situations:

$x > 0, y < 0,$ & $z < 0$ => $xyz > 0$ – Quantity A is greater than Quantity B

OR

$x < 0, y > 0,$ & $z > 0$ => $xyz < 0$ – Quantity A is less than Quantity B

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

487.

Quantity A	**Quantity B**
m	n

Given that $a(m-n)$ is non-negative as well as non-positive, we have $a(m-n) = 0$

Thus, we may have the following possibilities:

$m = n$ – Quantity A is equal to Quantity B

OR

$a = 0$; in this case m may be equal, less than or greater than n.

Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

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488.

Quantity A	**Quantity B**
x	2

We know that

$(|x| - 1)(|x| + 2) = 0$

=> $|x| - 1 = 0$ => $|x| = 1$

OR

$|x| + 2 = 0$ => $|x| = -2$

However, the absolute value of a number cannot be negative

=> $|x|$ cannot equal -2

Thus, we have

$|x| = 1$

=> $x = 1$ OR -1

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

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489.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$\sqrt{(a-b)^2}$	1

Since a and b are prime numbers and $a \times b = 6$, the possibilities are:

- $a = 2$ & $b = 3 \Rightarrow \sqrt{(a-b)^2} = \sqrt{(2-3)^2} = 1$
- $a = 3$ & $b = 2 \Rightarrow \sqrt{(a-b)^2} = \sqrt{(3-2)^2} = 1$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

Note: It would be wrong to say that $\sqrt{(a-b)^2}$ is simply $(a-b)$, since the radical (square root) sign always returns the positive root, whereas the value of $(a-b)$ can also be negative.

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490.

Quantity A	**Quantity B**
xy	1

We know that:

$2^{x+2y} = 8$

=> $2^{x+2y} = 2^3$

=> $x + 2y = 3$

Since x and y are positive integers, the only possible values of x and y are:

$x = 1$ & $y = 1$

=> $xy = 1$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

491.

Quantity A	**Quantity B**
k	0

We know that

$f(x) = x^2 + 2^x$

Since $f(k) = \frac{3}{2}$, we have

$k^2 + 2^k = \frac{3}{2}$

Let us work with a few values of k

- $k = 0$: $f(0) = 0^2 + 2^0 = 1 < \frac{3}{2}$
- $k = 1$: $f(1) = 1^2 + 2^1 = 3 > \frac{3}{2}$
- $k = -1$: $f(-1) = (-1)^2 + 2^{-1} = 1 + \frac{1}{2} = \frac{3}{2}$
- $k = -2$: $f(-2) = (-2)^2 + 2^{-2} = \frac{17}{4} > \frac{3}{2}$

Thus, we see that:

- For non-negative values of k:
  - For $k = 0$: LHS < RHS
  - For $k = 1$: LHS > RHS

=> There must be a fractional value of k between '0' and '1' satisfying the equation

- For non-positive values of k:
  - For $k = 0$: LHS < RHS
  - For $k = -1$: LHS = RHS
  - For $k = -2$: LHS > RHS

=> The only possible value of $k = -1$

Thus, there are two possible values of k:

- k is between '0' and '1'. For your curiosity, it is $k = 0.414$. It is not necessary for you to calculate this value. – Quantity A is greater than Quantity B

- $k = -1$ – Quantity A is less than Quantity B

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Thus, the relation between Quantity A and Quantity B cannot be determined.

The correct answer is option D.

492.

Quantity A	**Quantity B**
k	5

We know that:

$f(x) = 2x + k$

Since $f(3) = 2k + 1$, we have

$2 \times 3 + k = 2k + 1$

$\Rightarrow k = 5$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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493.

Quantity A	**Quantity B**
$a + b$	5

We know that:

$f(x) = x^2 + ax + b$

Since the two points where the graph of the quadratic function f intersects the X-axis are $x = 2$ and $x = 8$, we have

The roots of the quadratic function are 2 and 8

=> $f(x) = (x - 2)(x - 8)$

=> $x^2 + ax + b = x^2 - 10x + 16$

Since the coefficient of x^2 is the same on both sides, we can compare the coefficients of the other terms.

Thus, comparing the coefficients on either side, we have

- Comparing coefficients of x: $a = -10$
- Comparing the constant term: $b = 16$

=> $a + b$

$= (-10) + 16$

$= 6$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

494.

Quantity A	**Quantity B**
a	54

We know that In the sequence $a, b, x, y, \ldots$, each term of the sequence above is 9 more than $\frac{1}{3}$ the previous term.

Thus, we have

$b = 9 + \frac{a}{3} \ldots$(i)

$x = 9 + \frac{b}{3} \ldots$(ii)

$y = 9 + \frac{x}{3} \ldots$(iii)

We also know that:

$y = \frac{5}{6}x \ldots$(iv)

Thus, from (iii) and (iv), we have

$\frac{5}{6}x = 9 + \frac{x}{3}$

$=> \frac{x}{2} = 9$

$=> x = 18$

Thus, from (ii), we have

$18 = 9 + \frac{b}{3}$

$=> b = 27$

Thus, from (i), we have

$27 = 9 + \frac{a}{3}$

$=> a = 54$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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Alternate approach:

Let us assume that $a = 54$ and derive the value of y. If $y = \frac{5}{6}x$, the answer would be option C.

If $y > \frac{5}{6}x$, then we should have $a < 54$, thus the answer would be option B.

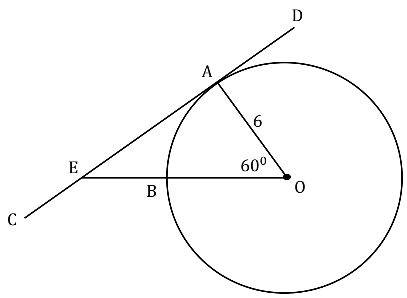
If $y < \frac{5}{6}x$, then we should have $a > 54$, thus the answer would be option A.

Given the relationship in the questions, there is no chance that this question have option D as an answer, we can surely compare Quantity A and Quantity B.

495.

Quantity A	**Quantity B**
Length of EO	$6\sqrt{3}$

Lets us bring out the figure.



Since CD is a tangent to the circle at A, $\angle OAE = 90^\circ$.

In right-angled triangle OAE, we have

$\angle AOE = 60^\circ$

Since $\angle OAE = 90^\circ$, we have

$\angle AEO = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

Thus, triangle OAE is a 60-90-30 triangle.

In a 60-90-30 triangle, the length of the sides opposite to 60°, 90° and 30° are in the ratio $\sqrt{3}\ : 2\ : 1$

Thus, we have

$AE : EO : OA = \sqrt{3}\ : 2\ : 1$

Since OA = 6, we have

$EO = 6 \times 2 = 12 > 6\sqrt{3}\ (= 10.38)$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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496.

Quantity A	**Quantity B**
Volume of the mixture that was removed	45 ml

Volume of the original mixture = 60 ml.

Volume of juice present

= 40% of 60 ml

= 24 ml

Let x ml of the mixture be removed.

Percent of juice present in x ml is also 40%

Thus, volume of juice removed

= 40% of x ml

$= \frac{2}{5}x$ ml

Thus, volume of juice remaining

$= \left(24 - \frac{2}{5}x\right)$ ml ...(i)

Total volume of the mixture left = $(60 - x)$ ml

Volume of water added = x ml

Thus, total volume of the final mixture

$= (60 - x) + x = 60$ ml

Final volume of juice

= 10% of 60 = 6 ml

Thus, from (i), we have

$24 - \frac{2}{5}x = 6 \implies x = 45$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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497.

Quantity A	**Quantity B**
The total number of cakes and biscuit packets purchased by Joe	3

Let the number of cakes and biscuit packets purchased be x and y, respectively.

$13x + 7y = 33$

The number of items purchased must be positive integers.

Let us try with a few positive integer values of x:

x	y
1	$13 + 7y = 33 \Rightarrow y = \frac{20}{7}$, which is not an integer – Not a valid solution
2	$26 + 7y = 33 \Rightarrow y = 1$, which is an integer – **A valid solution**
3	$39 + 7y = 33 \Rightarrow y$ is negative – Not a valid solution (Thus, for all other higher values of x, the value of y would be negative, and hence can be ignored)

Thus, the only solution is:

$x = 2$ & $y = 1$

$\Rightarrow x + y = 3$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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498.

Quantity A	**Quantity B**
$\frac{a}{b}$	2

We know that

$(a-2)^2 + |b-2| = 0$

We can observe that $(a-2)^2$ is a 'perfect square' term

$=> (a-2)^2 \geq 0$

Again, $|b-2|$ is an 'absolute value' term

$=> |b-2| \geq 0$

Thus, we observe that two 'non-negative' terms add up to '0', which is only possible if both terms are individually '0'.

Thus, we have

$(a-2)^2 = 0$

$=> a - 2 = 0$

$=> a = 2 \ldots$(i)

Also, we have

$|b-2| = 0$

$=> b - 2 = 0$

$=> b = 2 \ldots$(ii)

Thus, from (i) and (ii), we have

$\frac{a}{b} = \frac{2}{2} = 1$

Thus, Quantity B is greater than Quantity A.

The correct answer is option B.

499.

Quantity A	**Quantity B**
$\frac{1}{a} + \frac{1}{b}$	6

We know that

$$\frac{ab}{a+b} = \frac{1}{6}$$

Taking reciprocal on both sides:

$$=> \frac{a+b}{ab} = 6$$

$$=> \frac{a}{ab} + \frac{b}{ab} = 6$$

$$=> \frac{1}{b} + \frac{1}{a} = 6$$

Thus, Quantity A is equal to Quantity B.

The correct answer is option C.

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500.

Quantity A	**Quantity B**
$f(p)$	0

We know that

$f(x) = (a - x^n)^{\left(\frac{1}{n}\right)}$

$=> f(1) = (a - 1^n)^{\left(\frac{1}{n}\right)}$

Since $f(1) = p$, we have

$p = (a - 1)^{\left(\frac{1}{n}\right)}$

$=> f(p)$

$= \left[a - \left\{(a-1)^{\left(\frac{1}{n}\right)}\right\}^n\right]^{\left(\frac{1}{n}\right)}$

$= \left[a - (a-1)^{\left(\frac{n}{n}\right)}\right]^{\left(\frac{1}{n}\right)}$

$= [a - (a-1)]^{\left(\frac{1}{n}\right)}$

$= 1^{\left(\frac{1}{n}\right)}$

$= 1$

Thus, Quantity A is greater than Quantity B.

The correct answer is option A.

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Chapter 5

Talk to Us

Have a Question?

Email your questions to info@manhattanreview.com. We will be happy to answer you. Your questions can be related to a concept, an application of a concept, an explanation of a question, a suggestion for an alternate approach, or anything else you wish to ask regarding the GRE.

Please do mention the page number when quoting from the book.

Best of luck!

Professor Dr. Joern Meissner
& The Manhattan Review Team

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About Manhattan Review

Manhattan Review's origin can be traced directly back to an Ivy League MBA classroom in 1999. While teaching advanced quantitative subjects to MBAs at Columbia Business School in New York City, Professor Dr. Joern Meissner developed a reputation for explaining complicated concepts in an understandable way. Prof. Meissner's students challenged him to assist their friends, who were frustrated with conventional test preparation options. In response, Prof. Meissner created original lectures that focused on presenting standardized test content in a simplified and intelligible manner, a method vastly different from the voluminous memorization and so-called tricks commonly offered by others. The new methodology immediately proved highly popular with students, inspiring the birth of Manhattan Review.

Since its founding, Manhattan Review has grown into a multi-national educational services firm, focusing on preparation for the major undergraduate and graduate admissions tests, college admissions consulting, and application advisory services, with thousands of highly satisfied students all over the world. Our GRE material is continuously expanded and updated by the Manhattan Review team, an enthusiastic group of master GRE professionals and senior academics. Our team ensures that Manhattan Review offers the most time-efficient and cost-effective preparation available for the GRE. Please visit www.ManhattanReview.com for further details.

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Professor Dr. Joern Meissner has more than 25 years of teaching experience at the graduate and undergraduate levels. He is the founder of Manhattan Review, a worldwide leader in test prep services, and he created the original lectures for its first test preparation classes. Prof. Meissner is a graduate of Columbia Business School in New York City, where he received a PhD in Management Science. He has since served on the faculties of prestigious business schools in the United Kingdom and Germany. He is a recognized authority in the areas of supply chain management, logistics, and pricing strategy. Prof. Meissner thoroughly enjoys his research, but he believes that grasping an idea is only half of the fun. Conveying knowledge to others is even more fulfilling. This philosophy was crucial to the establishment of Manhattan Review, and remains its most cherished principle.

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